Chapter 5

DC Laws, E-I-R, Sources, Introduction to AC

Basic physical laws of electron current are covered here along with the relationship between potential E, current I, and resistance R. Analytical models of electron current sources are described along with elementary applications to later circuit modeling. A comparison of DC and AC is discussed as a lead-in to the next chapter.

General

Those who are familiar with basic DC and AC can skip this part, or at least review *sources* at the middle of the chapter. For the rest it is best to begin at the beginning.

Electric current always flowing in ***one*** direction is called *Direct Current* or ***DC*** in common usage. Electric current alternating direction of flow in a periodic cycle is called *Alternating Current* or ***AC*** in common terms. Current flow involves movement of *electrons*, always through a *conductor* from a circuit's *negative potential* (surplus of electrons) to a *positive potential* (lack of sufficient electrons). An often-used analogue is a water pipe (a conductor) with flowing water (electron current flow) having a water pressure (potential charge or voltage). The size of the water pipe can be related to resistance (of the conductor) in how much water can flow. This is only an analogue to what happens so we can dismiss the water example and concentrate on DC and AC relationships.

Ohm's Law of Resistance[1]

$$E = I \cdot R \qquad I = \frac{E}{R} \qquad R = \frac{E}{I} \qquad (5\text{-}1)$$

where:

E is potential in Volts   I is current in Amperes   R is resistance in Ohms

This law of physics is so basic that it should be engraved on one's synapses in indelible mnemonics. Scientifically the definitions are: A current of 1.0 Ampere flowing through a resistance of 1.0 Ohms will produce a *voltage drop* of 1.0 Volts.

Power

Current flow having a potential is capable of performing work. Normally, such work is

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[1] Colloquially the term *Ohm's Law* is a more common title.

defined as expenditure of energy over a specified time. In electronics power is more often used as a descriptor for the *heat* dissipated in a resistance element or in the capability of some source as heating up some resistive load.[2]

$$P = E \cdot I \qquad E = \frac{P}{I} \qquad I = \frac{P}{E} \qquad (5\text{-}2)$$

where:

P is the energy / work in Watts   E is potential in Volts   I is current in Amperes

By substitution with parts of (5-1) there result in some very useful identities:

$$P = I^2 R \qquad I = \sqrt{\frac{P}{R}} \qquad P = \frac{E^2}{R} \qquad E = \sqrt{P \cdot R} \qquad (5\text{-}3)$$

These are all very basic in design and should be memorized. One needs only to memorize the left-hand equations of (5-1) and (5-2); the remainder including (5-3) can be worked out by simple algebra.

Ideal Sources

For the purpose of analyzing a circuit response, all active devices can be *modeled* as *sources.* The two basic sources are shown in Figure 5-1. Figure 5-1 **A** illustrates a constant-voltage source, one that will maintain a specific output voltage regardless of the current through a load resistor. That current could range from below femtoAmperes to above TeraAmperes but the ideal voltage source would have exactly the same voltage potential. A battery can be said to be a limited-range constant voltage source.

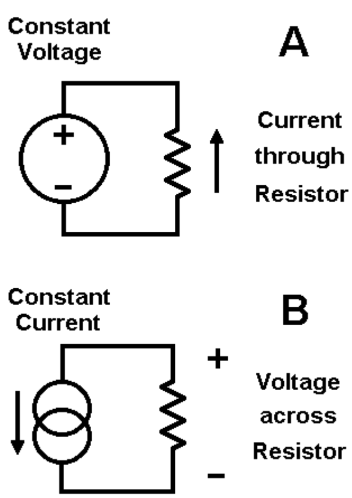

Figure 5-1 The two basic sources. Note symbolism.

Figure 5-1 **B** shows a constant-current source, one that produces exactly the same amount of current regardless of the load resistance. Obviously the output voltage can change if the load resistance is changed; the range over which the voltage can change is termed *compliance.* Nearly all active device outputs (vacuum tubes, transistors, FETs) are modeled basically as constant-current sources.

Dependent Sources

This is a special case for analytical purposes where a source's output is controlled by some external, unconnected condition. For an active device such as a triode vacuum tube, the cathode-to-plate current would be controlled by the voltage between cathode and grid. For a simple bipolar transistor, the

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[2] The author has seen Watts described as *Joule's Law* two other places, 1 Watt per second equal to 1 Joule (an SI unit of work). Technically, a Watt is the rate of work but the widely used equivalent in electronics is the heating of a resistance.

emitter-to-collector current would be controlled by the emitter-to-base current. Both of those special models have been termed *Dependent Current Sources* or *DCS.* More on those in later chapters.

Kirchoff's Voltage Law or KVL

The sum of all voltages about a closed loop in a circuit is equal to zero.

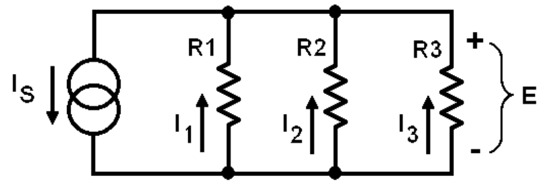

Figure 5-3 Simple circuit to show Kirchoff's Current Law.

It is common for most to be confused by that statement. It may be clarified by Figure 5-2 showing a constant voltage source in series with two resistors. There is only one current path and it is the same through both resistors. Therefore the addition of both voltage drops should be equal to the voltage of the constant-voltage source. Since the polarity of the sum of resistor voltage drops is opposite to the polarity of the constant-voltage source, the sum of **all** voltages is zero.[3]

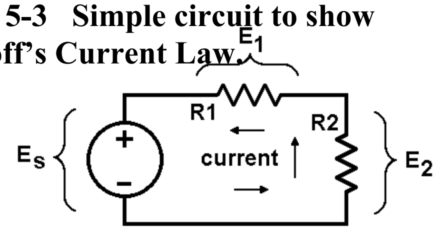

Figure 5-2 Circuit to illustrate Kirchoff's Voltage Law.

One might think of KVL as demonstrating a part of the Law of Conservation of Energy. Nothing is gained in the way of voltage, current doesn't flow anywhere else. The amount of current is governed by the total resistance of the resistors. KVL can be demonstrated mathematically relative to Figure 5-2:

$$E_S = E_1 + E_2 \qquad \text{therefore} \qquad E_S - E_1 - E_2 = 0$$

The second equation is the same expression as KVL.

An interesting sidelight of Figure 5-2 is that it can also demonstrate a useful arrangement of resistors commonly called a *voltage divider*. A voltage divider can provide a any fixed voltage that is always less than the source voltage. That voltage is proportional to the ratio of the two resistances.

$$E_S = E_1 + E_2 = I\left(R_1 + R_2\right) \qquad \text{and} \qquad E_2 = I \cdot R_2 \qquad \text{so:}$$

$$\frac{E_2}{E_S} = \frac{I\,R_2}{I\left(R_1 + R_2\right)} = \frac{R_2}{R_1 + R_2}$$

Kirchoff's Current Law or KCL

The sum of currents at any node in a circuit must equal zero.

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[3] The current through the constant-voltage source has to be in the direction shown by the arrows. ***Within*** the constant-voltage source current flows from positive to negative nodes; it must in order for current to flow at all through the circuit loop. Note: It is common to confuse current direction within a source to current direction in the circuit

It might be said that both KVL and KCL are fancier ways to say *what goes in must come out.* Figure 5-3 illustrates KCL in simple form. Note the word *node.* Node refers to a common connection point. Figure 5-3 has two nodes, the top-most junction of the three resistors and the constant current source, and the bottom-most junction. Voltage drop **E** is dependent on the total paralleled resistance and the amount of current available from the constant current source.

As with KVL, KCL can be demonstrated mathematically in reference to Figure 5-3:

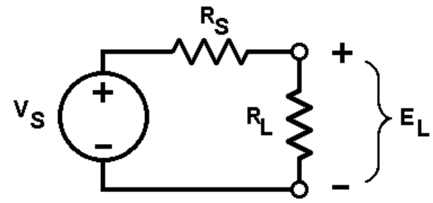

Figure 5-4 Using a constant voltage source and simulating a source impedance.

$$I_S = I_1 + I_2 + I_3 \qquad \text{so:} \qquad I_S - I_1 - I_2 - I_3 = 0$$

Note the direction of current flow within the source. It is opposite to the direction of flow in the resistors. The amount of current through each resistor is dependent on the voltage drop **E** across it. That can't be found unless the total parallel resistance is known. An easier way to find that total resistance is by using ***conductances*** rather than resistances.

Conductance as Inverse of Resistance

$$G = \frac{1}{R} \qquad \text{where G is in values of mhos, R is in values of Ohms} \qquad (5\text{-}4)$$

Conductances in parallel will add. For Figure 5-3 the identities and equation would be:

$$G_1 = \frac{1}{R_1} \qquad G_2 = \frac{1}{R_2} \qquad G_3 = \frac{1}{R_3} \qquad G_{PARALLEL} = G_1 + G_2 + G_3$$

$$R_{PARALLEL} = \frac{1}{G_{PARALLEL}} = \frac{1}{G_1 + G_2 + G_3} \qquad E = I_{SOURCE} \cdot R_{PARALLEL}$$

With any scientific pocket calculator this is any easy task: Invert each resistance to mhos[4], accumulate it; after all accumulation additions are done, invert the sum to obtain resistance in Ohms.

Conversion of Source Types to Near-Opposite Source Types

Testing of RF networks and circuits sometimes requires a specific resistive *source impedance* (nearly always purely resistive). That can be done with an ideal voltage source as shown in Figure 5-4. An ideal constant voltage source has *zero source impedance*; i.e., there is no change in voltage

[4] Yes, *mhos* is Ohms spelled backward. Conductance is the inverse and that could be a reason for the *inverted* spelling. The International Scientific name for conductance is *Siemens* but that has yet to take hold in U.S. literature. *RHdb* follows the U.S. literature style in retaining mhos.

regardless of the load resistance connected to it.[5]

By placing R_S in series with load resistance R_L a voltage divider action is introduced to lower E_L relative to V_S. With a very high source voltage and large series resistance (relative to load resistance), Figure 5-4 becomes a *quasi-constant-current* source. This can be useful in certain applications, particularly low-cost or simple test equipment.

As an example, a constant current of 1 mA is desired into any load resistance from zero to 100 Ohms. By making V_S a precision 100 VDC supply and R_S a precision 100 KOhm resistor, the DC currents through the load would be:

Load Resistance, Ohms	Total Resistance, Ohms	Current, µA
0	100,000	1.000000
1	100,001	0.999990
10	100,010	0.999900
25	100,025	0.999750
50	100,050	0.999500
100	100,100	0.999001

Worst-case error of the above is less than 0.25% relative to exactly 1.000 mA. If the load resistance had varied from 0 to 1 KOhm, the worst-case error would have been 0.990099 mA or -0.99%.

By holding the series resistance constant, varying the constant voltage source voltage would have the effect of varying the quasi-constant-current throughout the load resistance variations.[6] The disadvantage is that one must use a high voltage and high series resistance value for a small constant current. An advantage is simplicity and few parts relative to a more complex circuit using operational amplifiers and many passive components.

The circuit of Figure 5-5 could also be used in a computer analysis program to simulate any input source impedance by taking advantage of computer programs analyzing solely by mathematical values.[7] By making I_S something very large, say 1000 A and R_T something like 1.0 mOhm, the voltage at E_T would stay at 1.000 Volts over a wide range of total R_S and R_L values. In effect, E_T has become a *very stiff* voltage source, the equivalent of a constant-voltage type.

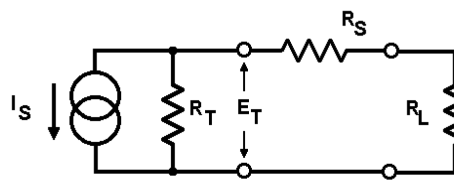

Figure 5-5 Variation for models available only as constant-current.

Matching Source Impedance or Resistance for Maximum Power Transfer

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[5] The concept of *impedance* will be introduced in the next chapter. For the moment, think of impedance as the equivalent of resistance. *Source impedance* is a common specification for test equipment signal sources.

[6] Such a simple constant-current source was used in Birtcher Instruments semiconductor test sets circa 1960s.

[7] Using *single-precision* floating point calculations (7 decimal digits maximum on results), the power-of-ten exponent range is ±38, easily capable of such multiplied and divided value entries.

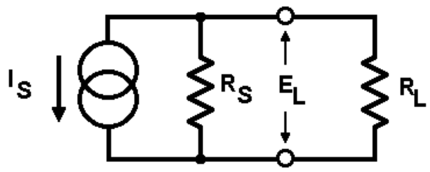

Figure 5-6 Example circuit to show maximum power transfer.

Every real-world signal source has a source impedance of some value. In order to maximize power transfer to a load the load should have the same impedance as the signal source. Figure 5-6 is one way to show that using a constant-current source. In this case R_S is 50 Ohms, the desired impedance match value. Source current I_S held constant at 2.0 A with R_L varied to examine load power.

R_L, Ohms	Total Load Current, A	Load Voltage	Power in R_L, Watts
12.5	1.6000	20.000	32.000
25	1.3333	33.333	44.444
50	1.0	50.000	50.000
100	0.66667	66.667	44.444
200	0.40000	80.000	32.000

In all fairness this is not the way to model an RF transmitter accurately. Such signal sources have complex internal models. The *unused, wasted* power doesn't happen as implied above. Figure 5-6 and the tabulation are used to show the *equivalent* of achieving maximum power transfer to a load.

Pi to/from Tee Equivalent Transformations

Some component connections can make circuit analysis difficult. Figure 5-7 shows two arrangements of three resistors. These are functionally equivalent, can be implemented in hardware or just left on paper as part of an analysis. Their names come from the shapes formed by the resistors: *Pi* and *Tee* are obvious; *Delta* comes from the shape of an inverted Greek Delta character; *Wye* from the shape formed if the two top resistors were both slanted upward.

Some circuit analysis problems can be eased in workload by applying these equivalents or, rather, *transformations.* First, the Pi to Tee transformation equations:

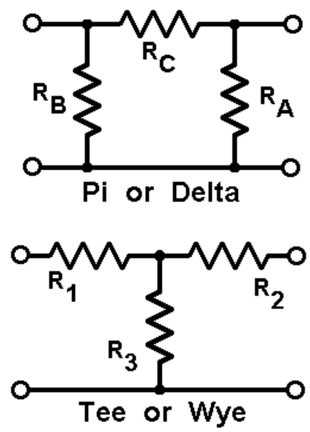

Figure 5-7 Delta - Wye equivalents.

$$R_1 = \frac{R_B \cdot R_C}{R_{DEN}} \qquad R_2 = \frac{R_A \cdot R_C}{R_{DEN}} \qquad R_3 = \frac{R_A \cdot R_B}{R_{DEN}} \qquad (5\text{-}5)$$

$$\text{where: } R_{DEN} = R_A + R_B + R_C$$

The Tee to Pi transformation equations:

$$R_A = \frac{R_{NUM}}{R_1} \qquad R_B = \frac{R_{NUM}}{R_2} \qquad R_C = \frac{R_{NUM}}{R_3} \qquad (5\text{-}6)$$

$$\text{where:} \quad R_{NUM} = R_1 R_2 + R_1 R_3 + R_2 R_3$$

Using a Tee to Pi Transformation to Solve an Analysis Problem

At first glance, Figure 5-8 solution would appear to be a voltage mid-way between +12 and +9 VDC. Not true. To ascertain that, one has to work from the constant voltage source potentials and solve for current, specifically through R_3.

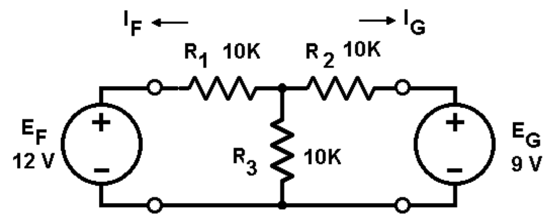

Figure 5-8 Analysis requiring voltage from common from to junction of all resistors (voltage across R_3).

The easy route is to transform the Tee arrangement into a Pi using (5-6). Since all the Tee resistor values are the same (10 KOhms) the Pi network resistors will also be the same, 30 KOhms. Figure 5-9 shows the process and rather straight-forward method of obtaining currents, voltages.

A common point or ground was added in Figure 5-9 as a point of reference. Current through R_A and R_B is direct since each is across a constant voltage source; the current up from common adds to 0.7 mA. Note the 0.1 mA current direction through R_C, from the 3.0 V across it. The total current into E_F is 0.4 mA + 0.1 mA but the total current into E_G is 0.3 mA - 0.1 mA. That could be determined in the Pi form (at left) of Figure 5-9 but it is also true based on voltage drops in the Tee form at right. One ***must pay attention to current direction.***

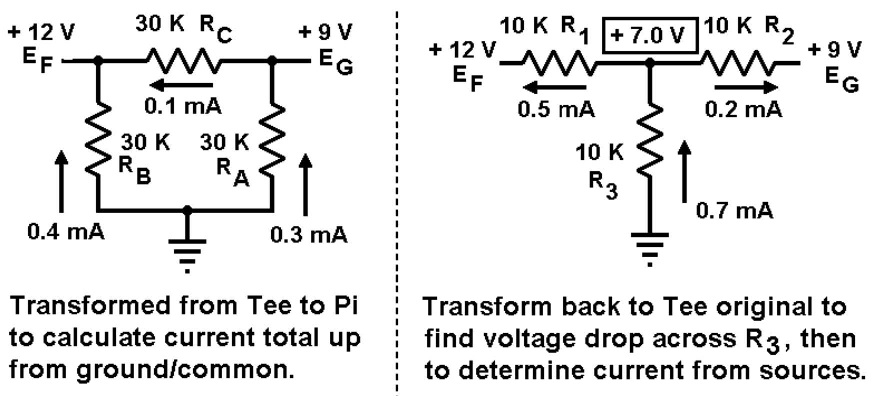

Figure 5-9 Transformation of Figure 5-8 for easy calculation.

A solution can be done without transformation, in simple algebra but is rather difficult.

Calculation of Ladder Networks

A *ladder network* is a chain of alternating shunt and series branches.[8] A simple one is shown in Figure 5-10 as an example. It shows the evolution of the right-to-left operations to find the input

[8] Please don't confuse that with an electrical *ladder diagram* found on appliances and machinery.

impedance. Figure 5-10 **A** is the original network.

In Figure 5-10 **B**, the parallel resistance of R_3 and R_4 has resulted in a new, temporary resistance called R_{34}.[9] Note: All shunt branches with common connections will add conductances.

In Figure 5-10 **C**, R_2 and R_{34} have been added to form a new, temporary resistance called R_{234}. Finally, in Figure 5-10 **D**, R_1 and R_{234} are calculated in parallel to form R_{1234}, the input resistance as presented to the constant current generator.

To find the output voltage relative to input voltage generated across R_{1234}, the operations would be in reverse, going from left to right. The first step is to determine the voltage across R_{34} via the voltage divider formed by R_2 and R_{34}. Since this is a simple network example, the sequence has ended here. In lengthy networks it is an iterative process of finding the voltage divisions, then multiplying all of them times the input voltage.

If all calculations stayed with resistor values in Ohms, the calculation equations can get quite large even with just four branches. They become downright difficult with longer networks. For just four branches of Figure 5-10**:**

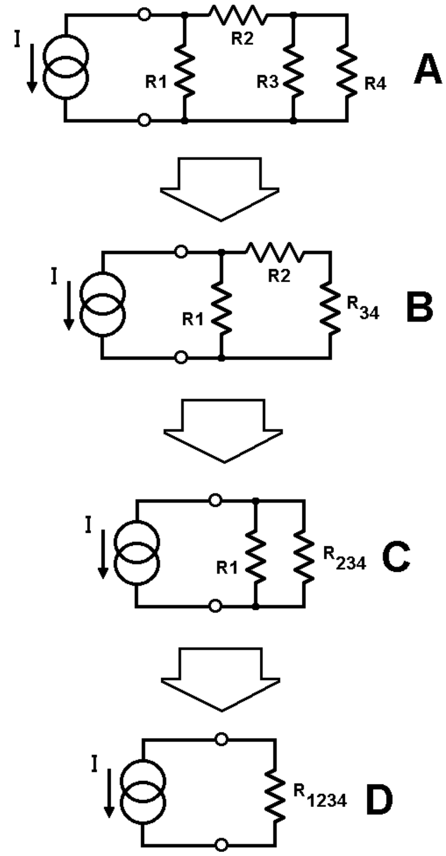

Figure 5-10 Simple ladder network to find input Z.

$$R_{34} = \frac{R_3 R_4}{R_3 + R_4} \qquad R_{234} = \frac{R_2\left(R_3 + R_4\right) + R_3 R_4}{R_3 + R_4} \qquad R_{1234} = \frac{R_1\left[R_2\left(R_3 + R_4\right) + R_3 R_4\right]}{\left(R_1 + R_2\right)\left(R_3 + R_4\right) + R_3 R_4}$$

It's fairly clear that the formal way results in some rather lengthy expressions with just a few parts. A much easier way is to use a scientific calculator directly, working from right to left or rather from the node farthest from the source to the source node. These steps are (using HP RPN):

1. Enter R_4, find its reciprocal.
2. Enter R_3, find its reciprocal, add to the reciprocal of R_4.
3. Find the reciprocal of step (2), jot it down; it is R_{34}.
4. Enter R_2, add to the result of step (3); the result is R_{234}. Jot that down, find its reciprocal.
5. Enter R_1, find its reciprocal, add to the result of step (4).
6. Take the reciprocal of step (5). That is R_{1234}.

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[9] The new, temporary resistance value can be called anything you like. This particular convention of using the original subscript identifiers is fairly standard in most texts. It allows fairly easy identification back to the original, individual branch values.

The resulting final resistance is then multiplied by the source current to find the input node voltage. The output node voltage is found by multiplying the input voltage by R_{34} (jotted down on notes) and dividing that by R_{234} (also jotted down on notes).

This method is simpler than the formal equation method and has less chances for error. It also illustrates a condition of ladder networks where voltage solutions can be found easily by working back-to-front. The final step's resistance is equivalent to the *input impedance* of the network.

Given: $I_{Source} = 0.5\text{ A},\ R_1 = 100\text{ Ohms},\ R_2 = 200\text{ Ohms},$
$R_3 = 300\text{ Ohms},\ R_4 = 600\text{ Ohms}$

Find input and output voltages using back-to-front calculator method

1. Find reciprocal of R_4, $1/600 = 0.0016667$
2. Find reciprocal of R_3, add to reciprocal of R_4, $1/300 = 0.0033333$, added to (1) $= 0.005$, reciprocal $= 200 = R_{34}$.
3. Add R_2 to R_{34}, find reciprocal of sum, $200 + 200 = 400 = R_{234}$, $1/400 = 0.0025$.
4. Find reciprocal of R_1, add to result of step (3), find reciprocal of sum, $0.010 + 0.0025 = 0.0125$, $1/0.0125 = 80.000 = R_{1234}$

Input voltage $= E_{IN} = I \cdot R_{1234} = 0.5 \cdot 80.000 = 40.000$ Volts

$$\text{Output voltage using voltage-divider} = E_{IN}\left(\frac{R_{34}}{R_{234}}\right) = 40\left(\frac{200}{400}\right) = 20.000\text{V}$$

An Example of A Current Source and A Voltage Source in a Circuit

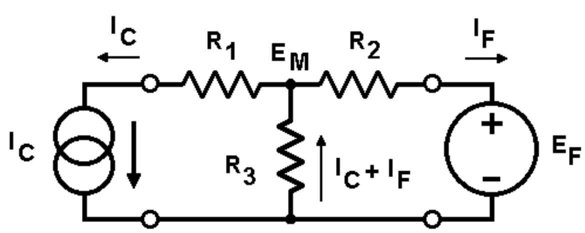

Figure 5-11 Example of combined constant current and voltage sources.

Figure 5-11 has a circuit with both a current source and a voltage source. The object here again is to find the mid-point voltage E_M. This example shows an algebraic solution.

By inspection, all of the current source will flow through R_1 and all of the current from the voltage source will flow through R_3. What is not known is the voltage across the current source or the current supplied by the voltage source; both are variable and as-yet undefined by the definition of ideal sources. From the directions of current flow, the sum of the current source and voltage source currents flows through R_2. Also from the directions of current flow E_M is more negative than E_F as well as more negative than E_C. With all of that the equations can be generated:

$$E_M = R_2(I_C + I_F) \qquad I_F = \frac{E_F - E_M}{R_3} \qquad \text{Therefore:}$$

$$E_M = R_2\left[I_C + \left(\frac{E_F - E_M}{R_3}\right)\right] = \frac{R_2R_3I_C + R_2E_F - R_2E_M}{R_3}$$

$$E_M = \frac{R_2(I_CR_3 + E_F)}{R_2 + R_3}$$

Given: $I_C = 15\text{ mA},\ E_F = 12\text{ V},\ R_1 = 2.2\text{K},\ R_2 = 3.9\text{K},\ R_3 = 18\text{K}$

$$E_M = \frac{3.9\,(1.5 \cdot 18 + 12)}{3.9 + 18} = \frac{3.9 \cdot 39}{219} = 6.94521\text{ V}$$

[note that mA and KOhms can eliminate their multipliers here]

$$I_F = \frac{E_F - E_M}{R_3} = \frac{12 - 6.94521}{18} = 0.280822\text{ mA}$$

$$E_C = E_M + I_CR_1 = 6.94521 + (1.5 \cdot 2.2) = 10.2451\text{ V}$$

What if the ideal voltage source was of opposite polarity? In that case the current flow through R_3 would be from right to left and the current through R_2 would be the difference of I_C and I_F. The equations would then be:

$$E_M = R_2(I_C - I_F) \qquad I_F = \frac{E_M - E_F}{R_3} \qquad \text{[Note polarities]}$$

Substituting right - hand equation into left - hand equation:

$$E_M = R_2\left[I_C - \left(\frac{E_M - E_F}{R_3}\right)\right]$$

Solving for E_M as before:

$$E_M = \frac{R_2(I_CR_3 + E_F)}{R_2 + R_3} = \frac{3.9(1.5 \cdot 18 - 12)}{21.9} = \frac{3.9 \cdot 15}{21.9} = 2.67123\text{ V}$$

Current through R_2 is $I_C - I_F = \frac{E_M}{R_2}$ and solving for I_F:

$$I_F = I_C - \left(\frac{E_M}{R_2}\right) = 1.5 - \left(\frac{2.67123}{3.9}\right) = 1.5 - 0.684931 = 0.815068 \qquad \text{[mA]}$$

It may seem incongruous to have the E_M solution equation exactly as before. The reason for that is the two preceding equations (for E_M and I_F) with changed polarities. This particular case resulted in the same solution but that isn't necessarily true for other circuits. Voltage differences and currents through the branches must be set up carefully for a good solution.

If a particular E_M could have been fixed, one of the resistors could be solved for a value to that. Fixing E_M at 3.00 Volts and solving for R_2 with all other values the same as the second example would have R_2 at exactly 4.5 KOhms.

This no-transformation algebraic method could have been used for the circuit of Figure 5-8. The choice is up to the designer, what they are comfortable in using. Any method or tool is acceptable as long as it yields the correct answer.

Alternating Current, AC

General

An AC voltage or current with a periodic *cycle* of polarity change has a *waveform* of some sort, an amplitude versus time periodicity.[10] The most basic of those is the *sinusoid.* With any *sinewave* the amplitude at any part of the cycle is a function of that part of the cycle described in degrees or radians; i.e., one full cycle is 360° or 2π radians. Each cycle of AC is so many Seconds long and the repetition time is the mathematical inverse of the frequency in Hertz. A sinewave has many *pure* qualities discussed a bit later.

Squarewaves don't have any amplitude changes. For half of a repetition cycle the voltage or current is one near-constant amplitude and in the next half the amplitude is opposite in polarity. A repetitive *pulse* will have the amplitude *on* to some *peak* value for a short duration followed by a long duration of no amplitude or *off* value. A repetitive pulse could have its *on time* lengthened until it is as long as half a cycle time; that would change it to a squarewave.

A repetitive audio or video signal is part of the *other* group that don't have a waveshape that is sinusoidal, on-off, or the pulse variety. Such a waveshape may seem random ups and downs with varying slopes of the curve but in a repetition of the same waveshape it has *harmonics* of the cyclic frequency that, when added linearly in voltage or current, form the waveshape.

Waveforms, their harmonic content was explained in more detail in Chapter 3.

Relationships Between AC and DC, the RMS Definition

AC voltages and AC currents are usually expressed as ***RMS*** or ***Root Mean Square*** values. Why RMS and what does *RMS* mean? The variation in waveshapes of AC is one reason. AC power expressed as the product of voltage and current is another reason. Power is basically a unit of work. Its measurements can be calibrated as the amount of heating of a pure resistor absorbing all the voltage and current. With DC the voltage potential and current flow are constant; DC has no waveshape over time. With AC both voltage and current can vary widely just within one cycle.

RMS is a statistical technique for describing an AC waveform characteristic such that the

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[10] Using *AC current* may seem like an oxymoron, such as saying *alternating current current.* In the early days of electricity no one came up with words differentiating *direct* and *alternating* that also differentiated voltage from current. In short form, using the acronyms **DC** and **AC** serve that purpose well enough for nearly all to comprehend.

resulting power for a work result from AC is ***the same as if the voltage or current was DC.*** Since such work results in heat dissipation for either AC or DC, the ***heat can be measured and compared to DC***. The name comes from taking the ***root*** of the ***mean*** value of ***squared*** periodic samples, thus abbreviated ***RMS.*** In simplified arithmetic form it is:[11]

$$e_{[RMS]} = \sqrt{\frac{e_1^{\,2} + e_2^{\,2} + e_3^{\,2} + e_4^{\,2} + e_5^{\,2} + \ \dots\ e_N^{\,2}}{N}}$$

$$i_{[RMS]} = \sqrt{\frac{i_1^{\,2} + i_2^{\,2} + i_3^{\,2} + i_4^{\,2} + i_5^{\,2} + \ \dots\ i_N^{\,2}}{N}} \qquad (5\text{-}7)$$

Where: e_n = Instantaneous voltage at point n along a period.
i_n = Instantaneous current at point n along a period
N = Number of points of instantaneous voltage or current in one period, all points being at equal time increments in that period.

Note: The numerator under the square-root sign could be of any length in time provided that the points of time are evenly-spaced.

As an example for a sinewave whose peak value is exactly 1.0, one can take 5° increments to five places from a Sine table between 0° and 90°, square each one, add the squares, then divide the sum by 19 (number of increments of angle) and get exactly 0.50000. Taking the square-root of that yields 0.70711 which, not surprisingly, is the RMS value of a sinewave having ± 1.0 Volt peaks.[12]

If several different waveforms, all symmetrical, are examined mathematically, there will be another AC definition: ***Crest Factor.*** Crest Factor is the peak voltage divided by the RMS voltage (or peak current divided by RMS current). The following table shows Crest Factors for various waveforms:[13]

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[11] The *correct* mathematical format is not used to show the summation as it really is without the math shorthand and to illustrate better how the formula could be applied in programming instructions. The root of the mean square is from Charles Proteus Steinmetz, his books and lectures, about 1897.

[12] Squaring a negative number results in a positive quantity. Since a sinewave is symmetrical, it will be numerically accurate to use just one-quarter of a period. That is considerably easier than repeating the 90° calculation three more times. Note: That is accurate only for a perfect, symmetric waveform such as a sinusoid.

[13] From Yokogawa on-line *Tutorial - Power Meter Calibration*, mid-2006, Yokogawa Electric Company.

Waveform	Crest Factor
Sinusoid or Full-Wave Rectifier output without filter*	1.41421
Symmetric Square-Wave	1.00000
Symmetric Triangle Wave	1.73205
Half-wave rectifier output without filter*	2.00000
Pulse of 10% duty-cycle	3.0
Pulse of 1% duty-cycle	10
Pulse of 0.1% duty-cycle	30

* Sinewave input to rectifiers

In AC the product of RMS voltage and RMS current is the value of Watts.[14] A pure resistor heated by that Watt value will be the same as if DC voltage and DC current was heating the same resistor. Once the AC voltage and AC current is measured as RMS, the same equations of DC will apply to AC: Volts are volts regardless of being DC or AC; Amperes are Amperes whether AC or DC. Watts are the same whether involving AC or DC since it is a unit of work.

A pure sinusoidal waveshape of AC turns out to have an RMS voltage value of exactly the *peak* voltage divided by the square-root of 2 or approximately the peak voltage multiplied by 0.707. Peak voltage is the maximum amplitude in one direction relative to the sinusoid's center-line of 0 Volts. Note: RMS values have no polarity signs since the positive and negative swings of a sinusoidal waveshape are symmetric. Non-sinusoidal waveshapes may be expressed as +peak or -peak relative to some baseline voltage stated.[15] That would be circuit-specific and is more in the way of identifying the voltage (or current) swings relative to the circuit itself.

Duty cycle refers to on-off or digital waveforms. It is the division of the on-time by the period, sometimes expressed as a percentage. In certain applications, such as a high-power search radar, the duty cycle times the peak power output will determine the *average power* consumed by the search radar transmitter. As an example, the peak power output may be 1.0 MW but the pulse width transmitted is only 1.0 μS wide. If the repetition rate is 400 Hz, the period would be 2.5 mS and thus the duty cycle would be $4 \cdot 10^{-4}$ and average power only 400 Watts. If some 100% absorbing material were placed in front of the radar antenna, it would have to absorb only 400 W, not the million Watts of power at peak output.[16]

Notational Convention Between AC and DC

[14] There's no such thing as *RMS Watts*. Power is just Watts whether DC or AC.

[15] This would be relevant on pulse or video waveforms which might have a fixed DC voltage bias present.

[16] The ubiquitous *microwave oven* operates at higher duty cycles but with lower peak output. Any material (such as food) placed inside it will absorb the *average* microwave power, heating the material to do the cooking. The author has heard and seen several *sea stories* from ex-sailors supposedly cooking steaks in just seconds using a ship's search radar; they are exaggerating by confusing peak power with average power.

It is convention to denote AC voltages and currents in lower-case letters such as ***e*** and ***i***, while retaining upper-case letters for DC. This is useful when circuits contain both DC and AC at the same node; each can be separated from the other for analysis, then rejoined for power determination. As has already been seen in Chapter 4 and earlier in this chapter, lower-case letters also apply to *instantaneous* values such as voltage or current at particular points in time along a periodic waveform. Instantaneous values are good when there are several AC waveforms superimposed on one another as in Chapter 4 modulation equations. The notation is merely a convention and does not change either AC or DC conditions. You are free to use any convention you wish *as long as you know which is which.*[17]

References for Chapter 5

It is convention to make references on important statements but Chapter 5 reviews such very basic topics in electrical current that all such notations would take up too much space. Kirchoff's Laws can be found in:

[11] *Radio Engineering Handbook,* Keith Henney editor-in-chief, McGraw-Hill Book Company Incorporated, Fourth Edition, 1950, specifically page 65.

Network theorems occupied much space in earlier texts to present a methodology for analyzing circuits. While that was fine for its day, having no modern computational tools such as Personal Computers, it is not considered necessary for the modern electronics hobbyist and may be a waste of time for many. Such methodologies as *superposition, reciprocity, Thevenin's theorems,* and *mesh equations* can be found in:

[12] *Radio Engineers' Handbook,* by Frederick Emmons Terman, Sc.D., McGraw-Hill Book Company Incorporated, 1943, beginning on page 198.

[17] Adhering to a convention standard keeps the mind focused, particularly when weeks or even months can elapse between using the same equations or notations.

Chapter 6

Passive R-L-C Components

Only three ideal types of passives exist in electronics: Resistors, inductors, and capacitors. Practical passive components are combinations of all of them. Ideal capacitors and inductors are characterized at a single frequency. Combinations of all components are described in terms of impedance or admittance, again at a single frequency.

Ideal Capacitors With Step Sources

An ideal capacitor is an electrostatic charge collector, two conductive plates separated by a dielectric. No direct current can flow between the plates but there can be transient current flow into and out of those plates.

If an ideal constant current source is suddenly connected to the capacitor, as in Figure 6-1, the potential between the plates will be a linear increase in voltage.[1] This voltage at any point in time from switch closure is calculated from:

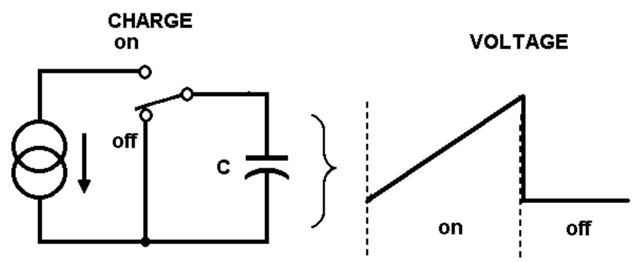

Figure 6-1 Ideal capacitor charged from a constant current source through a switch.

$$\frac{V}{T} = \frac{I}{C} \quad \text{and} \quad C = \frac{T\,I}{V} \qquad (6-1)$$

Where: V is capacitor plate potential, Volts
I is the constant current into / out - of C, Amperes
T is time of constant current flow in Seconds

The *rate of change* of capacitor charge voltage is directly proportional to the constant current. The *rate of change* of voltage is inversely proportional to capacitance value. A perfect capacitor will retain whatever electron charge it had accumulated from the source. However, there is always some *leakage* current reducing the electrostatic charge plus sources are not perfect and will possess some series resistance to the current flow. A more realistic condition is shown in Figure 6-2.

The series resistance controls the electron flow into and out of the plates dependent on the

---

[1] Note the *constant-current* source, not a constant-voltage source such as a battery.

voltage potential across it. With the capacitor at zero charge and the switch closed to the constant-current source, a maximum charging current will flow. As the capacitor charge voltage increases, the potential across the series resistor diminishes and the charging current reduces. When the capacitor charge voltage is almost equal to the constant-current source potential, the resistor current will be at virtual zero.

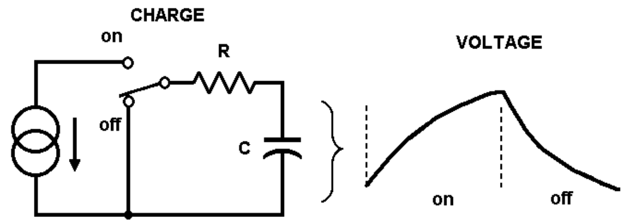

Figure 6-2 A resistor in series will cause a variable rate of charge on the capacitor, the slope of the voltage assuming an *exponential curve* shape.

With the switch made to ground potential, the capacitor begins to discharge at maximum electron flow current, then diminishes as the charge voltage drops closer to ground potential. Voltage across the capacitor can be expressed as:

$$v_c = V_S\left[1 - \varepsilon^{(-t/RC)}\right] \qquad (6\text{-}2)$$

Where: v_c = Instantaneous Voltage across capacitor

V_S = Voltage of source

t = time from start of charge (start = zero)

ε = base of natural logarithms = 2.718281828....

R = resistance in charge / discharge path, Ohms

C = capacitance being charged / discharged, Farads

Note that lower-case terms are used here to signify *instantaneous* values at one point in time. The general convention in electronics is to use lower-case for AC or RF, upper-case for steady-state DC.

The textbook formula of (6-2) can be a bit clumsy to use, so the following identities can simplify things:

$$\text{If:} \quad \rho = \frac{v_C}{V_S} \quad \text{and} \quad \tau = R\ C \quad \text{then:}$$

$$\rho = 1 - \varepsilon^{(t\,/\,\tau)} \qquad t = \tau\ \mathrm{Ln}\,(1 - \rho) \qquad R = \frac{1}{C\ \mathrm{Ln}\,(1 - \rho)} \qquad (6\text{-}3)$$

Greek letter *tau* (**τ**) is commonly used to denote time and, related to the product of R and C, it has the new description of *time constant,* sometimes referred to as *TC*. When tau has the value of unity, the charging capacitor voltage has reached 63.212 percent of the source voltage. Greek letter *rho* (**ρ**) will always be at or less than unity since the capacitor voltage cannot exceed the source potential.

Equations for discharge of C in Figure 6-2 become:

$$\rho = \varepsilon^{(t\,/\,\tau)} \qquad t = R\ C\ \mathrm{Ln}\,(\rho) \qquad R = \frac{1}{C\ \mathrm{Ln}\,(\rho)} \qquad (6\text{-}4)$$

A time constant of unity indicates capacitor voltage has decreased to 36.788 percent of its peak voltage charge (what it reached just before turn-off of the source). Do not mistake that for the voltage of the source.

Ideal Inductors With Stepped Sources

Capacitors store energy in the form of an electrostatic charge. Inductors store energy in the form of a magnetic field charge...but, that magnetic field is only present when *current flows through the inductor*. When the inductor current stops, the magnetic field collapses, generating an opposite-potential voltage spike across the inductor.[2] See Figure 6-3 for an inductor switched on and off a constant-current source.

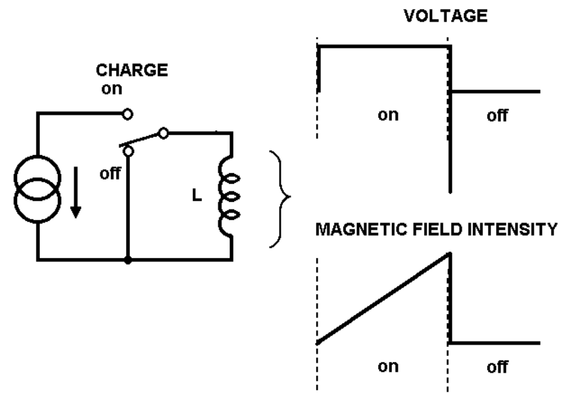

Figure 6-3 - Ideal inductor switched to a current source (switch on) or discharged (switch off). Note the high opposite-polarity spike of voltage on switch off.

Placing a resistor in series with the inductor and switch would make it an inductor duo to Figure 6-2. Equations would have the same form but with some terms changing definition:

- Voltage terms would become current terms.
- Tau would change to be equivalent to (L/R).
- Rho changes to instantaneous current divided by maximum source current.
- Stored electrostatic change is replaced by stored electromagnetic field.

Equations for magnetic field charging time then becomes:

$$\rho = 1 - \varepsilon^{(-t/\tau)} = \frac{i_T}{I_S} \quad \text{and} \quad \tau = \frac{L}{R} \quad \text{Where:} \qquad (6-5)$$

i_T is instantaneous current through inductor at some time.

I_S is maximum current available from source.

L is inductance, Henries

R is series resistance, Ohms

Equation for magnetic field discharging time is:

$$\rho = \varepsilon^{(-t/\tau)} \qquad \text{Where:} \qquad (6-6)$$

Terms are the same as in (6 - 5)

[2] This follows rather basic laws of electromagnetics: A conductor moving through a magnetic field (as in a generator rotor) will generate a current in the conductor; if the magnetic field is made to move past a stationary conductor, an electric current will be generated in the conductor.

Equation (6-6) does not apply if the inductor is open-circuited at switch-off; i.e., the off position has no connection. That was common in the first gasoline engines before the year 1900 and the heart of the gasoline engine *ignition system* called a *spark coil.* A low-voltage primary winding of a cylindrical transformer would be energized by a low-voltage battery. That current would be interrupted by *timing contacts* synchronized to the maximum compression position of a piston in a cylinder. On interruption the spark coil magnetic field would collapse, inducing a reverse-polarity voltage which was stepped up many times in the spark coil high-voltage secondary winding. That secondary winding's output would be coupled through *distributor* or rotary high-voltage switch to each cylinder's spark plug. Several KV of spark potential could be generated from a 6 VDC battery.[3]

The reverse-potential voltage spike on inductor turn-off has been both a boon and a bust for related electronics. Magnetic deflection cathode ray tubes (CRTs) used the linear magnetic field to create a sweep of the CRT's electron beam on the faceplate phosphor. At the end of the sweep the current was interrupted and the very high reversed-polarity potential spike would be rectified and filtered for the final CRT accelerating electrode. This reverse-polarity spike was called the *flyback pulse* and well-designed consumer TV color receivers could have 25 KVDC final accelerating electrode voltages for a bright picture. However, relays and solenoids using simple control circuits often encountered hundreds of volts of switch-off flyback, enough to pit control contacts and flash-over some wiring insulation, plus create considerable wideband RF noise to interfere with radios.[4]

Modern *switching power supplies* depend on inductors with currents switched on and off to generate the required voltages. Varying the on and off times can vary the DC voltage output and that is part of such a supply's control-regulating circuit.

Keeping Straight on Sources

A constant-current source into a capacitor produces a linear voltage change across that capacitor. A constant-current source into an inductor produces a linear magnetic field intensity change around that inductor. Any series resistance with either results in an exponential change of voltage (capacitor) or magnetic field (inductor). A constant-voltage source into a capacitor results in a constant voltage across the capacitor but has an enormous initial current surge dependent on the series resistance...a possible problem in simple AC rectifier circuits later.

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[3] This included miniature ignition systems for model aircraft of the late 1940s, each running off of two dry cells for 3 VDC...and on up to piston-engine aircraft of all types operating from 28 VDC electrical supply systems. Post WWII automobiles changed to 12 VDC electrical systems but the principle remained the same. Once the spark plug's applied voltage exceeded the insulation of its spark gap, the electrical arc would ignite the compressed air-gasoline mixture in the cylinder. The coil, timer, distributor would have done its job and the coil's primary would again be energized to build the magnetic field for the spark potential of the next cylinder's spark plug.

[4] One of the author's assignments at Hughes Aircraft Co., El Segundo, CA division, in 1957 was to test the flyback voltages of common 28 VDC relays. Most generated 500 V peak flyback pulses with a few peaking to 600 Volts! The common correction then (and still today) was to damp out the flyback was to connect a reverse-connected silicon diode across the relay coil. At normal control voltage polarity the diode would appear open across the relay coil. At switch-off the diode would forward conduct (flyback polarity was reverse of energize polarity) and damp out almost all of the flyback energy.

AC Applied to Capacitors or Inductors

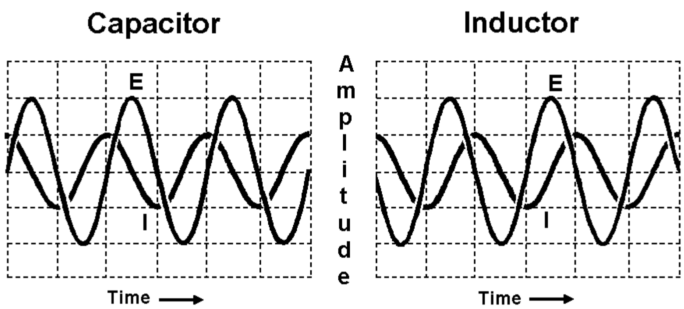

Figure 6-4 - AC current and voltage in ideal capacitors or inductors. Note that currents are 90° out of phase with voltages or *in quadrature.*

Applying an alternating current (AC) source to a pure resistor will result in both AC voltage and AC current to be in-phase. The pure resistor will then absorb all applied AC power. If the same AC power is applied to a pure capacitor, the AC voltage will be 90° *behind* of the AC current. If that AC is applied to a pure inductor, the AC voltage will be 90° *ahead* of AC current.

Obviously, at DC the pure capacitor will appear as an open circuit while the pure inductor appears as a short-circuit. Concentration here is on the effects of AC only, depicted in Figure 6-4.

The actual power flow, dissipation in pure capacitors and inductors is zero.[5] The combination of voltage and current, where a quadrature phase exists between them is termed *reactive power.* Practical capacitors and inductors aren't *pure*. Inductors will have some series resistance just from the wires in its coil. Capacitors will have a finite insulation resistance between plates which can be represented by a parallel resistance. These added resistances shift the voltage-current phase away from 90° quadrature and result in *losses* that can absorb some AC energy.

Models for Less Than Pure R-L-C Components

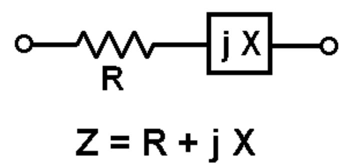

Figure 6-5 - Series model described by Impedance.

There can be two analytical models for practical inductors and capacitors: Series form described in terms of *impedance* as shown in Figure 6-5; parallel form described in terms of *admittance*, shown a little bit later. Both impedance and admittance are in *complex number notation* and may be expressed in either *rectangular* or *polar* form.[6]

In Figure 6-5 the *j X* box represents the *reactance* of either an inductor or capacitor and described in Ohms as follows:

$$X_{INDUCTIVE} = 2\pi fL \qquad X_{CAPACITIVE} = \frac{-1}{2\pi fC} \qquad (6\text{-}8)$$

Where: f is frequency, Hertz
L is inductance, Henries   C is capacitance, Farads

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[5] In examining Figure 6-4 it will be seen that the peak amplitude of AC voltage occurs when the AC current is zero. Similarly, when the AC current amplitude is maximum, the AC voltage is zero. Given that normal power expression is voltage times current, the net product is zero power.

[6] Please review Chapter 2, pp 2-11 to 2-17, if you are unfamiliar with complex numbers.

Note that *capacitive reactance* is always negative and *inductive reactance* always positive. That can be related to AC voltage phase *leading* AC current with inductors and AC voltage phase *lagging* AC current phase with capacitors in Figure 6-4. Note also that reactance value is valid *only at one frequency*. A simpler way to write [2 π f] is to use Greek letter omega [ω] as equivalent, ω occurs often in later equations and is termed *radian frequency* (from the 2 π sub-terms).

The two forms of series circuit impedance are:

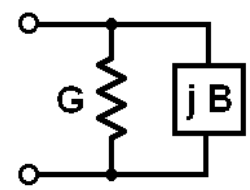
Rectangular: $$Z = R + j\,X$$

Polar: $$|Z| = \sqrt{R^2 + X^2} \qquad \theta = \mathrm{ArcTan}\left(\frac{X}{R}\right) \qquad (6\text{-}9)$$

Magnitude   Phase Angle

In the rectangular form, a negative reactance (capacitive) could be rewritten with the negative sign for the entire imaginary part. In polar form, magnitude is always positive; phase angle is expressed (in math textbooks) as *radians*.[7]

Turning It Upside Down



$$Y = G + j\,B$$

Figure 6-6 - Parallel model in Admittance.

Parallel models in complex notation need different terms such as *Admittance* of the whole, expressed in *mhos*.[8] A paralleled resistance now becomes a *conductance* with a value in mhos of the inverse of the resistance. The *j B* box in Figure 6-6 can be either a capacitor (usual case) or an inductor and both are termed *susceptance.* Susceptances are the negative inverse of reactances.

Conductance and susceptance equations:

$$G = \frac{1}{R} \qquad B_{CAPACITIVE} = \omega C \qquad B_{INDUCTIVE} = \frac{-1}{\omega L} \qquad (6\text{-}10)$$

where:

G is conductance, mho	B is susceptance, mho
R is paralleled resistance, Ohms	$\omega = 2\,\pi\,f$, f in Hertz
C is paralleled capacitance, Farads	L is paralleled inductance, Henries

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[7] Use depends entirely on the transcendental functions available on your computing device. Most scientific pocket calculators have an option of selection of degrees or radians but many high-level personal computer programs have angle functions only in radian arguments. To convert radians to degrees, multiply radians by 57.295 77951 [Chapter 2, page 2-19].

[8] Mho is *ohm* spelled backward. How that came about is irrelevant but it can be taken as a mnemonic jog that admittance is the complex inverse of impedance, thus mho is a sort of inverse of ohm.

The admittance equations look very much like those for impedance in (6-9):

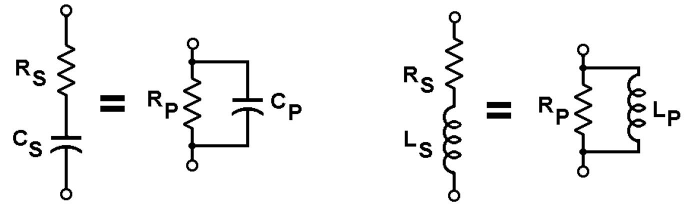

Figure 6-7 - Duals of series and parallel models at one frequency, one frequency *only*.

Rectangular	Polar		
$Y = G + jB$	$\lvert Y \rvert = \sqrt{G^2 + B^2}$	$\phi = \text{ArcTan}\left(\frac{B}{G}\right)$	(6-11)
	Magnitude	Phase Angle	

Conversion of Impedance to Admittance and Admittance to Impedance

Every series model and every parallel model may be converted to its dual counterpart *at one frequency*. These are shown in Figure 6-7.

It must be emphasized that such duals exist only as *equivalent circuits at one frequency*. Their usefulness is limited to narrowband applications such as interstage matching networks...also in seeing theory how resonance happens.

$$R_S = \frac{R_P\ X_P^2}{R_P^2 + X_P^2} \qquad R_P = \frac{R_S^2 + X_S^2}{R_S}$$

$$X_S = \frac{R_P^2\ X_P}{R_P^2 + X_P^2} \qquad X_P = \frac{R_S^2 + X_S^2}{X_S}$$

$$R_S = \frac{\omega^2\ R_P\ L_P^2}{R_P^2 + \omega^2\ L_P^2} \qquad R_P = \frac{R_S^2 + \omega^2\ L_S^2}{R_S} \qquad (6\text{-}12)$$

$$R_S = \frac{R_P}{\omega^2\ R_P^2\ C_P^2 + 1} \qquad R_P = \frac{\omega^2\ R_S^2\ C_S^2 + 1}{\omega^2\ R_S\ C_S^2}$$

$$L_S = \frac{R_P^2\ L_P}{R_P^2 + \omega^2\ L_P^2} \qquad L_P = \frac{R_S^2 + \omega^2\ L_S^2}{\omega^2\ L_S}$$

$$C_S = \frac{\omega^2\ R_P^2\ C_P^2 + 1}{\omega^2\ R_P^2\ C_P} \qquad C_P = \frac{C_S}{\omega^2\ R_S^2\ C_S^2 + 1}$$

where: $\omega = 2\pi f$, f in Hz R, X in Ohms, C in Farads, L in Hy

Complex Mathematics of Y to Z and Z to Y Conversion

Since admittance is the complex inverse of impedance and vice-versa, the complex inversion is a straightforward process:

$$Y = \frac{1}{Z} = \frac{1}{(R + jX)} = \left(\frac{R}{R^2 + X^2}\right) + j\left(\frac{-X}{R^2 + X^2}\right) = G - jB$$

$$Z = \frac{1}{Y} = \frac{1}{(G + jB)} = \left(\frac{G}{G^2 + B^2}\right) + j\left(\frac{-B}{G^2 + B^2}\right) = R - jX \qquad (6\text{-}13)$$

Note that the imaginary part of the rectangular-form conversion changes sign. This is a requirement of the complex number inversion process.

Polar-form conversion is numerically and operationally easier. From equation sets (6-9) and (6-11):

$$|Y| = \frac{1}{|Z|} \qquad \theta = -\phi \qquad (6\text{-}14)$$

While that is simpler and takes less computation time, the operations of obtaining magnitudes of admittance or impedance may take more program overhead.

Quality Factor *Q* of Inductors and Capacitors

To reiterate, there is no such thing as a *pure* inductor or capacitor. To gain a measure of the quality of an inductor, the *unloaded Q* was measured using a series circuit model containing only a resistance in series with the real inductance.[9] The resistance is equivalent to all losses of the inductor: wire resistance, core material (if other than air), proximity of shielding conductors (if not in toroidal form), length-to-diameter ratio, wire size, skin effect (at higher frequencies). Equivalent series resistance is then:

$$R_{SERIES} = \frac{X_L}{Q_U} = \frac{\omega L}{Q_U} = \frac{2\pi f L}{Q_U} \qquad [\text{ Ohms }] \qquad (6\text{-}15)$$

where: X_L is inductive reactance, Ohms
Q_U is measured Q of inductor, out-of-circuit
L is inductance, Hy   f is frequency, Hz

Q itself is a dimensionless quantity. It relates to the ratio of inductive reactance to equivalent series resistance.

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[9] *Unloaded Q* refers to the Q measured of only the inductor; i.e., not connected into any circuit.

A parallel resistance equivalent is sometimes useful. For values of Q greater than 10 it would equate to:

$$R_{PARALLEL} = Q_U X_L \qquad [\text{ Ohms }] \qquad (6-16)$$

where: Terms are the same as in equation set (6 - 15)

The equivalent loss resistance for capacitors is generally taken as the parallel equivalent resistor equal to:

$$R_{CAPACITY\ PARALLEL} = \frac{Q_C}{B_C} = \frac{Q_C}{\omega C} = \frac{Q_C}{2\pi f C} \qquad [\text{ Ohms }] \qquad (6-17)$$

where: Q_C is the Q of the capacitor
B_C is susceptance in mho   C is capacitance, Fd
f is frequency, Hertz

Electrolytic capacitors used in power supply filters are sometimes rated by manufacturers for *ESR* or Equivalent Series Resistance at 60 to 120 Hz (North American power grid frequency). In that case the manufacturer's values can be taken directly instead of measuring capacitance and Q, then converting from parallel-form to series-form.

Table 6-1
Percentage Charge/Discharge at Various Time Constants

Time Constant	Charge	Discharge
0.01	0.995	99.005
0.02	1.980	98.020
0.03	2.921	97.079
0.04	3.291	96.079
0.05	4.877	95.123
0.06	5.824	97.176
0.07	6.761	93.239
0.08	7.688	92.312
0.09	8.607	91.393
0.1	9.516	90.484
0.2	18.127	81.873
0.3	25.918	74.082
0.4	32.968	67.032
0.5	39.347	60.653
0.6	45.119	54.881
0.7	50.341	49.659
0.8	55.067	44.933

0.9	59.343	40.657
1.0	63.212	36.788
1.5	77.687	22.313
2.0	86.466	13.534
2.5	91.792	8.208
3.0	95.021	4.979
3.5	96.980	3.020
4.0	98.168	1.832
4.5	98.889	1.111
5.0	99.326	0.674

Note: The *Discharge* column is useful in capacitive-input rectifier filters to determine the AC ripple on the DC.

Appendix 6-1

Physical Structures of Passive Components

Inductance of a Straight, Round Wire

All components contain some resistance, capacitance, and inductance in addition to their stated component type. Those should all be considered as part of the design if such *inherent parasitic* elements might affect value and use of a particular component. One such is the leads of a component. The inductance in nanoHenries of a straight round wire may be calculated from:[10]

$$L_{(nHy)} = 5.08 \cdot \text{Length} \left[\text{Ln}\left(\frac{2 \cdot \text{Length}}{\text{Radius}} \right) - 0.75 \right] \qquad (6-16)$$

Where:

Length = Wire Length, inches

Radius = Wire radius, inches

Ln = natural logarithm

AWG Straight Round Wire Inductance, nanoHenries, versus Length

AWG	1 Inch	3/4 Inch	1/2 Inch	1/4 Inch	1/8 Inch
18	19.6	13.6	8.0	3.0	1.1
20	20.2	14.4	8.6	3.4	1.3
22	21.9	15.3	9.2	3.7	1.4
24	23.1	16.2	9.7	4.0	1.6
26	24.3	17.1	10.4	4.3	1.7
28	25.5	18.0	11.0	4.6	1.9
30	26.6	18.9	11.6	4.9	2.0
32	27.8	19.7	12.1	5.2	2.2
34	29.0	20.6	12.8	5.5	2.3
36	30.2	21.5	13.3	5.8	2.5

AWG is American Wire Gauge, a common industry standard for wire size.

Construction of Small Capacitors

Very small value capacitors can be made from double-sided PCB stock. For a two-plate fixed capacitor, this formula may be used:

[10] From reference [11], Henney, page 132.

$$C_{(pFd)} = 0.225 \cdot \varepsilon_d \cdot \left(\frac{\text{Area}}{\text{Thickness}} \right) \qquad (6\text{-}17)$$

Where:

ε_d = Dielectric Constant (Air = 1.0)

Area = Area of one plate in square inches (both plates assumed same)

Thickness = Dielectric thickness in inches

The dielectric constant can be found from tables of material. If the material is unknown, a larger piece of double-sided stock can be measured in area and thickness and measured directly on a Q Meter. By rearrangement of equation (6-17):

$$\varepsilon_d = \frac{C_{(pFd)} \cdot \text{Thickness}}{0.225 \cdot \text{Area}} \quad \text{and} \quad \text{Area} = \frac{C_{(pFd)} \cdot \text{Thickness}}{0.225 \cdot \varepsilon_d}$$

If the larger piece or PCB plate cannot be measured directly in picoFarads on a Q Meter or capacitance bridge, then a *Dipper* can be used to resonate with a known fixed capacitor and a known fixed inductor.[11] The procedure is to first measure the parallel resonance frequency of the known L-C pair by itself. The PCB test piece is then connected in parallel with the known fixed L-C pair and the new, lower resonance frequency is measured. Capacitance of the test piece is then:[12]

$$C_{Test\,(pFd)} = \frac{25330.3\left(f_1^{\,2} - f_2^{\,2}\right)}{L_{fixed} \cdot f_1^{\,2} \cdot f_2^{\,3}} \qquad (6\text{-}18)$$

Where:

L_{fixed} = Known fixed inductor in μHy

f_1 = Resonance frequency, MHz, of L_{fixed} and C_{fixed}

f_2 = Resonance frequency, MHz, of L_{fixed} and C_{fixed} with C_{Test} in parallel.

For an example, assume a fixed capacitor of 200 pFd and a fixed inductor of 3.3 µHy. The parallel resonance frequency is 6.19510 MHz. The test piece is then connected in parallel and the new resonance frequency is found to be 5.71517 MHz. The calculations are:

$$C_T = \frac{25330.3 \cdot (38.3793 - 32.6632)}{3.3 \cdot 38.3793 \cdot 32.6632} = \frac{25330.3 \cdot 5.71610}{3.3 \cdot 1253.59} = \frac{144.790}{4.13684} = 35.0002_{(pFd)}$$

---

[11] A *Dipper* is a small tunable oscillator that can loosely couple inductively to a parallel-LC circuit under test. Resonance of the LC circuit is indicated by the *dip* of oscillator current reading. If frequency needs to be read accurately, as in this case, such should be measured by loose coupling of a direct-reading *frequency meter*.

[12] The formula uses equation (7-1) from the next chapter solved for two frequencies and two values of capacitance.

The small residue of calculation (0.2 femtoFarads) represents a 0.0006 percent error and is negligible. That is due to entering values as 6 digits, calculating with 11-digit accuracy and getting some round-off error. Since the fixed capacitor does not appear in the equation and that frequency measurement can be done easily with 6 to 8 digit accuracy. The *method* of measurement is thus quite accurate and only the tolerances of the two fixed components would contribute any real error.

If the test piece had 2.0 square inches of foil area on one side, the dielectric substrate measured as 0.060 inches, then from the modified equation:

$$\varepsilon_d = \frac{35 \cdot 0.060}{0.225 \cdot 2.0} = \frac{2.1}{0.45} = 4.67 \qquad \text{[typical for phenolic paper laminate]}$$

If a 1.1 pFd capacitor was needed and the preceding PCB stock was available, then the needed area for that small size capacitor calculates from the second modification of equation (6-17):

$$\text{Area} = \frac{1.10 \cdot 0.060}{0.225 \cdot 4.67} = 0.0628 \text{ square inches or a square } \approx 0.25 \text{ inches per side}$$

The actual dimensions will be slightly smaller by approximately 5 percent due to *fringing capacitance*.[13] Fringing capacitance is that occurring from electrostatic lines of force curving around between plates at the edges. Normal and prevalent electrostatic force lines are direct through the dielectric material between plates. If the capacitor plate area is large relative to edge length, the effect of fringing capacitance is small.

Equation (6-18) is good for any unknown capacitor when you have a known L-C pair and a frequency meter to measure frequency accurately.

Determining Distributed Capacity of an Inductor

The usual method of testing inductors is to resonate it at a specific frequency using a calibrated variable capacitor. Since the distributed capacity is in parallel with the variable capacitor, the variable's reading will be *lower* by the amount of the distributed capacity. Most Q Meter variable capacitors are also calibrated in μHy or mHy (to be used only at specific frequencies), so their readings will be in error. Capacity measurement requires two frequencies, one twice that of the other, reading the two capacity dial indications.[14] Q Meter excitation frequency must be read accurately by loose coupling to a frequency meter. This method yields true inductance and distributed capacity.

---

[13] This is approximate and empirical based on construction of such small capacitors in the range of 0.7 to 4.1 pFd using a square of PCB with leads coming off perpendicular to the plane of the PCB square. Measurement of the finished product from a calibrated Q Meter capacitor dial by the *substitution* method. Substitution would use a standard or known inductor and the Q Meter capacitor tuned for maximum (resonance). The small capacitor is then connected in parallel with the variable Meter capacitor re-tuned for peak. The small capacitor total value would then be the difference between the two Q Meter variable capacitor readings..

[14] Reference [7], "Green Bible." pages 268, 269.

$$L_{true\ (\mu Hy)} = \frac{19000}{f_2^{\,2}(C_2 - C_1)} = \frac{76000}{f_1^{\,2}(C_2 - C_1)} \quad \text{and} \qquad (6\text{-}19)$$

$$C_{distributed\ (pFd)} = \frac{C_2 - 4C_1}{3} \qquad \text{Where:}$$

$f_1 = 2f_2$ and both are in MHz

C_1 = Calibrated variable capacitor (pFd) peaking resonance at f_1

C_2 = Calibrated variable capacitor (pFd) peaking resonance at f_2

Note: The numerator constants for true inductance are numerically 18997.7 and 75990.9 but the ones in the equation result in an inaccuracy of only 0.012 percent.

As an example, assume the higher frequency is 240 KHz, the lower 120 KHz. The inductor is approximately 3.3 mHy. The higher frequency resonating capacity is 513 pFd indicated and the lower frequency resonating capacity 113 pFd.. Calculations are then:

$$f_1^{\,2} = 0.0576 \quad f_2^{\,2} = 0.0144 \quad C_1 = 113\ \text{pFd} \quad 4C_1 = 452\ \text{pFd} \quad C_2 = 513\ \text{pFd}$$

$$(C_2 - C_1) = 400 \qquad (C_2 - 4C_1) = 513 - 452 = 61 \qquad C_{distr} = \frac{61}{3} = 20.33\ \text{pFd}$$

$$L_{true} = \frac{19000}{0.0144 \cdot 400} = \frac{19000}{5.76} = 3298.61\ \mu\text{Hy} \approx 3300\ \mu\text{Hy} \quad [\text{within } 0.042\%]$$

Note that the distributed capacity is at the mercy of the variable capacitor's calibration. That is due to the difference term in the numerator, particularly for the higher frequency (lower capacitance value) reading.

Air-Core Inductor Construction

Air-core coils (inductors) are the easiest components to make specifically for a value and shape of inductors. The *core* here refers to a thin-wall dielectric material form where 90+% of the center is air. A very low dielectric constant (2 or less) solid winding *mandrel* which may or may not have threading to hold the wires in place.

Following formulas are accurate to better than 5% for forms of one inch or less in diameter and form length less than 3 times the form diameter.[15] Inductance is in *free air* or unobstructed by any conductive plane within a 4 times diameter distance from any dimension.

[15] References say accuracy is ±2% but practical experience winding and measuring many inductors hedges accuracy to five percent based on how tightly the wire is wound, variations in diameter and so forth.

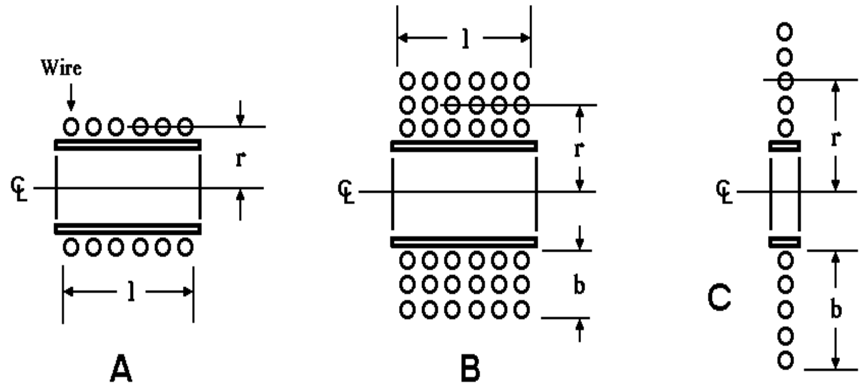

Figure 6-11 Cross-section view of the three most common air-core inductor styles: (A) single-layer cylindrical; (B) multi-layer cylindrical; (C) flat helix. The coil form is assumed to be a thin dielectric.

For the single-layer cylindrical form in (A):

$$L_{(\mu Hy)} = N^2 \left(\frac{r^2}{9\,r + 10\,l} \right) \qquad \text{Where:} \qquad (6\text{-}20)$$

N = Number of turns

r = Radius to wire center, inches (form radius plus half wire diameter)

l = Length of winding, inches

Since the design involves a known inductance, the number of turns is a rearrangement of equation (6-20):

$$N = \frac{\sqrt{L\,(\mu Hy) \cdot (9\,r + 10\,l)}}{r} \qquad (6\text{-}21)$$

For the multi-layer cylindrical form of (B):

$$L_{(\mu Hy)} = \frac{0.8\,(r \cdot N)^2}{6\,r + 9\,l + 10\,b} \qquad \text{Where:} \qquad (6\text{-}22)$$

r = Mean radius to winding center, inches

N = Number of turns

l = Length of winding, inches

b = Thickness of wire build - up, inches

Note that inductance is a function of the square of both radius and number of turns. The wire build-up must include any inter-layer dielectric film insulator, if used.

It is possible to wind one layer on another without any insulating layer. Enameled wire makes that possible. While that reduces the wire build-up dimension it may make the determination of that dimension difficult. An over-layer would have wires possibly laying in the groove of the wires of the under-layer. It also increases the distributed capacity of the inductor.

For the flat helix or spiral configuration of Figure 6-11 (C):

$$L_{(\mu Hy)} = \frac{(r\,N)^2}{8\,r + 11\,b} \qquad \text{Where:} \qquad (6-23)$$

r = Mean radius to center of wire heliz, inches

b = Distance between inner turn and outer turn, inches

N = Number of turns

For a single-turn round loop such as a loop antenna used in direction finding:

$$L_{(\mu Hy)} = \left(\frac{r}{100}\right)\left[7.353\ \mathrm{Log}_{10}\left(\frac{16\,r}{d}\right) - 6.386\right] \qquad (6-24)$$

r = Radius of loop, inches

d = Diameter of wire or tubing, inches

An approximation of the Q of a single-layer cylindrical inductor having a length to diameter ratio of between 0.7 and 1.5 is:

$$Q \approx 100\,d\,\sqrt{f_{(MHz)}} \qquad \text{Where:} \qquad (6-25)$$

d = Mean diameter of winding form plus wire diameter, inches

Q will increase slightly with longer winding length. This is only an approximation.

Both Q and inductance will be reduced by enclosing an air-core inductor in a shield can. The following is another approximation of the percentage reduction of both Q and inductance relative to measured values in free air:

Shield/Form Diameter Ratio	Percentage Reduction of Q, L
2:1	86
2.5:1	93
3:1	96

For square shield cans of one inch per side, the side dimension would be equal to a round shield diameter.

If in doubt, measure shielded inductors to determine the inductance and Q changes. Both Q and inductance may be increased by using powdered-iron cores. The effect of nearby conductors will be greatly reduced by using *toroidal* form cores, the toroid constructed of a specific powdered-iron composition.

Iron Powder Inductor Cores

Iron powder molded with a plastic binder into a *slug* shape (cylinder with an adjustment screw attached) is used for adjustable inductors. The inductance adjustment varies from 1.5:1 to 2:1, slug full in to slug full out. Q is also enhanced by an iron powder core. The types of iron powder *mixes* are frequency-sensitive, limited to a half-decade to full decade frequency span for optimum Q.

A common mistake is to call iron powder core material as *ferrite*. Ferrite material is bound together by *sintering* or baking at high heat. The basic ferrite material varies from the iron powder material in a binder that is molded into shape at relatively low heat. Ferrite material is used from UHF to microwaves; iron powder material is used from VLF to UHF.

Toroidal cores, a torus or doughnut shape with iron powder in the core has several advantages over cylindrical or helical windings. Most of the magnetic field is contained within the torus. This makes them the least susceptible of all inductor shapes to proximity of either conductors or a dielectric mass. With the magnetic field so contained the inductance is maximum and the Q is maximum (dependent on frequency range of the iron powder material).

Common iron powder mixes, in increasing frequency range is:

Material Mix #	**Color Code**	**Temp.Stability,PPM**	**Freq.Range,MHz**	**Mix Name**
-42	Blue/Red	550	0.0003-0.080	Hydrogen Reduced
-3	Grey/clear	370	0.02 - 1.0	Carbonyl HP
-8	Orange/clear	255	0.02 - 1.0	Carbonyl GQ4
-1	Blue/clear	280	0.15 - 3.0	Carbonyl C
-15	Red/White	190	0.15 - 3.0	Carbonyl GS6
-2	Red/clear	95	0.25 - 10.0	Carbonyl E
-7	White/clear	30	1 - 25	Carbonyl TH
-4	Blue/White	280	3 - 40	Carbonyl J
-6	Yellow/clear	35	3 - 40	Carbonyl SF
-10	Black/clear	150	15 - 100	Carbonyl W
-17	Blue/Yellow	50	20 - 200	Carbonyl
-12*	Green/White	170	30 - 250	Carbonyl Oxide
-0	Tan/Tan	—**	50 - 350	Phenolic (no iron)

* Inactive for new design, replaced by Mix 17.
** Phenolic form, no iron powder

Table information is from Micrometals, Inc. Color code has first color as a dot, second color that of the core itself (usually a grey color if clear). Temperature stability has a positive temperature coefficient; that is, inductance increases with temperature, roughly linear over a temperature range of -50°C to +125°C. Q has a negative temperature coefficient over the same range, approximately twice the magnitude of inductance stability in Parts Per Million.

Several other manufacturers use the same color coding and frequency range. For broadband applications, as in RF transformers, the frequency range extends at least another decade higher. The frequency tabulated is that for maximum Q.[16]

The number of turns of wire required is found from:

[16] Some independent core suppliers use Micrometals part numbers and one, Amidon Associates, includes sample Q curves and other information from Micrometals.

$$N_{(turns)} = \sqrt{\frac{desired\ L_{(nHy)}}{A_L}} \qquad \text{Where:} \qquad (6-27)$$

$$A_L = \text{Manufacturer's data per core in } (nHy / N^2)$$

Each core and mix material has a separate A_L value and must be obtained from catalog or datasheet information. An approximation for the maximum number of turns possible in a toroid is:

$$N_{(integer)} \approx \frac{5 \cdot (ID - d_w)}{2 \cdot d_w} \qquad \text{Where:} \qquad (6-29)$$

$$ID = \text{Inside diameter of toroid core}$$

$$d_w = \text{Wire diameter in same units as ID}$$

$$\text{N takes only integer part of result}$$

The above includes some fudge-factoring of not being able to close-space each wire turn.

A single wire turn is defined as a complete loop around one of the cross-sections of the toroid's core. Wire diameter in thousandths of an inch versus AWG number is:

AWG	**Diameter**	**AWG**	**Diameter**	**AWG**	**Diameter**
16	50.8	24	20.1	32	8.0
18	40.3	26	16.0	34	6.3
20	32.0	28	12.6	36	5.0
22	25.4	30	10.0		

Resistance of Short Wires

These are useful as *shunt resistors* for measuring high Amperages with *milliammeters* or as current-sensing resistors to develop a voltage drop across them at a specific current. The can be derived from copper wire tables' Ohms Per Thousand Feet values or interpolated from the following listing of three short wire lengths in inches and resistance in milliOhms:

AWG	**1 Inch**	**2 Inch**	**3 Inch**
18	0.53	1.06	1.63
20	0.84	1.68	2.53
22	1.35	2.70	4.05
24	2.14	4.28	6.43
26	3.42	6.84	10.3
28	5.44	10.9	16.3
30	8.67	17.3	26.0
32	13.5	27.0	40.5
34	21.8	43.5	65.3
36	34.6	69.2	104

Printed circuit board trace lines are really flattened wires. Low-value resistances can be made from such trace lines in *2 ounce foil* PCB by the approximation:[17]

$$L \approx 4000 \cdot w \cdot R \qquad \text{where:} \qquad (6-30)$$

L = Length in inches w = Width of trace line R = Desired resistance in Ohms

That is at best about 30% accurate due to foil thickness variations. Wire is more accurate at about 10% since the diameter is controlled but may be subject to some stretching in handling. One problem with wire as a resistor is the positive temperature coefficient of 0.0042%/°C (0.22233%/°F). A temperature range of -25°C to +75°C would cause a total resistance variation of about 0.42%.

Skin Effect

Skin Effect is a peculiar property of conductors that causes current to flow more near the surface at higher frequencies. The resistance of a conductor will increase approximately proportional to the increase in the square root of the frequency increase. In general, Skin Effect can be disregarded below about 100 MHz. For very small PC Board trace widths (under 0.05 inches) above HF, that may be a factor to consider. The effective RF resistance of a copper conductor trace in Ohms per inch:

$$R_{RF} = \frac{1.31 \cdot 10^{-4}\sqrt{f}}{W + H} \qquad \text{Where:} \qquad (6-31)$$

f is frequency in MHz W is trace width in inches

H is trace height in inches or about 1.4 x Ounce rating of PC Board stock foil; 2 Ounce foil = 0.0028 inches

As an example, a trace width of 0.1 inch on 2 ounce foil at 50 MHz would be about 9 milliOhms per inch. At 500 MHz, that same trace would have an RF resistance of 28.5 milliOhms per inch. Going to 1 ounce foil would double the RF resistance. Doubling the trace width would halve the RF resistance.

As a general rule at high HF and upward, make all wire leads and PC Board traces as big as possible and as short as possible. That avoids any extra frequency-dependent series resistance that might upset Q or impedance-matching problems.

[17] PCB copper foil is graded in *ounces per square foot*. 2-ounce copper foil would be 0.0028 inches thick. 1-ounce foil is 0.0014 inches thick. Some consumer-grade PCBs are made with half-ounce foil.

References for Chapter 6

[13] NBS Circular C74 as reproduced in [7] for inductor construction and formulas, also [4].

[14] "Radiotron Designer's Handbook," F. Kingston-Smith editor, 4th Edition 1952 (published in USA by RCA Corporation, 1960) particularly Chapter 11 for shield can effect on inductance and Q; also in RCA Application Note No. 48, 12 June 1935.

[15] "Practical R.F. Design," by James M. Bryant, 13 April 1994, included in Analog Devices Fall 2000 Communications Solutions CD-ROM, for suggestions on looking at components as they really are and for a reminder of Murphy's Ubiquitous Laws.

[16] NBS Paper 468, 1923, by F. W. Grover, reproduced in Henney [11] on page 132, for straight wire inductance. Wire resistance obtained from wire tables available from any wire and cable manufacturer.

[17] Design Note 122, Linear Technology Corporation, Milpitas, CA, for the approximation of PCB trace line formula (6-30) for low-Ohm PCB resistors. That formula was based on a copper volume resistivity of 1.7241 x 10^{-6} Ohms per cubic centimeter, 0.0027 inch foil thickness, and 97% conductivity to account for electro deposited copper foil versus annealed copper.

[18] Micrometals Inc., 5615 E. La Palma, Anaheim, CA, 92807, Catalog 3 and "Q Curves for Iron Powder Cores," Issue G, December 1997 for information on iron powder materials. On the Internet in 2005 at http://www.micrometals.com.

[19] National Semiconductor, Inc., *Controlling Noise and Radiation in Mixed-Signal and Digital Systems*, by Nicholas Gray, National Semiconductor Web Seminar, 10 December 2002, available from National website http://edge.national.com for data on skin effect of PC Board traces.

Chapter 7

Resonance, Single and Multiple

Resonance occurs when an R-L-C circuit exhibits no phase shift between AC current and AC voltage at an AC frequency. At resonance frequency the network takes the appearance of a pure resistance. *Single resonance* occurs with only one capacitor and one inductor for the in-phase condition at only one frequency. *Multiple resonance* occurs when there exist more than one capacitor or inductor for the in-phase condition at more than one frequency.

The L-C Condition For Resonance

Resonance occurs when the magnitude of inductive reactance is equal to the magnitude of capacitive reactance but their signs are opposed. The following formula applies to all resonances:

$$f = \frac{1}{2\pi\sqrt{LC}} \quad f^2 = \frac{1}{4\pi^2 LC} \quad L = \frac{1}{\omega^2 C} \quad C = \frac{1}{\omega^2 L} \qquad (7\text{-}1)$$

$$\text{where: } \omega = 2\pi f \quad f \text{ in Hz, } L \text{ in Hy, } C \text{ in Fd}$$

If an inductive reactance is positive and capacitance reactance is negative but their magnitudes are equal, they will add to zero.

Series Single Resonance

All components connected in series must be considered first as resistances and reactances, the total described as an impedance. Figure 7-1 shows a simple single series resonance R-L-C circuit. The impedance of that circuit is expressed as:

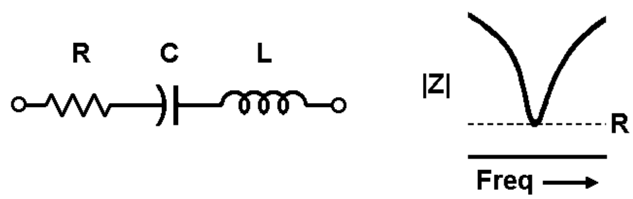

Figure 7-1 - Series resonant circuit (left) and magnitude over frequency (right).

$$Z = R + j\left[(X_L) + (X_C)\right] = R + j\left[(\omega L) - \left(\frac{1}{\omega C}\right)\right] = R + j\left(\frac{\omega^2 LC - 1}{\omega C}\right) \qquad (7\text{-}2)$$

$$\text{where: } Z, R \text{ in Ohms, } L \text{ in Hy, } C \text{ in Fd, } f \text{ in Hz, } \omega = 2\pi f \text{ or radian frequency}$$

At resonance frequency the imaginary part of **Z** reduces to zero and the impedance is purely *resistive*. The series circuit impedance magnitude is minimum at series resonance and the phase angle is zero.

At any frequency below resonance frequency, capacitive reactance magnitude is larger than inductive reactance magnitude and the imaginary part of **Z** is finite and ***negative***. The series R-L-C impedance is said to be ***capacitive below resonance.*** At any frequency above resonance, the inductive reactance magnitude is larger than the capacitive reactance magnitude and the imaginary part is finite and ***positive***. The series R-L-C impedance is said to be ***inductive above resonance***.

Parallel Single Resonance

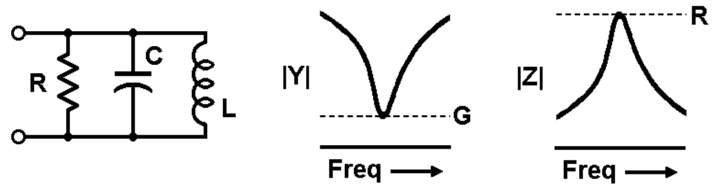

Figure 7-2 - Parallel-resonant circuit (left) with frequency response depicted at right.

All parallel connection components must first be considered as conductances and susceptances with the total described as an admittance. Figure 7-2 shows a simple R-L-C connected for parallel resonance. The magnitude of the circuit is shown as the inverse of admittance magnitude. The admittance of that circuit is expressed as:

$$Y = G + j(B_C + B_L) = G + j\left[(\omega C) - \left(\frac{1}{\omega L}\right)\right] = G + j\left(\frac{\omega^2 CL - 1}{\omega L}\right) \qquad (7-3)$$

where: Y, G, B are in mhos, C in Fd, L in Hy, $\omega = 2\pi f$, f in Hz

At resonance frequency the parallel R-L-C circuit's susceptances add up to zero; admittance magnitude is equal to conductance. Since impedance is the inverse of admittance, the equivalent impedance at resonance is resistive and of maximum magnitude.

At frequencies below resonance capacitive susceptance is larger than inductive susceptance and the imaginary part remains positive. At frequencies above resonance the inductive susceptance is larger than capacitive susceptance and the imaginary part remains negative.

$$Y = G + jB \qquad Z_{Parallel} = \frac{1}{Y} = \left(\frac{G}{G^2 + B^2}\right) - j\left(\frac{B}{G^2 + B^2}\right) \quad \text{so:}$$

$$\left|Z_{Parallel}\right| = \frac{1}{\sqrt{G^2 + B^2}} \qquad \text{If } B = 0 \text{ then: } \left|Z_{Parallel}\right| = \frac{1}{G}$$

At resonance the equivalent parallel R-L-C circuit impedance magnitude is simply the inverse of conductance. That is an important factor for higher-impedance tuned circuits.

The equivalent parallel R-L-C circuit impedance is ***inductive below resonance*** and ***capacitive above resonance***, just the opposite of the series R-L-C circuit.

Effect of Q on a Parallel R-L-C Circuit

If the magnitude of a parallel R-L-C at resonance is equal to the inverse of conductance and that conductance is equal to Q times the magnitude of inverse susceptance then:[1]

$$R_{PL} = \text{equivalent parallel resistance from inductor} = Q_L \, \omega L_P$$

and

$$R_{PC} = \text{equivalent parallel resistance from capacitor} = \frac{Q_C}{\omega C_P}$$

and Q_L is inductor Q and Q_P is capacitor Q, then:

$$R_P = \text{total parallel resistance at resonance} = \omega L_P \left(\frac{Q_L Q_C}{Q_L + Q_C} \right) \qquad (7\text{-}4)$$

What may be more important insofar as selectivity is concerned is ***bandwidth***. Applying a constant AC current across a parallel R-L-C will produce an AC voltage that is maximum at resonance, then drops to lower values on either side of resonance. Bandwidth has been defined as the difference between the ***-3 db*** voltage response frequencies with voltage at resonance being the 0 db reference. Those -3 db frequencies correspond to the equivalent parallel impedance magnitude as 70.7 percent of the magnitude at resonance.[2] Relationship of Q and bandwidth is simple:

$$f_{BW} = \frac{f_{resonance}}{Q_{total}} \qquad \text{Where both f terms have the same multiplier} \qquad (7\text{-}5)$$

and

$$Q_{total} = \frac{Q_C \, Q_L}{Q_C + Q_L}$$

As with equation (7-4) the total Q can be taken as inductor Q alone if the capacitor Q is at least 20 times higher than inductor Q.

Bandwidth can also be defined as the difference between the high and low frequencies of the -3 db response points but those two frequencies are not linear about the resonance frequency. Instead, the resonance frequency can be expressed as the ***geometric center of the bandwidth.*** Those relationships are:

---

[1] Reactance is the negative inverse of susceptance and the resulting two parallel resistances are expressed as Q times the reactance magnitude for convenience (and common expression use). If capacitor Q is at least 20 or more times the inductor Q it can be neglected within 5% accuracy and equivalent parallel resistance be expressed as inductor Q times inductive reactance. The final expression of (7-4) at resonance is equivalent to two resistors in parallel as indicated by the two Q term in the bracket.

[2] The 70.7 percent corresponds to a fraction equal to half the square root of 2 which is equal to a -3.0103 db for purists; *-3 db* for the rest of us. The relationship of Q to bandwidth frequency can be derived independently using the *-3 db* response criterion and may be good math exercise to some. It isn't really needed and the hobbyist designer can accept the Q and bandwidth relationship and use it in full confidence.

$$
\begin{aligned}
f_H &= \text{Higher frequency of } -3 \text{ db response} \\
f_L &= \text{Lower frequency of } -3 \text{ db response} \qquad (7\text{-}6) \\
f_B &= \text{bandwidth between } -3 \text{ db responses } = f_H - f_L \\
f_C &= \text{geometric center frequency of bandwidth } = \text{ resonance frequency}
\end{aligned}
$$

$$f_C = \sqrt{f_H \, f_L} \qquad \text{Then:} \qquad [\text{all frequencies in same units}]$$

$$f_H = \frac{f_C^{\,2}}{f_L} = f_L + f_B \qquad f_L = \frac{f_C^{\,2}}{f_H} = f_H - f_B \qquad \text{and:}$$

$$f_B = \frac{f_C^{\,2} - f_L^{\,2}}{f_L} = \frac{f_H^{\,2} - f_C^{\,2}}{f_H}$$

$$f_C = \sqrt{f_L (f_L + f_B)} = \sqrt{f_H (f_H - f_B)}$$

These bandwidth and geometric center frequency relationships apply equally to series R-L-C circuits.

Effect of Q on a Series R-L-C Circuit

As can be expected, the Q and bandwidth relationships are the same as with a parallel circuit but the impedance magnitude shape is inverted. Consider a constant voltage AC source feeding the series R-L-C circuit. The AC current through the series circuit will be maximum when the impedance magnitude is minimum. That occurs when the magnitudes of reactances are equal. The frequencies at which the current magnitudes are 70.7 percent of the peak at resonance are the low and high frequencies for bandwidth. The major difference lies in the resistance due to Q losses:

If R_{SL} is the series resistance due to inductor having Q_L and
R_{SC} is the series resistance due to capacitor having Q_C and
$R_{Stotal} = R_{SL} + R_{SC}$ for a series circuit impedance at resonance and

$$R_{SL} = \frac{|X_L|}{Q_L} = \frac{\omega L}{Q_L} \qquad R_{SC} = \frac{|X_C|}{Q_C} = \frac{1}{Q_C \, \omega C} \qquad \text{Then:}$$

$$R_{Stotal} = \frac{Q_C \cdot \omega^2 LC + Q_L}{Q_L \cdot Q_C \cdot \omega C} \approx \frac{\omega L}{Q_L} \quad \text{if } Q_C \gg Q_L \qquad (7\text{-}7)$$

The approximation for total series resistance can be used as a trial value or be within 5% of true value if capacitor Q is at least 20 times higher than inductor Q.

Characteristics Away From Resonance

Impedance magnitude at single-circuit resonance frequency is Q times X for parallel resonance, X divided by Q for series resonance. Bandwidth is defined by resonant frequency divided by Q. There are situations where the impedance magnitude needs to be known at other frequencies. The most accurate way is to calculate the impedance magnitude directly but that may be cumbersome. An approximation of response away from resonance is:[3]

$$\text{Relative } |Z| \text{ fraction} = \left| \frac{\gamma}{Q_\gamma\left(\gamma^2 - 1\right)} \right| \qquad (7\text{-}8)$$

Where: Q_γ is total circuit Q at resonance

$$\gamma = \frac{\text{Frequency away from resonance}}{\text{Frequency at resonance}} \qquad \text{[same frequency units]}$$

There are two ways to use this formula. Voltage or current response in db can be found by taking 20 times the logarithm of the fraction. Or the fraction can be multiplied by impedance magnitude at resonance to find the impedance magnitude away from resonance.

The magnitude bars are necessary for frequencies below resonance since the arithmetic result will be negative for a gamma lower than unity.

Let's suppose a parallel circuit has a resonance impedance of 15 KOhms and a total Q of 40. It is desired to find the response at 4 times resonance and 1/4th resonance frequency. On the high side of resonance gamma is 4 and the fraction is 1/150. The impedance would be 100 Ohms and voltage response to a constant current source would be -43.5 db. At 1/4th resonance frequency, gamma would be 0.25 but the fraction is the same: 1/150 or 43.5 ***db down***.[4]

If this were a series resonant circuit with the same total Q, the reactance at resonance would be 375 Ohms inductive but the impedance magnitude would be 9.375 Ohms (375 divided by 40). While a series resonant circuit increases impedance magnitude, the AC current through it is maximum at resonance, tapering off to lesser current on either side of resonance. Use of equation (7-8) depends on what function is desired by a resonant circuit.

Active devices are nearly all constant current sources. A parallel resonant circuit would then *peak* its voltage response at resonance and that voltage could be the control of the next stage if and only if that next stage had a very high impedance input (vacuum tube grids or FET gates).[5] On the other hand, bipolar transistor base-emitter junctions have a rather low input impedance and might be better fed by a series resonant circuit. That seeming incongruity can be solved by ***reactive matching circuits*** that are covered in the next chapter. For now it is useful to show how three reactances may be arranged to be both parallel and series resonant.

---

[3] From Terman, reference [12], page 144, re-arranged for fewer terms.

[4] Using *db down* is the same thing as saying *minus db* in relation to some known peak response. This is common and will occur many places in the rest of this book.

[5] This is getting ahead of the game since tuned amplifiers are discussed in more detail later. This small example is there mainly to show that both parallel and series resonance have a place in tuned circuit design.

Multiple Resonance Circuits

Series-Low in Parallel With a Capacitor

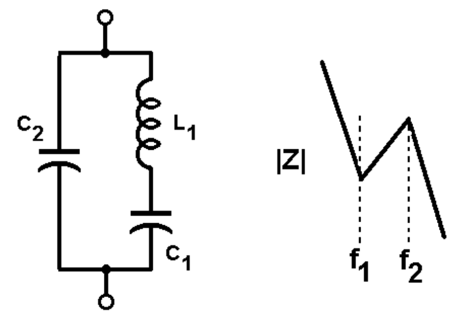

Figure 7-3 Series-resonant pair in parallel with a capacitor.

Figure 7-3 shows a circuit which is series resonant at a lower frequency through L_1-C_1. That arm of the circuit is inductive above its series resonance. That inductance in parallel with C_2 forms a parallel resonant circuit, parallel resonance frequency being above series resonance frequency. Impedance magnitude is approximate to show relative response.

This circuit provides a low impedance at the low frequency and a high impedance at the higher frequency. Component values are:

$$f_2 \rangle f_1 \qquad k = \frac{f_2^{\,2}}{f_1^{\,2}} \quad \text{[k greater than unity]} \quad \text{[all f in Hz]} \qquad (7\text{-}9)$$

$$L_1 = \frac{1}{\omega_1^{\,2}\, C_1} = \frac{1}{\omega_1^{\,2} C_2 (k-1)} \qquad C_1 = \frac{1}{\omega_1^{\,2}\, L_1} = (k-1)\, C_2 \qquad \omega = 2\,\pi\, f$$

$$C_2 = \frac{C_1}{(k-1)} = \frac{1}{\omega_1^{\,2}\, L_1 (k-1)} \qquad \text{L in Hy, C in Fd}$$

As an example, assume the low frequency as 8 MHz and high frequency as 10 MHz. Constant **k** will be 100/64 or 1.56250. Choose C_1 as 100 pFd. The calculations are:

$$\omega_1 = 2\pi \cdot 8 \cdot 10^6 = 50.2655 \cdot 10^6 \qquad \omega_2 = 2\pi \cdot 10 \cdot 10^6 = 62.8319 \cdot 10^6$$

$$L_1 = \frac{1}{\omega_1^{\,2}\, C_1} = \frac{1}{2.52662 \cdot 10^{15} \cdot 100 \cdot 10^{-12}} = \frac{1}{252.662 \cdot 10^2} = 3.95786 \cdot 10^{-6}$$

$$C_2 = \frac{C_1}{k - 1} = \frac{100 \cdot 10^{-12}}{0.56250} = 177.778 \cdot 10^{-12} \qquad \text{[k is dimensionless]}$$

$$C_1 = 100 \text{ pFd} \qquad C_2 = 177.778 \text{ pFd} \qquad L_1 = 3.95786\ \mu\text{Hy}$$

Note that C_2 can be calculated in the same units as C_1 since the only other equation term is **k**, a dimensionless constant dependent on the square of the ratio of the two frequencies.

The two frequencies of resonance are somewhat far apart in this example (a ratio of 1.25:1). Their impedance magnitudes may be taken as values obtained for individual single resonance circuits. At closer frequency spacings this cannot be done since the components will interact more. Using the tools of the preceding chapter, the Figure 7-3 circuit impedances and admittances (for inductive Q of 50 and capacitive Q of 500) are:

At the 8 MHz frequency:

$$Z_{L1} = 3.97887 + j\,198.944 \qquad Z_{C1} = 0.397887 - j\,198.944$$

$$Z_{\text{series arm total}} = 4.37676 + j\,0.0 \qquad \text{[series arm resonance, zero reactance]}$$

$$Y_{\text{series arm total}} = 0.228480 + j\,0.0 \text{ mhos} = Y_1$$

$$Z_{C2} = 0.223811 - j\,111.906 \text{ Ohms} \qquad Y_{C2} = 17.8721\cdot10^{-6} + j\,8.93606\cdot10^{-3} \text{ mhos}$$

$$Y_1 + Y_{C2} = 0.228480 + j8.93606\cdot10^{-3} \text{ mhos} = Y_T$$

$$Z_T = \frac{1}{Y_T} = 4.36973 - j\,0.170891 \text{ Ohms} = 4.37307 \text{ Ohms}\angle -2.23958°$$

Note that the total impedance is slightly reactive. That is due to C_2 in parallel. Minimum impedance is very slightly off, 0.084% lower than the series arm. The phase angle is slightly negative, not quite zero. The phase angle would become zero at a slightly higher frequency if phase is of importance.

At the 10 MHz frequency:

$$Z_{L3} = 4.97359 + j\,248.680 \qquad Z_{C1} = 0.318310 - j\,159.155$$

$$Z_1 = 5.29190 + j\,89.525 \text{ Ohms} \qquad \text{[series arm total impedance]}$$

$$Y_1 = 657.974\cdot10^{-6} - j\,11.1312\cdot10^{-3} \text{ mhos}$$

$$Y_{C2} = 22.3402\cdot10^{-6} + j\,11.1701\cdot10^{-3} \text{ mhos}$$

$$Y_T = 680.315\cdot10^{-6} + j\,38.9291\cdot10^{-9} \text{ mhos}$$

$$Z_T = 1465.11 - j\,83.8368\cdot10^{-3} \text{ Ohms} = 1467.51 \angle -3.27502°$$

Again, this is not a perfect solution but the error is quite small and shows up mainly in the relatively small phase angle. Parallel resonance is still very close to the specified, much closer than the tolerances of available components.

In a practical application it is advisable to make one component in the series arm fixed, then make the other two components adjustable for exact series and parallel resonance..

Series-Pair in Parallel With an Inductor

This version shown in Figure 7-4 takes advantage of the series arm being capacitive at frequencies below series resonance. The second inductor, L_2, parallel resonates with that capacitance. Impedance magnitudes over frequency will be a left-for-right version of the circuit of Figure 7-3.

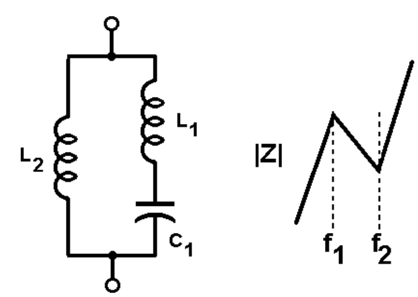

Figure 7-4 Series resonant pair in parallel with an inductor.

As with the preceding circuit, fixing one component value allows calculating the remaining two components. The component calculations are:

$$f_2 \rangle f_1 \quad m = \frac{f_1^2}{f_2^2} \quad \text{[m is less than unity]} \quad \text{[frequencies in Hz]} \qquad (7-10)$$

$$\omega_1 = 2\pi f_1 \qquad \omega_2 = 2\pi f_2$$

$$L_1 = \frac{1}{\omega_2^2 C_1} = \frac{m L_2}{(1-m)} \qquad C_1 = \frac{1}{\omega_2^2 L_1} = \frac{1-m}{m \omega_2^2 L_2}$$

$$L_2 = \frac{(1-m) L_1}{m} = \frac{(1-m)}{m \omega_2^2 C_1} \qquad \text{[L in Hy, C in Fd]}$$

For an example, assume the same frequencies as before, 8 and 10 MHz, the single capacitor at 100 pFd. Constant **m** is less than unity and equals 0.64 (constants picked for the parallel resonance):

$$L_1 = \frac{1}{\omega_2^2 C_1} = \frac{1}{3.94784 \cdot 10^{15} \cdot 100 \cdot 10^{-12}} = \frac{1}{394784} = 2.53303 \cdot 10^{-6}$$

$$L_2 = \frac{(1-m) L_1}{m} = \frac{(1-0.64)\ 2.53303 \cdot 10^{-6}}{0.64} = \frac{911.891 \cdot 10^{-9}}{0.64} = 1.42483 \cdot 10^{-6}$$

At 10 MHz the series resonance dominates total impedance:

$$Z_{L1} = 3.18310 + j\ 159.155 \text{ Ohms},\ Z_{C1} = 0.31831 - j\ 159.155 \text{ Ohms, so:}$$

$$Z_1 = 3.5014.1 + j0 \text{ Ohms and } Y_1 = 0.285599 + j0 \text{ mhos}$$

$$Z_{L2} = 1.79049 + j\ 89.5247 \text{ Ohms and } Y_{L2} = 223.313 \cdot 10^{-6} - j\ 11.1656 \cdot 10^{-3} \text{ mhos}$$

$$Y_{total} = Y_1 + Y_{L2} = 285.822 \cdot 10^{-6} - j\ 11.1656 \cdot 10^{-3} \text{ mhos}$$

$$Z_{total} = 3.49335 + j\ 136.467 \cdot 10^{-3} \text{ Ohms} = 3.49601 \angle 2.23711°$$

At 8 MHz the impedances and admittances are:

$$Z_{L1} = 2.54648 + j\ 127.324 \text{ Ohms},\ Z_{C1} = 0.397887 - j\ 198.944 \text{ Ohms}$$

$$Z_1 = 2.94437 - j\ 71.620 \text{ Ohms and } Y_1 = 573.047 \cdot 10^{-6} + j\ 13.9390 \cdot 10^{-3} \text{ mhos}$$

$$Y_{L2} = 279.141 \cdot 10^{-6} - j\ 13.9570 \cdot 10^{-3} \text{ mhos}$$

$$Y_{total} = 952.188 \cdot 10^{-6} - j\ 18.0489 \cdot 10^{-6} \text{ mhos}$$

$$Z_{total} = 1172.92 + j\ 24.8420 \text{ Ohms} = 1173.19 \angle 1.21332°$$

Again zero phase angle occurs at a slight frequency difference from the specified resonance frequencies. For this example those frequencies are 8.00184 MHz (+0.023%) and 9.9957 MHz (-0.043%), not a gross error. Note that this configuration has a DC path through L_2.

Parallel Pair in Series with Capacitor

The Figure 7-5 version takes advantage of the parallel resonant portion being inductive below resonance. The series capacitor will resonate with that inductance at a lower frequency. Impedance will be maximum at the higher frequency and minimum at the lower frequency. Calculations are:

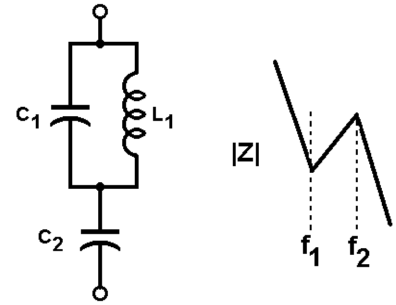

Figure 7-5 Parallel resonant pair in series with a capacitor.

$$f_2 \rangle f_1 \qquad k = \frac{f_2^2}{f_1^2} \quad \text{[k will be greater than unity] [f in Hz]} \qquad (7\text{-}9)$$

$$\omega_1 = 2\pi f_1 \qquad \omega_2 = 2\pi f_2$$

$$C_1 = \frac{1}{\omega_2^2 \cdot L_1} = \frac{C_2}{(k-1)} \qquad L_1 = \frac{1}{\omega_2^2 \cdot C_1} = \frac{(k-1)}{\omega_1^2 \cdot C_2}$$

$$C_2 = (k-1)\,C_1 = \frac{(k-1)}{k\,\omega_1^2\,L_1} \qquad \text{[C in Farads, L in Henries]}$$

As an example, assume the previous example frequencies with C_1 as 100 pFd, capacitive Qs of 500 and inductive Q of 50. Radian frequency values are as before and **k** is 1.5625. L_1 will be 2.53303 µHy and C_2 calculates as 56.25 pFd. Impedances and admittances are then:

At 10 MHz:

$$Y_{C1L1} = 138.230 \cdot 10^{-6} + j\,0 \quad \text{[susceptances cancel each other]}$$
$$Z_{C1L1} = 7234.32 + j\,0 \text{ Ohms}$$
$$Z_{C2} = 0.565884 - j\,282.942 \text{ Ohms}$$
$$Z_{total} = 7234.88 - j\,282.942 \text{ Ohms} = 7240.41 \angle -2.23958°$$

At 8 MHz:

$$Y_{C1} = 10.0531 \cdot 10^{-6} + j\,5.02655 \cdot 10^{-3} \text{ mhos}$$
$$Y_{L1} = 157.080 \cdot 10^{-6} - j\,7.85398 \cdot 10^{-3} \text{ mhos}$$
$$Y_{C1L1} = 167.133 \cdot 10^{-6} - j\,2.82743 \cdot 10^{-3} \text{ mhos}$$
$$Z_{C2} = 0.565884 - j\,282.942 \text{ Ohms}$$
$$Z_{total} = 21.5409 - j\,1.23142 \text{ Ohms} = 21.5761 \angle -3.27184°$$

Again, the phase angles are not quite zero but the minimum and maximum impedance magnitude frequencies are less than a tenth percent from those specified.[6]

Parallel Pair in Series with Inductor

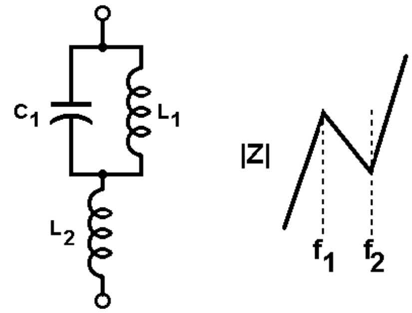

Figure 7-6 Parallel resonant pair in series with an inductor.

The Figure 7-6 version takes advantage of the parallel resonant portion being capacitive above resonance. The series inductor will resonate with that capacitance at a higher frequency. Impedance will be maximum at the lower frequency and minimum at the higher frequency.

This network provides a DC path through the inductors. Calculations are:

$$f_2 \rangle f_1 \quad m = \frac{f_1^{\,2}}{f_2^{\,2}} \quad \text{[m is less than unity]} \quad \text{[f in Hz]} \qquad (7\text{-}12)$$

$$\omega_1 = 2\pi f_1 \qquad \omega_2 = 2\pi f_2$$

$$L_1 = \frac{1}{\omega_1^{\,2}\, C_1} = \frac{(1-m)\, L_2}{m} \qquad C_1 = \frac{1}{\omega_1^{\,2}\, L_1} = \frac{m}{(1-m)\,\omega_1^{\,2}\, L_2}$$

$$L_2 = \frac{m\, L_1}{(1-m)} = \frac{m}{(1-m)\,\omega_1^{\,2}\, C_1} \qquad [\,\text{L in Hy, C in Fd}\,]$$

Note that the term groupings of all four 3-component circuits are similar but are arranged differently. Some care must be exercised in selecting the right equation group for its circuit.

As another example the same two frequencies are kept, the same Qs, and the same C_1. The component values, admittances and impedances are then:

$$m = 0.64 \quad C_1 = 100\,\text{pFd} \quad L_1 = 3.95786\,\mu\text{Hy} \quad L_2 = 7.03619\,\mu\text{Hy}$$

$$Q_C = 500 \quad Q_L = 50$$

At 10 MHz:

$$Y_{C1\text{-}L1} = 138.230\cdot 10^{-6} + j\,0 \text{ mhos} \qquad Z_{C1\text{-}L1} = 7234.32 + j\,0 \text{ Ohms}$$

$$Z = 0.565884 - j\,282.942 \text{ Ohms}$$

$$Z = 7234.88 - j\,282.942 \text{ Ohms} = 7240.41 \angle -2.23958°$$

---

[6] Some slight differences arise due to the calculation digit accuracy. All of these examples used 10-digit precision calculations with 6-digit accuracy component values. The frequency error, small as it is, comes largely from the models to show effect of Q and the rather simple derivation of the constants for calculating unknowns. The errors are very small and will not show up in practical circuits using one percent tolerance components. The method of derivation of the component calculations are given in Appendix 7-1.

At 8 MHz:

$$Y_{C1} = 10.0531 \cdot 10^{-6} + j\,5.02665 \cdot 10^{-3} \text{ mhos}$$
$$Y_{L1} = 157.080 \cdot 10^{-6} - j\,7.85398 \cdot 10^{-3} \text{ mhos}$$
$$Y_{C1L1} = 167.133 \cdot 10^{-6} - j\,2.82743 \cdot 10^{-3} \text{ mhos}$$
$$Z_{C1L1} = 20.8335 + j\,352.447 \text{ Ohms}$$
$$Z_{C2} = 0.565884 - j\,282.942 \text{ Ohms}$$
$$Z_{total} = 21.5409 - j\,1.23142 \text{ Ohms} = 21.5761 \angle -3.27184°$$

Example calculations appear to be number-crunching drudgery. A frequency-domain computer network analysis program could have found the answers almost immediately with much more frequency response detail. Frequencies far from any resonances may be important to overall circuit or system block performance.

Two Parallel Resonants in Series

The circuit of Figure 7-7 provides a peak |Z| at two frequencies with a slight |Z| dip in-between; impedance magnitude well below, well above resonances is low. Impedance of the L_1 and C_1 pair will be capacitive above its parallel resonant frequency. The impedance of the L_2 and C_2 pair will be inductive below its parallel resonant frequency. The combination produces a series resonant condition in the geometric center between the two parallel resonance frequencies.

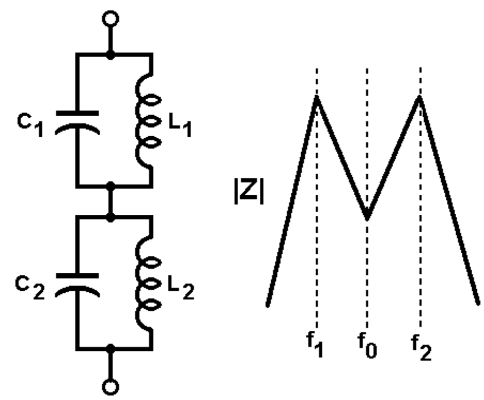

Figure 7-7 Two parallel resonant pairs in series provide a center dip.

$$f_2 \rangle f_0 \rangle f_1 \qquad k = \frac{\omega_0^2}{\omega_1^2} = \frac{\omega_2^2}{\omega_0^2} \qquad \omega_0 = \sqrt{\omega_1\,\omega_2} \qquad (7\text{-}13)$$

$$C_1 = \frac{1}{\omega_1^2\,L_1} = \frac{1}{\omega_0^2\,L_2} = \frac{k}{\omega_0^2\,L_1} = k\,C_2$$
$$C_2 = \frac{1}{\omega_2^2\,L_2} = \frac{1}{\omega_0^2\,L_1} = \frac{1}{k\,\omega_0^2\,L_2} = \frac{C_1}{k}$$
$$L_1 = \frac{1}{\omega_1^2\,C_1} = \frac{1}{\omega_0^2\,C_2} = \frac{k}{k\,\omega_0^2\,C_1} = k\,L_2$$
$$L_2 = \frac{1}{\omega_2^2\,C_2} = \frac{1}{\omega_0^2\,C_1} = \frac{1}{k\,\omega_0^2\,C_2} = \frac{L_1}{k}$$

Note that **k** must be greater than unity. Note also that substitution of the inverse of **k** (1/**m**) would result in equation set (7-13). This is not unusual in such symmetrical networks having an equal number of capacitors and inductors.

As an example, assume the same frequencies as before and also the same component values. Constant **k** will be 1.5625. The impedances in Ohms at the three frequencies will be:

$$\text{at } f_1: \quad Z_1 = 24.13\text{K}\angle +0.32° \; = \; 24.13\text{K} + j\,135.2$$
$$\text{at } f_0: \quad Z_0 = \; 117.9\angle +0.01° \; = \; 117.9 + j\,0.0264$$
$$\text{at } f_2: \quad Z_2 = 24.14\text{K}\angle -2.41° \; = \; 24.12\text{K} - j\,1.016\text{K}$$

Two Series Resonants In Parallel

Figure 7-8 has the L_1 and C_1 pair series resonant at the lower frequency so they will be inductive above that frequency. The L_2 and C_2 pair are series resonant at the higher frequency and will be capacitive at a frequency lower than series resonance. The combination provides a parallel resonance at geometric center frequency.

In general this is not a practical configuration but the equations are included to show its duality to Figure 7-7:

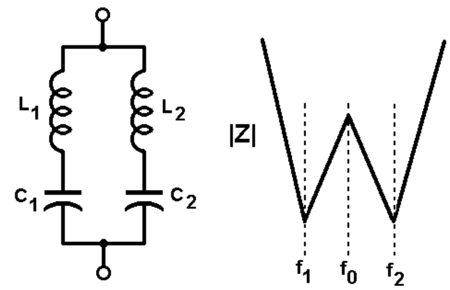

Figure 7-8 Two series resonants in parallel for a center peak.

$$f_2 > f_0 > f_1 \qquad m = \frac{\omega_1^{\,2}}{\omega_0^{\,2}} = \frac{\omega_0^{\,2}}{\omega_2^{\,2}} \qquad \omega_0 = \sqrt{\omega_1\,\omega_2} \qquad\qquad (7\text{-}14)$$

$$C_1 = \frac{1}{\omega_1^{\,2}\,L_1} = \frac{1}{\omega_0^{\,2}\,L_2} = \frac{1}{m\omega_0^{\,2}\,L_1} = \frac{C_2}{m}$$

$$C_2 = \frac{1}{\omega_2^{\,2}\,L_2} = \frac{1}{\omega_0^{\,2}\,L_1} = \frac{m}{\omega_0^{\,2}\,L_2} = m\,C_1$$

$$L_1 = \frac{1}{\omega_1^{\,2}\,C_1} = \frac{1}{\omega_0^{\,2}\,C_2} = \frac{1}{m\,\omega_0^{\,2}\,C_1} = \frac{L_2}{m}$$

$$L_2 = \frac{1}{\omega_2^{\,2}\,C_2} = \frac{1}{\omega_0^{\,2}\,C_1} = \frac{m}{\omega_0^{\,2}\,C_2} = m\,L_1$$

$$[\,\text{C in Fd}, \;\; \text{L in Hy}\,]$$

To use (7-14) requires picking two of the three frequencies and then selecting one component value. The other three component values will then calculate out as given. Note that **m** must be less than unity. As an example, assume a parallel resonance at 3.75 MHz and a series resonance at 3.0 MHz plus C_1 fixed at 100 pFd:

f_0 = 3.75 MHz (center frequency) and f_1 = 3.0 Mhz so $k = \dfrac{f_1^2}{f_0^2} = \dfrac{9}{14.0625} = 0.640$

If C_1 = 100 pFd, then C_2 = 64 pFd. That yields L_1 = 28.1448 µHy for the low frequency resonance. The high frequency series resonance is 4.6875 MHz and comes from: $\varpi_2 = \dfrac{\omega_0^2}{\omega_1}$ or $f_2 = \dfrac{f_0^2}{f_1} = \dfrac{14.0625}{3} = 4.5875$

That yields 18.0127 µHy for L_2 resonant with 64 pFd C_2 at 4.6875 MHz.

If the inductive Q is 50 and capacitive Q is 500, the impedances in Ohms will be:[7]

$$\text{at } f_1 \quad Z_1 = 11.66 - j\,0.2753 = 11.66 \angle -1.35°$$
$$\text{at } f_0 \quad Z_0 = 2388 - j\,43.48 = 2388 \angle -1.04°$$
$$\text{at } f_2 \quad Z_2 = 11.66 + j\,0.2808 = 11.66 \angle +1.38°$$

Impedance magnitude will increase below the lowest frequency and again beyond the highest frequency. Bandwidth is approximately equal to that resulting from combined Q times the inductive reactance of the L_1 and C_1 pair at 3.75 MHz. This may not be good for a high-impedance application but a circuit as a series network as a divider with a very low impedance load will pass maximum current at two frequencies.

Inductor Self-Resonance Problems With Decoupling

All inductors have some *distributed capacity* between turns, between windings, and to the core material (if used). This takes the effect of a capacitance in parallel with the inductor to make it *self resonant* at a frequency well above the range it is used. Above the self-resonant frequency the combination is capacitive, just the opposite of what was desired.

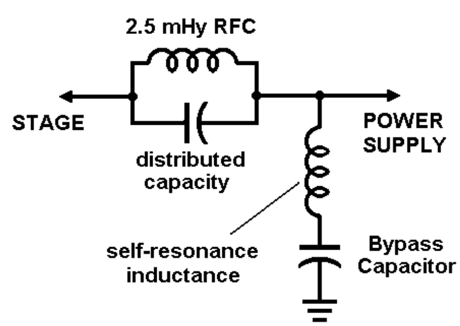

Figure 7-9 A power supply decoupler with some problems.

In very early radio the term *RF Chokes* or *RFCs* were used for DC coupling while blocking RF flow, as in the plate lead of a vacuum tube to HV supply. A *bypass* capacitor was added at the junction of the RFC and HV supply lead to direct any RF current flow to ground. This common arrangement is known as a *decoupling network* to isolate a stage from any RF going to other stages or vice-versa. An example is that of Figure 7-9.

A typical RFC of an older radio might use 2.5 mHy RFCs of special construction to lessen

[7] Obtained from modeling circuit in LINEA, a frequency-domain circuit analysis program whose solution answers are in 14-digit accuracy..

their intrinsic distributed capacity.[8] If the distributed capacity is as little as 4 pFd, it would resonate with 2.5 mHy as low as 1.6 MHz, the top of the AM BC band.[9] Above self-resonance, the component would appear to be a capacitor, not an inductor, so it would not block any RF signals. One reason why such circuits managed to work is that the distributed capacity became part of the interstage tuned circuits to the following stage...and decoupling per se depended entirely on the bypass capacitor.[10]

Capacitors aren't pure in that they have some internal series inductance. Monolithic *chip* (SMD) and *disk* (very high dielectric-constant insulation, minimum layers of conductors) capacitors have the least internal inductance. Monolithic mica-dielectric capacitors are next, followed by tubular-structured multi-layer capacitors. See Figure 7-10 for more on tubulars.

The effectiveness of the decoupling can be measured in terms of decibels between a constant input RF voltage versus the RF voltage at the power supply. This is tabulated following for two values of bypass capacitor: 0.001 µFd and 0.01 µFd.

The reason for decoupling can be seen by considering the circuit block versus other circuit blocks in the system. If the circuit block is an amplifier which is preceded by another amplifier, the second amplifier should not feed back any signal to the first. If such feedback happens, the total gain can be reduced through *negative feedback* (signal is out of phase) or oscillation can happen if there is *positive feedback* (signal in phase).[11] Designers should anticipate such possibilities.

Approximate Decoupling in db of Circuit in Figure 7-9

Frequency, MHz	0.001 µFd Bypass	0.01 µFd Bypass
1.0	-50	-60
1.3	-58	-74
1.6	-90	-98
2.0	-59	-76
2.4	-55	-73
3.4	-50	-69
16	-48	-67

[8] Two methods were used: *Scramble-winding* of wire on multiple-layer inductors (as opposed to neat, linear windings) greatly lessened distributed capacity between adjacent winding layers; *pie-layering* of several scramble-wound inductors on the same form had the same action as using several lower-inductance coils in series.

[9] Most of the old 2.5 mHy *4-pie* RFCs had distributed capacities on the order of 1 pFd or self-resonant at about 3.2 MHz.

[10] The colloquial name of *RF* ***Choke*** took root with some hobbyists who sincerely believed it would be a perfect inductor way, way up in frequency. Too many were hung up on the name instead of realizing that an RFC was just an inductor like all inductors. The *name* only applied to how the inductor was used

[11] There is more on *feedback* beginning in the basic amplifier chapter.

Capacitor Self-Resonance Through Construction

Inductors have only three major component concerns: Distributed capacity, effect of core material or shielding (if used), and wire resistance affecting Q. Capacitors have more and depend largely on the materials used and the fabrication structure itself. A capacitor is merely two conductor plates separated by a dielectric (air space or a physical insulating material), but the component structure varies considerably depending on the type. Capacitor types from worst to best RF applications are: Electrolytic or tantalum, tubular, disk ceramic, silver-mica or square ceramic, air or vacuum. The type names refer generally to the internal dielectric material.

Electrolytic and tantalum capacitors should be restricted to frequency applications below about 50 KHz. Behavior above audio range is unpredictable due to the wide variety of manufacturing techniques. Manufacturers do specify an *ESR* or Equivalent Series Resistance in Ohms, usually at a primary AC power line frequency or multiple thereof. Some electrolytics may appear as inductors with a series resistance above the audio frequency range. If electrolytics are considered as bypasses for RF circuits they should have additional parallel ceramic or mica dielectric capacitors in parallel to minimize self resonances. The terms *electrolytic* or *tantalum* refer to the dielectric material with additive to increase capacity. The additive, or electrolyte, makes them *polarized*; i.e., the proper voltage potential must exist between the plates. So-called *unpolarized* electrolytics are really two in opposing-polarity series.

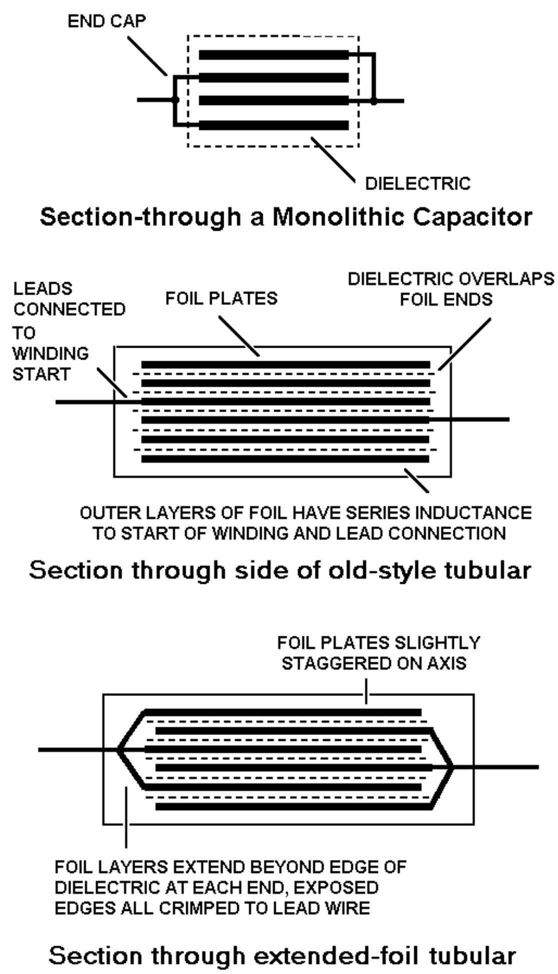

Figure 7-10 Comparison of monolithic and tubular capacitor type construction.

Tubular capacitors are made with metal foil plates separated by a variety of different dielectric film material: Wax-impregnated paper, cellulose acetate (acetate-impregnated paper), Mylar, Teflon, polystyrene, polyethylene. That order represents the worst-to-best temperature stability. The foils and dielectric film is wrapped into a tubular cylinder shape, then encased in a dielectric sleeve. The act of wrapping adds an internal distributed inductance.

Figure 7-10 shows a comparison of monolithic structure capacitors (ceramic, mica) and two types of tubulars. The middle section-through represents the old style where the dielectric film was extended to aid in insulation between the foil plates. That structure also has the highest distributed inductance. To mnimize the inductance, the *extended-foil* structure lets the plate foils extend on each end and crimps the leads to the foil extension. The leads can now make contact with each plate at all layers of winding and distributed inductance is at a minimum.

Ceramic disk capacitors are inexpensive but their leads come out in-line and connect to the fired-on plate in the center. That results in more *lead inductance* than the axial-lead monolithic capacitors. Consider a 0.01 µFd disk capacitor with one inch of total lead length of #22 AWG bare wire. According to the straight-wire inductance table of the preceding chapter (Table 6-3) those leads add 21.9 nanoHenry series inductance. Such a wired-in disk capacitor would have a series resonance frequency of about 10.8 MHz. While that would be good for an FM receiver IF subsystem (typically at10.7 MHz), that series circuit would appear as an *inductance* of 14.7 Ohms reactance at 108 MHz. That would not be good for bypassing the tuner (88 to 108 MHz) subsection.

The obvious cure is to use shorter leads for bypass capacitors. Table 7-1 shows the self-resonance of some typical capacitor values with two different total lengths of #22 AWG straight wire.

Table 7-1

Series Resonant Frequencies in MHz of Capacitors with #22 Straight Wire

Capacitance, µFd	1 Inch Length	1/2 Inch Length	1/4 Inch Length
0.1	3.4	5.2	8.3
0.047	5.0	7.7	12.1
0.022	7.3	11.2	17.6
0.01	11	16.6	26
0.0047	16	24	38
0.0022	23	35	56
0.001	34	53	83
0.00047	50	77	121
0.00022	73	112	176
0.0001	107	166	262

The thought might come to mind now that two capacitors with short leads should be better. The answer to that is both yes and no. Two capacitors of the same value will lower the impedance magnitude but the series resonance remains and the impedance will go inductive above the series resonance frequency. So, what will happen with two capacitors of different values?

Two capacitors of different values but with approximately the same lead length will form a two series resonant circuit described in Figure 7-7. While the impedance magnitude is minimum at two frequencies there is a magnitude maximum between the two series resonance frequencies. With three capacitors of successively decreasing value there will be three minima but two maxima.

The saving grace in decoupling is that one can pick the series component as inductive or resistive or both to fit the shunt impedance of the bypass capacitor. Caution must be exercised there. With the improper selection, an inductor in series with supply line will show some capacitive impedance at one frequency. If the bypass capacitor has an inductive impedance (due to lead length) at that frequency, the decoupling network has become a highpass filter that will not be an effective decoupler. There is more on decoupling under amplifier chapters later.

What should be noted is that all capacitors have some *parasitic* conditions of inductance at higher frequencies. As with inductors and their parasitic capacitance, all must be considered in any circuit use at frequencies above intended bands.

A Few Problems With Resistors

Wire-wound resistors should not be used at RF regardless of winding methods attempting to reduce inductance, especially so with DC resistances of about a few hundred Ohms or lower. In some cases of about 25 Watt dissipation and lower wire-wounds, their outer metal casings reduce the inductance but add a capacitance between resistor and mounting surface.

Some metal film resistors have an inductance due to the method of achieving resistance. Such a resistor blank is fired with a solid surface of resistance material and the end caps and leads attached. Either a grinder, air blast, or high power laser then grooves the resistance surface to change the conduction path. That results in several turns of resistance material, more for higher DC resistance, fewer for less DC resistance. The process may be automated for a precise resistor tolerance. The protective coating usually applied makes it difficult to determine the number of turns. Those have to be measured on a Q Meter to determine the inductance; the DC resistance material will be in parallel with the inductance when so measured..

Metal film resistors in surface mount packages generally have a solid, no-groove structure that minimizes any internal inductance. Some axial lead metal film resistors are made with a continuous resistance film. Both of those types are good at RF as resistors.

Carbon composition resistors are constructed of a solid cylinder of resistance material, end caps and leads attached, the whole encapsulated in a hard plastic. The end cap capacity, in parallel to the resistor, may be 0.2 to 0.5 pFd for 1/10th to 1/2 Watt dissipation, twice that for 1 Watt versions, almost twice more for 2 Watt types. The variation depends on the style of end caps. Such case capacitances can be easily measured on high DC resistance value resistors. Low DC resistance values from the same manufacturer will generally have common end cap style so a measurement on high value models should suffice. Although an older resistor type, carbon composition resistors exhibit the least internal reactance and are good at RF.

Reisistance-film potentiometers introduce an extra inductance from the variable-position contact structure. This inductance will vary with potentiometer setting. Multi-turn rotary types (for manual control) generally use wire-wound resistance elements. Trimmer potentiometers not for manual control are generally of the metal-film or carbon variety.

Appendix 7-1

Derivation of the Multiple-Resonance Circuit Formulas

These were done as close approximations to lessen design calculation work. Exact calculations aren't necessary unless one is bound to a classroom desk. The purpose here, and throughout the book is to get *in the ballpark* as the colloquial expression goes. The two-frequency circuit of Figure 7-5 is used as an example of the method used to obtain equation set (7-11).

Defined: $f_2 \rangle f_1$ $\qquad k = \dfrac{f_2^{\,2}}{f_1^{\,2}} = \dfrac{\omega_2^{\,2}}{\omega_1^{\,2}}$

and $\omega_2^{\,2} L_1 C_1 = 1$ for the parallel resonance at f_2.

So far, that relates L_1 and C_1 but the relationship of C_2 must be found. That is obtained by a conjugate match of C_2 reactance to the reactance of the L_1 and C_1 combination at f_1. Neglecting the conductances, the susceptance match is:

$$\omega_1 C_2 = \frac{1 - \omega_1^{\,2} L_1 C_1}{\omega_1 L_1} \qquad \text{[right side negative for conjugate match]}$$

$$\omega_1^{\,2} C_2 L_1 = 1 - \omega_1^{\,2} L_1 C_1 \qquad \text{since } \omega_1^{\,2} = \frac{\omega_2^{\,2}}{k} :$$

$$C_2\, \omega_2^{\,2} L_1 = k - \omega_2^{\,2} L_1 C_1 \qquad \text{but } \omega_2^{\,2} L_1 C_1 = 1 \quad \text{so:}$$

$$C_2\, \omega_2^{\,2} L_1 = k - 1 \qquad \text{substituting} \quad \omega_2^{\,2} L_1 = \frac{1}{C_1} :$$

$$C_2 = (k-1)\, C_1$$

With C_2 and C_1 relationship established, the relationships to either frequency are through identities.

Note that the conjugate match deliberately ignored the conductances of admittance. That makes the derivation one of an *approximation* rather than a complete solution. The approximation works very well in the three-component, two-resonance circuits provided the capacitor Qs and higher than inductor Qs and the lowest Q is above about 30 and the frequency separation has a 1.2:1 or greater ratio. That fits most applications involving discrete capacitors and inductors. Closer frequency spacing is possible but requires much higher Qs possible only with quartz crystal resonators.

A Note On Simple Conjugate Matching

The *lossless* matching technique (neglecting real-part values) works well with simple circuits to obtain good approximations. The derivation of the Figure 7-5 circuit formulas used the susceptance combination of the parallel L-C pair in a conjugate match with the susceptance of the series capacitor. That's really a matter of convenience in term reductions. In the lossless case derivations susceptance is the negative inverse of reactance and vice-versa.

Derivation of the Figure 7-4 circuit equation set (7-10), the reactance of the series pair was equated to the reactance of the parallel inductor by:

$$\omega_1 L_2 = \frac{1 - \omega_1^2 L_1 C_1}{\omega_1 C_1} \quad \text{[right - hand term negative for conjugate match]}$$

Note the denominator term to the right and how it differs from the derivation for equation set (7-11):

$$\omega_1 C_2 = \frac{1 - \omega_1^2 L_1 C_1}{\omega_1 L_1}$$

For equation set (7-11) the negative susceptance of the parallel pair was equated with the susceptance of the series capacitor. While the numerator on the right sides are the same, the denominator differs.

The real difference comes from the way the pairs are expressed. For a series pair the reactance is:

$$X_{total} = \omega L - \left(\frac{1}{\omega C}\right) = \frac{\omega^2 L C - 1}{\omega C}$$

Whereas for a parallel pair the susceptance is:

$$B_{total} = \omega C - \left(\frac{1}{\omega L}\right) = \frac{\omega^2 L C - 1}{\omega L}$$

It is possible to get confused between reactance and susceptance, winding up with an unworkable equation. The slight (apparent) difference in the denominators changes the polarities on either side of resonance. It must be remembered that a susceptance is the *negative inverse* of reactance and vice-versa.

Chapter 8

Two-Component Matching Networks

Two-component matching networks behave similarly to series or parallel resonant circuits in that both work at a single frequency and seek to make RF current in phase with RF voltage. The difference between them and resonant circuits is that simple matching circuits can use reactances to both match a complex impedance or to achieve an impedance step-up or step-down or do both at one frequency. These two-component matching networks are at the heart of modern automatic-tuning antenna matching subsystems as well as being used for interstage and interblock matching.
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The General Matching Case

The whole purpose of a matching network is to maximize power transfer from a source to a load.. Reactances do not dissipate power in an ideal component model. Maximum power transfer occurs when the load is made to ***appear resistive*** to the source and also to appear as ***the same resistance*** as the source. In either ideal case, load power will be the ***same*** as the source power when matched, maximum power transferred from source to load.

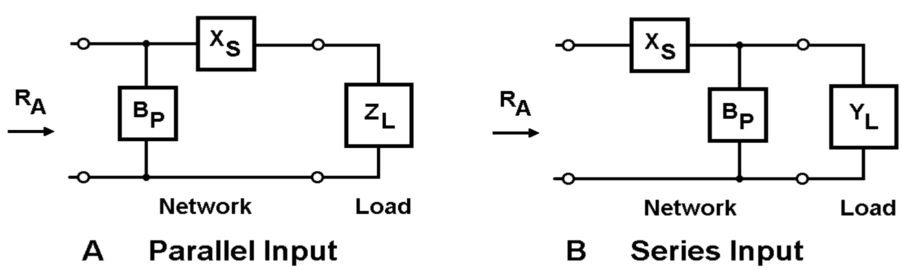

Figure 8-1 The two possible 2-reactance matching configurations. (A) is used when load impedance resistive part is higher than source resistance. (B) is used when load impedance resistive part is lower than source resistance. Each matches at only one frequency.

Figure 8-1 (A) shows a two-component reactance configuration used for cases when the load impedance resistive part is ***higher*** than the source resistance. This is referred to as a *parallel input* network Figure 8-1 (B) is the *series input* used for cases when the load impedance resistive part is ***lower*** than source resistance. The configuration must be one way or the other.

Note that L and C components are represented as blocks which are reactances if in series connection or susceptances if connected in parallel or shunt. This is for ease of calculation. In Figure 8-1 (A) there could be a shunt capacitor and series inductor or a shunt inductor and a series capacitor. The arrangement would depend on the load impedance. The same is true for Figure 8-1 (B).

There are four more possible 2-component matching circuits, each all-inductor or all-capacitor. Those have been omitted because their *range* of matching is much less than L-C or C-L combinations. If needed they can be derived in the methods explained.

Parallel-Input L-Section Matching Network

Figure 8-2 shows the configuration when the load impedance resistive part is lower than the amplifier (source) resistance.[1]

$$R_A \geq R_L$$

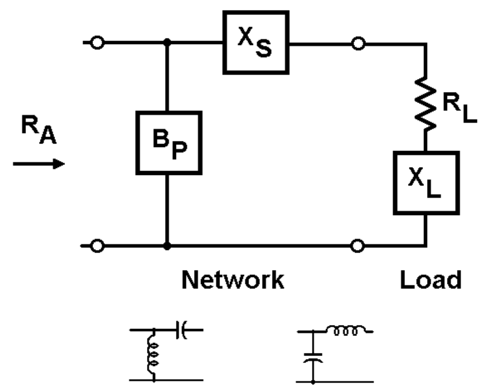

Figure 8-2 Parallel input network when load resistance is lower than source. Small diagrams at bottom show the two possible configurations of L and C for parallel input.

If the load reactance is capacitive, then the series matching section reactance must be inductive. If the load reactance is inductive, then the series matching section reactance must be capacitive. The magnitudes can be calculated by:

$$|X_S| = \sqrt{R_L(R_A - R_L)} + |X_L|$$

Resistance and reactance are in same units of Ohms.

If the load reactance is slightly capacitive then it is still possible to have a series capacitor. Most cases will have a capacitor if the load reactance is inductive and vice-versa. The parallel component at the input will be the opposite of the series component: If a series inductor then the parallel is a capacitor; if a series capacitor then the parallel component is an inductor. Parallel component magnitude will be:

$$|B_P| = \sqrt{\frac{R_A - R_L}{R_A^{\,2} \cdot R_L}} \quad \text{or} \quad |X_P| = R_A\sqrt{\frac{R_L}{R_A - R_L}} \qquad (8\text{-}1)$$

$$|X_S| = \sqrt{R_L(R_A - R_L)} + |X_L| \qquad \text{When:}$$

$$R_A \geq R_L \qquad \text{R, X in Ohms, B in mhos}$$

All of the required equations were combined in one set, (8-1), for later reference.

As an example, assume the load impedance to be 25 - j 100 Ohms and to be matched to a resistive impedance of 50 Ohms.. Since the load reactance is capacitive, the series component will be an inductor. The series component is then:

[1] The source resistance is subscripted A rather than S for source to avoid confusion with series reactances. The source will probably be an amplifier stage, thus justifying the A.

$$|X_S| = \sqrt{25\,(50-25)} + |-100| = \sqrt{25 \cdot 25} + 100 = 125 \text{ Ohms}$$

The total impedance of the load and series inductor is now 25 + j 25 Ohms. The admittance of this total becomes 0.02 - j 0.02 mhos. The parallel component must be a capacitor for a conjugate match of the inductive susceptance. The series component calculation is:

$$|B_P| = \sqrt{\frac{50-25}{2500 \cdot 25}} = \sqrt{\frac{25}{2500 \cdot 25}} = \sqrt{\frac{1}{2500}} = \frac{1}{50} = 0.02 \text{ mho}$$

The total admittance *looking into* the network from the amplifier (source) end is 0.02 + j 0 mhos. It is purely conductive.[2] The negative inverse of admittance is purely resistive at 1/0.02 or 50 Ohms. A match has been achieved.

Network Bandwidth

Few loads behave to make matching easy. In the case of antennas there can be a wide variation of impedance values and it may be desired to fix-tune a matching network at one frequency. Fixed tuning over a band is questioned in a four-element Yagi antenna at 28, 28.35, and 28.7 MHz:[3]

At 28.00 MHz Z = 23.70 - j 7.59 Ohms
At 28.35 MHz Z = 26.04 - j 1.47 Ohms
At 28.70 MHz Z = 20.39 + j 3.99 Ohms

Fixing the matching values at 28.35 MHz with a parallel-input L-Section for a 50 Ohm resistive source impedance would result in a series reactance of +j 26.35 Ohms (148 nHy) and a parallel capacitance of +j 19.19 mmho (103.7 pFd). Impedance at the source terminals would then be:

Frequency, MHz	Impedance, Ohms	VSWR (approximate)[4]
28.00	37.62 + j 2.911	1.33:1
28.35	50.00 + j 0	1.00:1
28.70	50.14 + j 8.233	1.14:1

That example had a fairly benign impedance range and some antennas may be quite different. From the same reference, another antenna is:

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[2] The phrase *looking into* occurs often and means a condition at a particular location in the circuit. In this case the amplifier would "see" the total of the load and the matching network.

[3] From the L. B. Cebik column on Antenna Modeling (#26) at the antenna experimental website of www.antennex.com.

[4] VSWR graphically determined on a Smith Chart.

At 28.00 MHz Z = 20.24 + j 6.85 Ohms
At 28.35 MHz Z = 10.00 + j 20.49 Ohms
At 28.70 MHz Z = 3.56 + j 41.45 Ohms

Another parallel-input L-Section, fixed-tuned at 28.35 MHz to match a resistive source of 50 Ohms would calculate out to a series capacitor of -j 40.5 Ohms (138.7 pFd) and parallel susceptance of -j 40.0 mmho (140.4 nHy). Over the same band the source would see:

Frequency, MHz	Impedance, Ohms	VSWR (approximate)
28.00	24.73 + j 36.24	3.3:1
28.35	50.00 + j 0	1.00:1
28.70	3.920 + j 0.9576	13:1 (!)

Obviously the high VSWR on the highest frequency would preclude fixed tuning. The fixed-tuned bandwidth would be fairly narrow. It's possible to sense such conditions by the impedance remaining inductive and the uneven resistive-part load impedance over frequency. This may also happen on an interstage matching situation where the load (another stage's input) also varies about the same way. The same thing can happen with the series-input L-Section.

Series-Input L-Section Matching Network

Figure 8-3 shows the configuration when the load impedance magnitude is higher than the amplifier (source) magnitude. The limits of matching are:

$$R_A \le R_L \quad \text{or}$$
$$G_L \cdot R_A \le 1$$

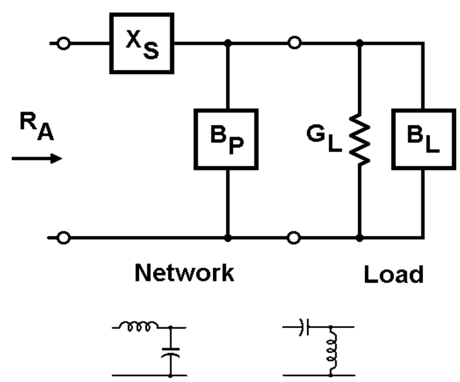

Figure 8-3 Series input network when load resistance is larger than source resistance.

Here the load impedance must be changed to an admittance for ease of calculations. Since the parallel component is in parallel with the load, it is more natural to think of it as a susceptance in parallel with load admittance.

As with the parallel-input network, if the load susceptance is inductive, the parallel component of the network is a capacitor. If the load susceptance is capacitive, the parallel network component is an inductor. The series component is the opposite of the parallel component. The formulas are:

$$|B_P| = |B_L| + \sqrt{\frac{G_L(1-R_A G_L)}{R_A}} \qquad |X_S| = \sqrt{\frac{R_A(1-R_A G_L)}{G_L}} \qquad (8\text{-}2)$$

Where: $Y_{Load} = G_L + jB_L$ Y, G, B in mhos R, X in Ohms

As an example suppose the load impedance is 19.231 + j 96.154 Ohms. The complex inverse of that is 0.002 - j 0.01 mhos. A resistive match to 50 Ohms is desired. The load admittance magnitude of 10.198 mmho times 50 Ohms is 0.509902 and that satisfies the condition for a series-input network.

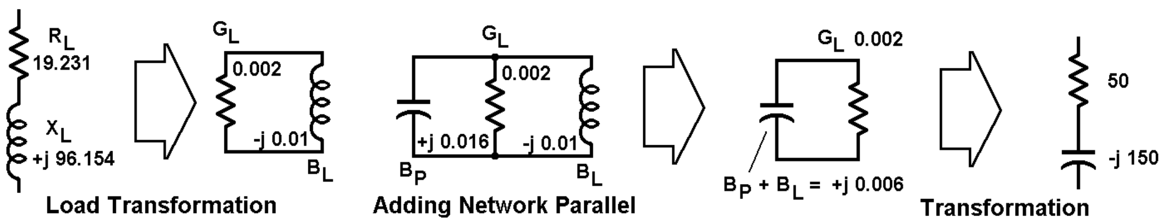

Figure 8-4 Example illustrated to show progression from load to source. Load impedance is first transformed into an admittance. Then the network parallel capacitor is added to change the admittance. Final transformation back to an impedance for series addition of network inductor.

$$|B_P| = 0.01 + \sqrt{\left(\frac{0.002}{50}\right) - (0.002)^2} = 0.01 + \sqrt{\left(40 \div 10^{-6}\right) - \left(4 \cdot 10^{-6}\right)} =$$

$$0.01 + \sqrt{36 \cdot 10^{-6}} = 0.01 + 0.006 = \quad 0.016 \text{ mhos}$$

Since B_L is inductive, B_P will be positive, a capacitor

$$|X_S| = \sqrt{\frac{50(1 - 50 \cdot 0.002)}{0.002}} = \sqrt{\frac{50(0.9)}{0.002}} = \sqrt{22500} = 150 \text{ Ohms}$$

Since B_P is capacitive, X_S will be an inductor

The matching example check operations are shown above. In the final transformation to an impedance at right in Figure 8-4, the resistive part is shown as 50 Ohms. It became so as the result of adding enough positive susceptance so that the transformation from admittance to impedance resulted in the desired resistance. If the network parallel capacitor was too small, such as +j 0.015 mho, the final impedance would have been 172.414 - j 68.9655 Ohms. If the network capacitor was too large at +j 0.017 mhos, the final impedance would have been 37.7358 - j 132.075 Ohms. As a general rule the network component closest to the load is the most sensitive in achieving a desired resistive match.

After the right-hand transformation in Figure 8-4 it should be clear that a series inductor of +j 150 Ohms will be a conjugate match value and the load impedance has been transformed to a resistive 50 Ohms impedance at the source end.

This may seem as a bit of black magic at work but it isn't. The total impedance at the source end is 50 + j 0 Ohms. If a 1.0 Ampere constant RF current is supplied the network by the source then the RF voltage at the source is 50 V RMS. Now consider two impedances connected as a voltage divider. The first impedance is the series component, a lossless inductor having an impedance of 0 + j 150 Ohms. The second impedance is the parallel component itself in parallel with the load. That total impedance is 50 - j 150 Ohms.

From Figure 8-5 the output voltage is the product of input voltage times the quantity of the second impedance divided by the sum of both impedances. The sum of the two impedances at one frequency is resistive at 50 Ohms; magnitude is 50 Ohms with zero phase angle. The developed RF voltage is equal to the impedance times current. Since the constant current can be assumed as having zero phase angle, the voltage also has a zero phase angle. The input voltage is 50 V RMS.

It may be more convenient to operate with the polar form here. The numerator of the divider equation would be 158.114 ∠-71.5651° Ohms. The denominator is 50 ∠ 0° Ohms. The output voltage is then 158.114 ∠ -71.5651° Volts RMS, higher in magnitude than the source but also of a different phase angle.

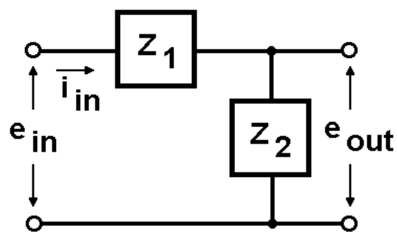

Figure 8-5 A voltage divider from two impedances as in the example case.

If the capacitor and inductor of the network are considered lossless, the current delivered to the resistive part of the load is output voltage divided by 500 Ohms, the equivalent parallel resistance at the single match frequency (inverse of 0.002 mho). The current is then 0.316228 ∠ -71.5651° Amperes RMS, in phase with the voltage. Output power is voltage times current or 50.0001 ∠ 36.8698° Watts relative to input power.[5]

A phase shift of power? That is only relative to the source power. As far as power transfer is concerned, the RF voltage and current *magnitudes* result in 50 Watts and both voltage and current there are *in phase* to satisfy the ideal case of full power transfer at one frequency. The network components are lossless in this case so that all they do is a changing of branch voltage and current *phases*. That phase change accounts for the voltage step-up as well as the current step-down. No power is changed. The parallel input network would have a voltage step-down and a current step-up with different phase shifts in the series input network.

Power Losses Due to Non-Ideal Components

In any simple network such as the L-Section, component losses contribution to lesser power transfer can be a straightforward calculation. Assume the previous series input network values and assume the inductor Q is 50 and the capacitor Q is 500. The parallel capacitor would then be a parallel admittance of 0.032 + j 16 mmho and the admittance across the load terminals would become 2.032 + j 6.000 mmho (load admittance plus the parallel capacitor with its loss). As an impedance that would be 50.6367 - j 149.518 Ohms or, in polar form, 157.859 ∠ -71.2905°.

The series inductor would have an impedance of 3.00000 + j 150.000 Ohms. In series with the impedance across the load terminals, the sum is 53.6367 + j 0.482000 Ohms or 53.6389∠0.5149°. The constant current of exactly 1 A RMS would then produce a voltage of 53.6389 ∠ 0.5149° across the source (input) terminals. The voltage divider action would then

[5] The apparent 100 microWatt extra is due to six-digit numeric input, an error of only 0.0002%. Such very small errors can be expected anywhere in calculations and should be neglected. There is no power gain through any passive-component circuit. Power delivered depends on the product of the voltage magnitude times the current magnitude and the phase shift between voltage and current.

calculate as:

$$e_{out} = e_{in}\left(\frac{Z_2}{Z_1 + Z_2}\right) = (53.6389\angle +0.5149°)\left(\frac{157.859\angle -71.2905°}{53.6389\angle 0.5149°}\right) =$$
$$157.859\angle -71.2905° \text{ Volts}$$

Note that with a constant current of unity, the developed voltage across the load has the same numeric value as the sum of the two impedances. That happens *only* at unity current.

The current in the resistive part of the load is then:

$$i_{out} = \frac{e_{out}}{\text{(parallel load resistance)}} = \frac{157.859\angle -71.2905°}{500\angle 0°} = 0.315718\angle -71.2905° \text{ A}$$

$$P_{out} = e_{out} \cdot i_{out} = (157.859\angle -71.2905°)(0.315718\angle -71.2905°) =$$
$$49.8389\angle 142.5810° \text{ Watts}$$

$$P_{in} = e_{in} \cdot i_{in} = 53.6389\angle 0.5149° \text{ Watts} \qquad \frac{P_{out}}{P_{in}} = 0.929156 \text{ or } -0.319 \text{ db}$$

It should be fairly obvious that the lossy inductor is absorbing most of the wasted power. The capacitor loss can be calculated as:

$$P_{caploss} = \frac{e_{out}{}^2}{R_{caploss}} = \frac{(157.859\angle -71.2905°)^2}{(1 \ / \ 0.000032)} = \frac{24919.5\angle -144.581°}{31250} =$$
$$0.797423\angle -144.581° \text{ Watts}$$

Recall that squaring a polar form complex number squares the magnitude and multiplies the phase angle by two. The capacitor loss can, for all practical purposes, work only on magnitudes here; the phase angle is not important in this case.

Capacitor loss is fairly substantial at nearly 0.8 W considering the Q is ten times better than the inductor. However, this is an impedance step-up network and load terminal voltage is higher than input terminal voltage. The inductor loss would be input current squared (still unity) times the equivalent series loss resistance or 3.0 Watts. All networks handling power should be calculation-checked for their possible losses.

It is possible to reduce the losses very slightly by accounting for them in calculations, then calculating the required components. However, while that does allow a tiny increase in efficiency, the increase is too slight to be worth the effort. A much better alternative is to use the lowest-loss component possible.

The Two-Component Network in Antenna Matching

Several automatic antenna tuners appeared on the amateur radio market in the early 1990s, all using the two-component basic network configuration. The inductor and capacitor were relay step-switched, generally in a 1-2-4-8-16-... sequence, all under the control of a microprocessor. The configuration had the inductor in the series arm, the capacitor in either parallel arm; an extra relay did the capacitor switching to either the source or load side.

The microprocessor input was from a *Bruene detector* that sensed RF voltage and current in the coaxial line from transmitter to antenna. A comparison of the relative voltage and current phases is at the heart of the microprocessor decision-making. The best match is done when both RF voltage and current are in phase.

An advantage for relatively narrowband transmitters is compactness. The stepped inductors can be toroids with core material optimized for the band (usually in the HF spectrum). Stepped toroids through relay switching is less expensive and more robust than a large roller-contact continuously-variable inductor. The stepped capacitors can be high voltage mica dielectric. The microprocessor's input detector can be made sensitive enough to do decisions on relatively low power, minimizing interference to others during tune-up. Using a microprocessor with *flash memory* data storage allows retention of matching settings.

Chapter 9

Three-Component Matching Networks

Three-component passive matching networks are able to achieve a combination of impedance matching and the ability to appear as resonant circuits at their center design frequency. Most solutions allow using two specific component values with the third calculated. Impedance step-up or step-down can be done without transformers over a narrow frequency band.

General Case of Three-Component Networks

These are an outgrowth of the two-component networks of the previous chapter. They allow both resonance with selectivity and a resistive-impedance input-to-output change. In all cases the output resistance is larger than the input resistance. The five configurations shown here also permit adjustment for stray shunt capacity at each resistive-termination end. Equations are given for lossless components and for a Q value that defines the bandwidth around resonance.

Computation involves *precalculation* of common terms, then a choice of exact match or a choice of three values fixed with a solution for the remaining two unknown values.[1] In the first four networks, Q is selected for bandwidth and has no relation to component Q. Component Q of the output shunt elements may be used to modify the actual output resistance value in all but the second case (Figure 9-2 network).

All equations assume values in Hz, Fd, Hy, Ohms, and mhos. All shunt branches are treated as susceptances. All series branches are treated as reactances. The stray shunt capacitance at input or output may be included or neglected by assigning it zero value. Where there are two inductors there is no inductive coupling between them.

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[1] These formulas were derived for inclusion in HP-67 programmable calculator program cards, reference [19], hence the precalculation phrasing. Each program had two calculation sections, the precalculation part being common to the section that allowed fixing two reactances and one end resistance with a solution for the third reactance and the other end resistance.

One-Capacitor, Two-Inductor Series Input Network

In Figure 9-1 R_i is less than R_o, C_i and C_o are input and output shunt capacitances, respectively. Stray capacity at ends may be included in an exact solution but is seldom required for later two-fixed-component calculations.

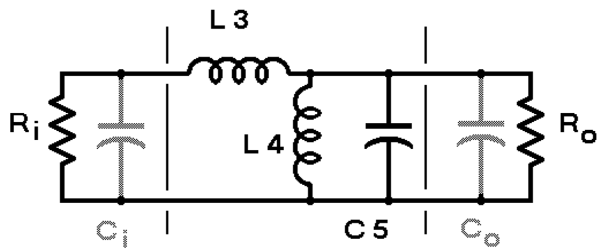

Figure 9-1 2-L, 1-C series input resonant network. Stray capacity shown in grey.

Precalculation requires:

$$B_i = w\,C_i \qquad B_o = w\,C_o \qquad \text{[zero if no stray capacity]} \qquad (9\text{-}1A)$$

$$a = 1 + R_i^2 B_i^2 \quad [a = 1 \text{ if no } C_i] \qquad b = \sqrt{R_i\,(a\,R_o - R_i)}$$

$$B_5 = w\,C_5 \qquad B_{10} = B_5 + B_o \qquad B_4 = \frac{1}{w\,L_4} \qquad X_3 = w\,L_3$$

Note that alpha and beta are required precalculations. For an exact match:

$$X_3 = \frac{R_i^2 B_i + b}{a} \quad \text{- or -} \quad L_3 = \frac{R_i^2 B_i + b}{w\,a} \qquad (9\text{-}1B)$$

$$X_4 = \frac{R_i R_o}{2Q \cdot R_i - b} \quad \text{- or -} \quad L_4 = \frac{R_i R_o}{w\,(2Q \cdot R_i - b)}$$

$$B_5 = \frac{2Q - R_o B_o}{R_o} \quad \text{- or -} \quad C_5 = \frac{2Q - R_o B_o}{w\,R_o}$$

Formulas are given so that inductance values are reactance divided by ω, capacitance values are susceptance divided by ω. If the stray capacities were zero the following simplifications apply:

$$X_3 = \frac{\beta}{\alpha} \qquad B_5 = \frac{2Q}{R_o}$$

As an example, assume a center frequency of 3.8 MHz, Q of 30 for a -3 db bandwidth of about 127 KHz, input resistance is 50 Ohms with 12 pFd of stray capacity, and output resistance is 5 KOhms with 15 pFd stray capacity. The following may be precalculated:

$$\omega = 23.8761 \cdot 10^{6} \quad B_i = 286.513 \cdot 10^{-6}\,\text{mho} \quad B_o = 358.142 \cdot 10^{-6}\,\text{mho}$$

$$\alpha = 1 + 205.225 \cdot 10^{-6} = 1.00021 \qquad \beta = \sqrt{50\,(5001.03 - 50)} = \sqrt{247551} = 497.545$$

Exact-match calculations would then be:

$$X_3 = \frac{2500 \cdot 286.513 \cdot 10^{-6} + 497.545}{1.00021} = \frac{498.262}{1.00021} = 498.159 \text{ Ohms}$$

$$L_3 = \frac{498.159}{23.8761 \cdot 10^{6}} = 20.8643 \cdot 10^{-6} \text{Hy}$$

$$X_4 = \frac{50 \cdot 5000}{60 \cdot 50 - 497.545} = \frac{250000}{2502.45} = 99.9019 \text{ Ohms} \quad L_4 = 4.18418 \cdot 10^{-6} \text{Hy}$$

$$B_5 = \frac{60 - \left(5000 \cdot 358.142 \cdot 10^{-6}\right)}{5000} = \frac{60 - 1.79071}{5000} = 11.6419 \cdot 10^{-3} \text{mho}$$

$$C_5 = 487.595 \cdot 10^{-12} \text{ Fd}$$

If there was no stray capacity X_3 would be 497.443 Ohms, B_5 would be 12 mmho. For each set of unknowns:

$$X_3 = \omega L_3 \quad X_4 = \omega L_4 \quad B_4 = \frac{1}{X_4} \quad B_5 = \omega C_5 \tag{9-1C}$$

Given R_i, L_3, L_4:

$$R_o = \frac{X_3^2 + R_i^2}{R_i} \quad \beta = \sqrt{R_i (R_o - R_i)} \quad 2Q = R_o B_4 + \frac{\beta}{R_i} \quad C_5 = \frac{2Q}{\omega R_o}$$

Given R_i, L_3, C_5:

$$R_o = \frac{X_3^2 + R_i^2}{R_i} \quad \beta = \sqrt{R_i (R_o - R_i)} \quad 2Q = R_o B_5 \quad L_4 = \frac{R_i R_o}{\omega(2Q R_i - \beta)}$$

Given R_i, L_4, C_5:

$$\rho = B_5 - B_4 \quad R_o = \frac{1 + \sqrt{1 - 4 R_i^2 \rho^2}}{2 R_i \rho^2} \quad 2Q = R_o B_5 \quad L_3 = \frac{\sqrt{R_i (R_o - R_i)}}{\omega}$$

Given R_o, L_3, L_4:

$$R_i = \frac{R_o - \sqrt{R_o^2 - 4 X_3^2}}{2} \quad 2Q = \left(\frac{R_o}{X_4}\right) + \sqrt{\frac{R_o - R_i}{R_i}} \quad C_5 = \frac{2Q}{\omega R_o}$$

Given R_o, L_3, C_5:

$$R_i = \frac{R_o - \sqrt{R_o^2 - 4 X_3^2}}{2} \quad \beta = \sqrt{R_i (R_o - R_i)} \quad 2Q = R_o B_5 \quad L_4 = \frac{R_i R_o}{\omega(2Q R_i - \beta)}$$

Given R_o, L_4, C_5:

$$2Q = R_o B_5 \quad R_i = \frac{R_o}{1 + R_o^2 (B_5 - B_4)^2} \quad L_3 = \frac{\sqrt{R_i (R_o - R_i)}}{\omega}$$

While this seems to be a prodigious group of equations, several expressions are common and they can be incorporated into a computer or calculator program as functions or subroutines. Several square-root terms exist. These should be trapped out as unworkable solutions if negative.

For examples, the fixed component values were 22 µHy, 4.3 µHy, or 490 pFd (220 in parallel with 270), all available in 5% tolerance parts. With the given end conditions the solutions (to 4 digits) were:

Given R_i, L_3, L_4: R_o = 5568 Ohms Q = 32.37 C_5 = 487.0 pFd

Given R_i, L_3, C_5: R_o = 5568 Ohms, Q = 32.57 L_4 = 4.268 µHy

Given R_i, L_4, C_5: R_o = 5160 Ohms, Q = 30.19 L_3 = 21.17 µHy

Given R_o, L_3, L_4: R_o = 55.81 Ohms Q = 29.06 C_5 = 486.8 pFd

Given R_o, L_3, C_5: R_o = 55.81 Ohms, Q = 29.25 L_4 = 4.266 µHy

Given R_o, L_4, C_5: R_o = 51.57 Ohms, Q = 29.25 L_3 = 21.16 µHy

The variation is rather slight but it can make a difference when one component is variable as a trimmer and the other two are fixed but at extreme ends of their tolerance. Small changes may result in large variations elsewhere. For example, with R_i at 50 Ohms, L_4 at 4.3 µHy, C_5 fixed but at 10 pFd steps, the R_o, Q, and L_3 would be:

C5 (pfd)	B5 (mmho)	B5-B4 (mmho)	Ro (Ohms)	Q	X3 (Ohms)	L3 (µHy)
510	11.96	2.222	4001	23.93	444.5	18.62
500	11.94	2.198	4090	24.41	449.4	18.82
490	11.70	1.959	5160	30.19	505.5	21.17
480	11.46	1.720	6707	38.44	577.0	24.16
470	11.22	1.482	9061	50.84	671.2	28.11

Frequency response well away from resonance follows a single parallel resonant circuit on the high frequency side of resonance. On the low side, well away from resonance, the response becomes that of an inductive voltage divider of L_3 in series with L_4 shunted by R_O.

Two-Inductor, One-Capacitor Parallel Input Network

This network is quite similar to that of Figure 9-1 and, at first glance, might seem like the inductors are really one that is tapped for the input. Both inductors are separate and have no inductive coupling.

Output resistance is higher than input resistance. An option is to include stray capacitances at each end. Bandwidth is again determined by center frequency divided by Q.

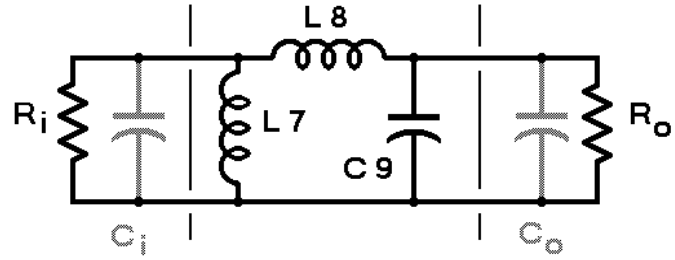

Figure 9-2 2-L, 1-C parallel input resonant network. Stray end capacities shown in grey.

All equation values are Hz, Ohms, mhos, μHy, and pFd. Precalculation requires:

$$\beta = 1 + 4Q^2 \quad B_i = \omega C_i \quad B_o = \omega C_o \qquad (9\text{-}2A)$$

$$B_7 = \frac{1}{\omega L_7} \quad X_8 = \omega L_8 \quad B_9 = \omega C_9$$

For an exact match:

$$B_7 = \sqrt{\frac{\beta R_i - R_o}{R_i^2 R_o}} \quad X_7 = \frac{1}{B_7} \quad L_7 = \frac{X_7}{\omega} \qquad (9\text{-}2B)$$

$$X_8 = \frac{2Q\,R_o - \sqrt{R_o\,(b\,R_i - R_o)}}{b} \qquad L_8 = \frac{X_8}{w}$$

$$B_9 = \left(\frac{2Q}{R_o}\right) - B_o \qquad C_9 = \frac{B_9}{w}$$

As an example, the previous end resistances and capacities can apply along with center frequency:

$$f = 3.8\text{ MHz} \quad \omega = 23.8761\cdot 10^6 \quad Q = 30 \quad 2Q = 60 \quad R_i = 50 \quad R_o = 5000$$

$$C_i = 12\text{ pFd} \quad B_i = 286.513\cdot 10^{-6} \quad C_o = 15\text{ pFd} \quad B_o = 358.142\cdot 10^{-6}$$

$$\beta = 1 + (2Q)^2 = 3601.00 \qquad \omega = 23.8761\cdot 10^6 \qquad \text{Then:}$$

$$B_7 = \sqrt{\frac{3601\cdot 50 - 5000}{2500\cdot 5000}} + 286.513\cdot 10^{-6} = \sqrt{\frac{175050}{12.5\cdot 10^6}} + 286.513\cdot 10^{-6} =$$

$$118.338\cdot 10^{-3} + 286.513\cdot 10^{-6} = 118.625\cdot 10^{-3}\text{ mho} \qquad X_7 = 8.42993$$

$$L_7 = \frac{8.42993}{\omega} = 353.070\cdot 10^{-9}\text{ Hy} = 0.353\ \mu\text{Hy}$$

$$X_8 = \frac{60 \cdot 5000 - \sqrt{5000 \cdot (3601 \cdot 50 - 5000)}}{3601} = \frac{300000 - \sqrt{8.75250 \cdot 10^6}}{3601} =$$

$$\frac{270415}{3601} = 75.0945 \text{ Ohms} \qquad L_8 = \frac{75.0945}{w} = 3.14517 \cdot 10^{-6} \text{Hy}$$

$$B_9 = \frac{60}{5000} - 358.142 \cdot 10^{-6} = 12 \cdot 10^{-3} - 358.142 \cdot 10^{-6} = 11.6419 \cdot 10^{-3} \text{mho}$$

$$C_9 = \frac{11.6419 \cdot 10^{-3}}{23.8761 \cdot 10^6} = 487.595 \cdot 10^{-12} \text{ Fd} = 488 \text{ pFd}$$

This results in some impractical values of 0.353 µHy, 3.15 µHy, and 488 pFd. Standard 5% tolerance components of 0.33 µHy, 3.30µHy, and 490 pFd (220 in parallel with 270) could be tried with one end resistance fixed at the desired value. Again, with two components and one end resistance fixed, the solution for the third component results in a new Q and opposite-end resistance.

Stray capacities at the ends can be neglected to make calculations easier. Any stray capacity at the output end can be compensated for by reducing the value of C by that amount.

In each case the formulas must be calculated in the order given. Precalculation of reactance and susceptance is required for the fixed values:

$$B_7 = \frac{1}{w L_7} \quad X_8 = w L_8 \quad B_9 = w C_9 \quad \text{and, after each new Q value:} \quad b = 1 + 4Q^2$$

Given R_i, L_7, C_8: **(9 - 2C)**

$$\alpha = 1 + R_i^2 B_7^2 \quad 2Q = \frac{R_i^2 B_7 + \alpha X_8}{R_i} \quad R_o = \frac{\beta R_i}{\alpha} \quad C_9 = \frac{2Q}{\omega R_o}$$

Given R_i, L_7, C_9:

$$\alpha = 1 + R_i^2 B_7^2 \quad R_o = \frac{\alpha + \sqrt{\alpha^2 - 4 R_i^2 B_9^2}}{2 R_i B_9^2} \quad 2Q = R_o B_9 \quad L_8 = \frac{2Q R_o - \sqrt{R_o (\beta R_i - R_o)}}{\omega \beta}$$

Given R_i, L_8, C_9:

$$R_o = \frac{R_i + \sqrt{R_i^2 - 4 X_8^2 (1 - B_9 X_8)^2}}{2 (1 - B_9 X_8)^2} \quad 2Q = R_o B_9 \quad L_7 = \left(\frac{R_i}{\omega}\right)\sqrt{\frac{R_o}{\beta R_i - R_o}}$$

Given R_o, L_7, L_8:

$$\alpha = 1 + B_7 X_8 \quad 2Q = \frac{R_o(2\alpha - 1) - \sqrt{R_o^2 - 4 X_8^2 \alpha^2}}{2 \alpha X_8} \quad R_i = \frac{R_o(R_o - 4 Q X_8) + X_8^2 \beta}{R_o} \quad C_9 = \frac{2Q}{\omega R_o}$$

Given R_o, L_7, C_9:

$$2Q = R_o B_9 \quad R_i = \frac{\beta + \sqrt{\beta^2 - 4 R_o^2 B_7^2}}{2 R_o B_7^2} \quad L_8 = \frac{2Q R_o - \sqrt{R_o (\beta R_i - R_o)}}{\omega \beta}$$

(9 - 2C) continued....

Given R_o, L_8, C_9:

$$2Q = R_o B_9 \qquad R_i = \frac{R_o(R_o - 4\,Q\,X_8) + X_8^{\,2}\beta}{R_o} \qquad L_7 = \left(\frac{R_i}{\omega}\right)\sqrt{\frac{R_o}{\beta R_i - R_o}}$$

Using the original example of 3.8 MHz center frequency, input resistance of 50 Ohms, output resistance of 5000 Ohms, no stray capacity on the ends, and a choice of two of three components of 0.36 µHy, 3.0 µHy, and 500 pFd, the results from equation set (9-2C) would be:

Given R_i, L_7, L_8: R_o = 4458 Ohms Q = 27.86 C_9 = 523.5 pFd

Given R_i, L_7, C_9: R_o = 4888 Ohms, Q = 29.17 L_8 = 3.158 µHy

Given R_i, L_8, C_9: R_o = 2270 Ohms, Q = 13.55 L_7 = 0.5371 µHy

Given R_o, L_7, L_8: R_o = 56.35 Ohms Q = 31.24 C_9 = 523.3 pFd

Given R_o, L_7, C_9: R_o = 51.22 Ohms, Q = 29.85 L_8 = 3.157 µHy

Given R_o, L_8, C_9: R_o = 106.0 Ohms, Q = 29.85 L_7 = 0.5142 µHy

Choosing fixed values of L_8 and C_9 as given run into some discrepancies. In the fixed R_i case there is sensitivity to the value of L_8 as shown by:

L8 (µHy)	C9 (pFd)	Ro (Ohms)	Q	L7 (µHy)
3.0	500	2270	13.55	0.5371
3.1	500	3578	21.36	0.4230
3.2	500	6354	37.93	0.3147
3.3	500	14.05K	83.88	0.2103
3.0	510	2955	17.99	0.4578
3.1	510	5018	30.55	0.3480
3.2	510	10.19K	62.03	0.2426

Some cases happen to be sensitive to component tolerances and the choice is either use a different network or pin down the fixed component tolerances.

At frequencies far from resonance this network appears quite close to a single parallel resonant circuit response, reasonably symmetrical about the center frequency.

One-Inductor, Two-Capacitor Series Input Network

The circuit of Figure 9-3 is similar to that of Figure 9-1 except that there is more attenuation below the resonant frequency than above. Above resonance the attenuation far from resonance is due largely to the voltage-divider action of C_3 and C_4.

Stray end capacities are shown in grey and may be omitted from an exact solution if desired. Precalculation for an exact solution requires:

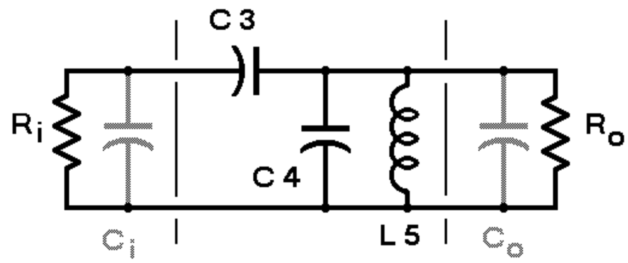

Figure 9-3 1-L, 2-C Series input network. stray end capacities are shown in grey.

$$B_i = \omega C_i \quad B_o = \omega C_o \quad \text{[optional, may be zero]} \qquad (9\text{-}3A)$$

$$\alpha = 2Q - R_o B_o \qquad \beta = \alpha - R_o B_4$$

For an exact solution:

$$C_3 = \frac{\beta^2 + 1}{\omega R_o (\beta - R_i B_i)} \qquad \text{If no } C_i \text{ then: } C_3 = \frac{\beta^2 + 1}{\omega R_o \beta} \qquad (9\text{-}3B)$$

$$C_4 = \frac{\alpha - \sqrt{R_i R_o B_i^2 + (R_o / R_i) - 1}}{\omega R_o} \qquad \text{If } C_i = 0\text{:} \quad C_4 = \frac{\alpha - \sqrt{(R_o / R_i) - 1}}{\omega R_o}$$

$$L_5 = \frac{R_o}{2Q\omega} \qquad B_4 = \omega C_4 = \frac{\alpha - \sqrt{R_i R_o B_i^2 + (R_o / R_i) - 1}}{\omega R_o}$$

For an example, assume the resonant frequency is 7.15 MHz, input resistance 50 Ohms, output resistance 5000 Ohms, stray input capacity is 10 pFd, and stray output capacity is 25 pFd. Precalculation results in:

$$\omega = 44.9248 \cdot 10^6 \qquad B_i = 449.248 \cdot 10^{-6} \qquad B_o = 1.12312 \cdot 10^{-3}$$

$$\alpha = 2 \cdot 30 - 5000 \cdot 1.12312 \cdot 10^{-3} = 60 - 5.61560 = 54.3844$$

Beta cannot be found until B_4 is calculated.

$$B_4 = \frac{54.3844 - \sqrt{50 \cdot 5000 \cdot 201.824 \cdot 10^{-9} + 100 - 1}}{5 \cdot 10^3} = \frac{54.3844 - \sqrt{99.0505}}{5 \cdot 10^3} = 8.88640 \cdot 10^{-3}$$

$$\beta = 54.3844 - 5000 \cdot 8.88640 \cdot 10^{-3} = 9.95240 \qquad C_4 = \frac{B_4}{\omega} = 197.806 \cdot 10^{-12} \text{ Fd}$$

$$C_3 = \frac{\beta^2 + 1}{\omega R_o (\beta - R_i B_i)} = \frac{99.0503 + 1}{\omega \cdot 5000 \left(9.95240 - 22.4624 \cdot 10^{-3}\right)} = \frac{100.050}{2.23050 \cdot 10^{12}} = 44.8555 \cdot 10^{-12} \text{ Fd}$$

$$L_5 = \frac{R_o}{2\,Q\,\omega} = \frac{5000}{60 \cdot 44.9248 \cdot 10^6} = 1.85495 \cdot 10^{-6}\ \text{Hy} = 1.85495\ \mu\text{Hy}$$

To pick two fixed component values, one end resistance, the remaining component, other end resistance and Q, with no stray end capacities, precalculate the fixed values' susceptances and reactance:

Given R_i, C_3, C_4: (9-3C)

$$R_o = \frac{R_i^2 B_3^2 + 1}{R_i B_3^2} \qquad 2Q = R_o B_4 + \sqrt{(R_o / R_i) - 1} \qquad L_5 = \frac{R_o}{2Q\,\omega}$$

Given R_i, C_3, L_5:

$$R_o = \frac{R_i^2 B_3^2 + 1}{R_i B_3^2} \qquad 2Q = \frac{R_o}{X_5} \qquad C_4 = \frac{2Q - \sqrt{(R_o / R_i) - 1}}{\omega R_o}$$

Given R_i, C_4, L_5:

$$\rho = (B_4 X_5 - 1)^2 \qquad R_o = \frac{X_5\left(X_5 + \sqrt{X_5^2 - 4\,R_i^2 \rho}\right)}{2\,R_i\,\rho} \qquad 2Q = \frac{R_o}{X_5}$$

$$\beta = 2Q - R_o B_4 \qquad C_3 = \frac{\beta^2 + 1}{\omega R_o\,\beta}$$

Given R_o, C_3, C_4:

$$R_i = \frac{2}{B_3\left(R_o B_3 + \sqrt{R_o^2 B_3^2 - 4}\right)} \qquad 2Q = R_o B_4 + \sqrt{(R_o / R_i) - 1} \qquad L_5 = \frac{R_o}{2Q\,w}$$

Given R_o, C_3, L_5:

$$R_i = \frac{2}{B_3\left(R_o B_3 + \sqrt{R_o^2 B_3^2 - 4}\right)} \qquad 2Q = \frac{R_o}{X_5} \qquad C_4 = \frac{2Q - \sqrt{(R_o / R_i) - 1}}{w\,R_o}$$

Given R_o, C_4, L_5:

$$2Q = \frac{R_o}{X_5} \qquad b = 2Q - R_o B_4 \qquad R_i = \frac{R_o}{b^2 + 1} \qquad C_3 = \frac{b^2 + 1}{w\,b\,R_o}$$

Using equation set (9-3C) and fixed values of 43 pFd, 220 pFd, 1.8 µHy, the following result for the six possible combinations.

Given R_i, C_3, C_4:	$R_o = 5409$ Ohms	$Q = 31.71$	$L_5 = 1.899$ µHy
Given R_i, C_3, L_5:	$R_o = 5409$ Ohms	$Q = 33.45$	$C_4 = 234.3$ pFd
Given R_i, C_4, L_5:	$R_o = 3193$ Ohms	$Q = 19.75$	$C_3 = 56.15$ pFd
Given R_o, C_3, C_4:	$R_i = 54.18$ Ohms	$Q = 29.49$	$L_5 = 1.887$ µHy
Given R_o, C_3, L_5:	$R_i = 54.18$ Ohms	$Q = 30.92$	$C_4 = 232.7$ pFd
Given R_o, C_4, L_5:	$R_i = 32.23$ Ohms	$Q = 30.92$	$C_3 = 55.63$ pFd

As with the other two networks this shows some sensitivity with the third and sixth combinations.

One-Inductor, Two-Capacitor Parallel Input Network

The circuit of Figure 9-4 is the analogue to that of Figure 9-2. Inductors and capacitors have switched locations. At frequencies far from resonance the response is more like a single parallel resonant circuit. Stray end capacities are optional for an exact solution and are shown in grey.

This network has been used as a substitute for a link-coupled or tapped inductor parallel resonator where the tapping is difficult to achieve.

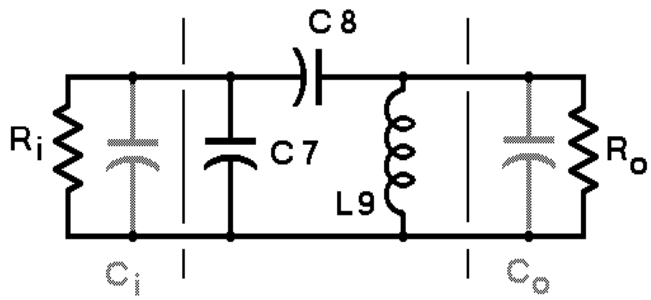

Figure 9-4 1-L, 2-C parallel input network. Stray end capacities shown in grey.

Precalculation for exact solution:

$$B_i = \omega C_i \quad B_o = \omega C_o \quad \text{[optional, may be zero]} \tag{9-4A}$$

$$\alpha = 2Q - R_o B_o$$

Exact solution:

$$B_7 = \sqrt{\frac{R_i\left(\alpha^2 + 1\right) - R_o}{R_i{}^2 R_o}} \qquad C_7 = \frac{B_7}{\omega} \tag{9-4B}$$

$$C_8 = \frac{\alpha^2 + 1}{\omega R_o\left[\alpha - R_i\left(B_i + B_7\right)\right]} \qquad L_9 = \frac{R_o}{2Q\,\omega}$$

As an example, assume resonance at 7.15 MHz, input resistance is 50 Ohms, output resistance is 5 Kohms, Q is 30 for a bandwidth of approximately 238 KHz, stray input capacity is 10 pFd, and stray output capacity is 25 pFd. The calculation is:

$$\omega = 2\pi \cdot 7.15 \cdot 10^6 = 44.9248 \cdot 10^6 \quad B_i = 449.248 \cdot 10^{-6} \quad B_o = 1.12312 \cdot 10^{-3}$$

$$\alpha = 2 \cdot 30 - 5 \cdot 10^3 \cdot 1.12312 \cdot 10^{-3} = 60 - 5.61560 = 54.3844$$

$$B_7 = \sqrt{\frac{50 \cdot 2958.66 - 5000}{2500 \cdot 5000}} = \sqrt{11.4347 \cdot 10^{-3}} = 0.106933 \quad C_7 = \frac{0.106933}{\omega} = 2380.27 \cdot 10^{-12}$$

$$C_8 = \frac{2958.66}{\omega\, 5000 \left[54.3844 - 50\,(107.382 \cdot 10^{-3})\right]} = \frac{2958.66}{\omega\, 5000\,(54.3844 - 5.36911)} =$$

$$\frac{2958.66}{245076\,\omega} = \frac{12.0724 \cdot 10^{-3}}{44.9248 \cdot 10^6} = 268.725 \cdot 10^{-12} \text{ Fd}$$

$$L_9 = \frac{5000}{60\,\omega} = \frac{83.3333}{\omega} = 1.85495 \cdot 10^{-6} \text{ Hy}$$

If there were no stray capacitances, then C_7 would become 2634.15 pFd and C_8 would be 296.419 pFd. There is no change for the L_9 value.[2] It remains at 1.85495 µHy.

For two fixed components and one end resistance cases without stray capacitances:

Given R_i, C_7, C_8: (9-4C)

$$R_o = \frac{1 + R_i^2 (B_7 + B_8)^2}{R_i B_8^2} \quad 2Q = \sqrt{R_i R_o B_7^2 + (R_o / R_i) - 1} \quad L_9 = \frac{R_o}{2Q\,\omega}$$

Given R_i, C_7, L_9:

$$\beta = X_9^2 \left(R_i^2 B_7^2 + 1\right) \qquad R_o = \frac{\beta + \sqrt{\beta^2 - 4 R_i^2}}{2 R_i} \qquad 2Q = \frac{R_o}{X_9}$$

$$C_8 = \frac{4Q^2 + 1}{\omega R_o \left(2Q - R_i B_7\right)}$$

Given R_i, C_8, L_9:

$$\rho = (X_9 B_8 - 1)^2 \qquad R_o = \frac{R_i B_8^2 X_9^2 + \sqrt{(R_i B_8^2 X_9^2)^2 - 4 X \rho}}{2\rho}$$

$$2Q = \frac{R_o}{X_9} \qquad B_7 = \sqrt{\frac{R_i\,(4Q^2 + 1) - R_o}{R_i^2 R_o}} \qquad L_7 = \frac{B_7}{\omega}$$

[2] The reason for that is the solution as explained in Appendix 9-1.

Given R_o, C_7, C_8: (9 - 4C continued)

$$R_i = \frac{2}{R_o B_8^2 - \sqrt{\left(R_o B_8^2\right)^2 - 4\left(B_7 + B_8\right)^2}}$$

$$2Q = \sqrt{R_i R_o B_7^2 + (R_o / R_i) - 1} \qquad L_9 = \frac{R_o}{2Q\,\omega}$$

Given R_o, C_7, L_9:

$$2Q = \frac{R_o}{X_9} \qquad \beta = 4Q^2 + 1$$

$$R_i = \frac{2\,R_o}{\beta - \sqrt{\beta^2 - 4\,R_o^2 B_7^2}} \qquad C_8 = \frac{\beta}{\omega R_o\left(2Q - R_i B_7\right)}$$

Given R_o, C_8, L_9:

$$2Q = \frac{R_o}{X_9} \qquad R_i = \frac{\left(2Q - R_o B_8\right)^2 + 1}{R_o B_8^2} \qquad B_7 = \sqrt{\frac{R_i(4Q^2 + 1) - R_o}{R_i^2 R_o}} \qquad C_7 = \frac{B_7}{\omega}$$

By example, using fixed components of 2700 pFd (C_7), 300 pFd (C_8), or 1.8 µHy (L_9):

Given R_i, C_7, C_8: R_o = 5110 Ohms Q = 31.06 L_9 = 1.831 µHy
Given R_i, C_7, L_9: R_o = 4941 Ohms Q = 30.55 C_8 = 305.7 pFd
Given R_i, C_8, L_9: R_o = 7245 Ohms Q = 44.80 C_7 = 3284 pFd
Given R_o, C_7, C_8: R_i = 48.87 Ohms Q = 30.40 L_9 = 1.831 µHy
Given R_o, C_7, L_9: R_i = 61.83 Ohms Q = 30.92 C_8 = 305.7 pFd
Given R_o, C_8, L_9: R_i = 61.83 Ohms Q = 30.92 C_7 = 3225 pFd

Lowpass Pi-Network

The *Pi-Network* is in a category of semi-symmetric arrangements (C-L-C or L-C-L) which has a mild magnitude peak at resonance. The primary definition of resonance is achieved by the output voltage being in phase with output current but the selectivity (magnitude response) is definitely one-sided.

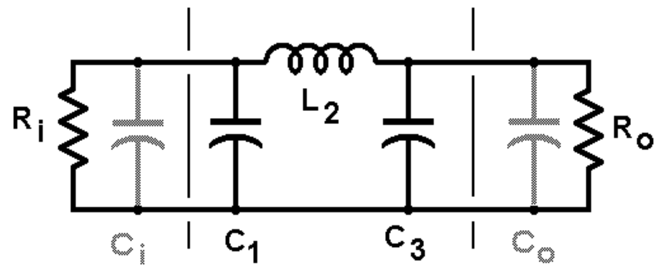

Figure 9-5 Lowpass Pi-Network with stray capacities shown in grey.

In the circuit of Figure 9-5 all frequencies on the low side of design frequency will pass on through, thus the term *lowpass*. At DC the output resistance is presented directly to the input.[3] As frequency is raised towards the design frequency, the input sees a progressivly higher impedance until it peaks at the resistive impedance value of Ri. Above the design frequency there is a attenuation increasing with frequency.

The generic pi-network of Figure 9-6 is the basis for the component value calculation formulas. If the output is terminated in a resistance of Ro, the impedance looking into the input will appear as Ri at the design frequency and vice-versa. Let the following be true:

$\omega = 2\pi f$, where f is the design frequency in Hz

R_i is the desired source resistance, Ohms, and

R_o is the load resistance in Ohms.

$R_i = k R_o,\ R_i > R_o \qquad X_1 = m X_3$

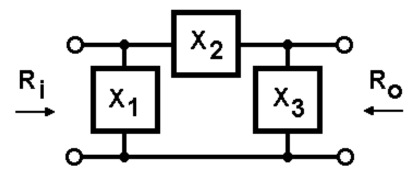

Figure 9-6 Generic pi-network composed of reactances.

The solution for the reactance box magnitudes and component values is then:

$$X_1 = R_o \sqrt{\frac{k\left(k - m^2\right)}{k-1}} \qquad X_2 = \frac{R_o \sqrt{k\left(k - m^2\right)(k-1)}}{k-m} \qquad X_3 = \frac{X_1}{m} \qquad (9\text{-}5A)$$

$$C_1 = \frac{1}{\omega X_1} \qquad L_2 = \frac{X_2}{\omega} \qquad C_3 = \frac{1}{\omega X_3}$$

All three component values are inter-related through Ro, k, and m. What is interesting here (with a semi-symmetric arrangement, C-L-C) is that there is a great range of m values possible, all of which result in the proper resistive match for any given k ratio!

The only requirements are:

---

[3] At DC the inductor has no reactance, only a small amount of internal resistance in series with Ro. The capacitors are essentially non-existent at DC.

$$k \text{ must be greater than } 1 \qquad m \text{ must be less than } \sqrt{k}$$

There is no Q as in the conventional resonant circuit case. However, an approximation of Q may be obtained by dividing the inductor reactance by the equivalent series resistance due to the terminating resistances. That equivalent resistance may be obtained from that in Figure 9-7.

$$R = \frac{2\,k\,R_o\left(k - m^2\right)}{k^2 - m^2} \qquad Q_{\text{series-resonant}} = \frac{X_2}{R} = \left(\frac{k+m}{2\,k}\right)\sqrt{\frac{k\,(k-1)}{k-m^2}} \qquad (9\text{-}5B)$$

Since Ro, k, and m are all inter-related, it is useful to find m given a particular Q:

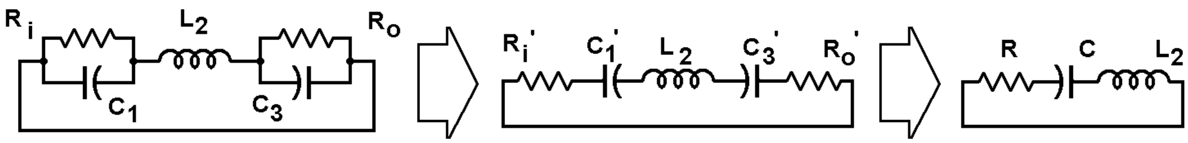

Figure 9-7 Transformation of lowpass pi-network to an equivalent series-resonant circuit.

$$m = k\left[\frac{2Q\sqrt{4Q^2k - (k-1)^2} - (k-1)}{4Q^2k + (k-1)}\right] \qquad (9\text{-}5C)$$

where Q is the equivalent Q-series obtained in (9-5B)

The Q values given as optimum for transmitter power amplifier pi-networks vary from 2 through 16, apparently with little consistency.[4] Any of these Q values will work fine for relatively narrowband impedance matching. Since (9-5C) is a bit formidable, calculated values of m for various k and Q values are included on Table 9-1.

[4] ARRL 1999 Handbook CD Version 3.0, Chapter 13. The final choice of "Q" depends more on the practical components that are closest to calculated values. Terms such as "L/C ratio" are relatively meaningless when the pi-network is analyzed as a network.

Table 9-1

Calculated Values of m for Selected k, Q Values of a Pi-Network

k	Q=5	Q=7.5	Q=10	Q=12.5
1.5	1.21467	1.22026	1.22222	1.22313
2	1.39371	1.40507	1.40907	1.41092
2.5	1.54971	1.56713	1.57325	1.57609
3	1.68920	1.71295	1.72129	1.72516
4	1.93288	1.97007	1.98315	1.98921
5	2.14286	2.19453	2.21268	2.22109
6	2.32851	2.39561	2.41916	2.43007
8	2.64727	2.74787	2.78310	2.79941
10	2.91526	3.05262	3.10062	3.12282
12	3.14598	3.32316	3.38490	3.41344
15	3.43900	3.68135	3.76541	3.80420
20	3.82215	4.18688	4.31233	4.37004
30	4.31479	4.97441	5.19675	5.29839
40	4.54465	5.56795	5.90454	6.05729
50	4.56332	6.02725	6.49465	6.70508
60	4.38584	6.38288	6.99753	7.27186
70	4.00236	6.65275	7.43136	7.77559
80	3.37076	6.84809	7.80803	8.22806
100	0.41655	7.04167	8.42059	9.00983
120	*	6.99424	8.87818	9.66050
140	*	6.70985	9.20553	10.2058
160	*	6.16826	9.41768	10.6626
200	*	3.99641	9.52855	11.3528
240	*	*	0.17666	11.7892

* Impossible solution due to requirement of a square-root of a negative.

Highpass Pi-Network

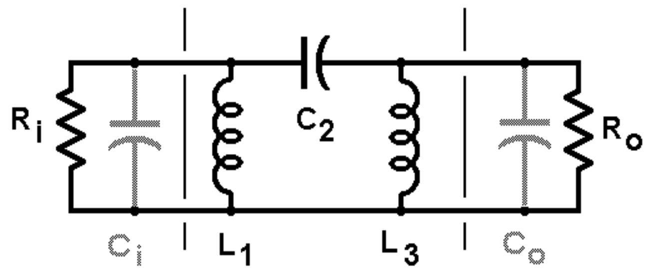

Figure 9-8 Highpass Pi-Network. Stray capacities are shown in grey.

Figure 9-8 shows the dual of the lowpass pi-network. Exchanging the capacitors and inductors from that of Figure 9-5, but keeping the same reactance magnitudes, yields a highpass matching network. At frequencies above the design frequency, the inductor reactances become higher and the capacitor reactance becomes lower. At some higher frequency the source will see only Ro. At frequencies below the match design frequency the attenuation increases inversely proportional to frequency until infinite attenuation occurs at DC.

The same k, m, and Q constants used with the lowpass pi-network can be used here along with the reactance magnitude formulas of (9-5A). The difference is that the component values are:

$$L_1 = \frac{X_1}{\omega} \qquad C_2 = \frac{1}{\omega X_2} \qquad L_3 = \frac{X_3}{\omega} \qquad (9\text{-}6)$$

Perhaps the main reason this network has seen little application is that it passes nearly all higher frequency signal components.[5] That is true if the load, Ro, remains resistive at higher frequencies. That is seldom the case in the real world of antennas or even for a following-stage input impedance.

The highpass pi-network may be of some advantage in coupling to a frequency multiplier stage
where high harmonic content is preferred. The match design frequency could be at one of those harmonics while the fundamental frequency would present a very low, reactive impedance to the source.

Cascading Pi-Networks

The semi-symmetry (C-L-C or L-C-L arrangement) plus the match being to a resistive source and load allows simple cascading. Assume a 5 KOhm source is to be matched to 50 Ohms at 7.15 MHz. Assume a Q of 10 and two lowpass pi-networks. The first network (closest to source) has an Ri of 5 KOhms but an Ro of 500 Ohms. The second network has an Ri of 500 Ohms and an Ro of 50 Ohms. The value of k is 10 and m for a Q of 10 would be 3.10062. From (9-5):

First network: $X_1 = 327.514$ and $C_1 = 67.9648$ pFd $\quad C_3 = m\, C_1 = 210.733$ pFd

$X_2 = 422.231$ and $L_2 = 9.50991\ \mu$Hy

[5] All radio regulation administrations in the world have specifications on the maximum level of harmonics of a transmitter signal. If the highpass pi-network were used in the transmitter output coupling network it would not attenuate any harmonics generated by the transmitter power amplifier.

Second network: $X_1 = 32.7514$ and $C_1 = 679.648$ pFd $\quad C_3 = 2107.33$ pFd (call that C_5)

$X_2 = 42.2231$ and $L_2 = 0.950991\ \mu$Hy (call that L_4)

Mid - point capacitance is the sum of C_3 of first network and C_1 of second network or 210.733 pFd + 679.648 pFd = 890.381 pFd (call that one C_3)

Figure 9-9 shows the evolution of two separate networks joined in series for a single network. In this particular case the fixed values shown in the combined network worked out well provided that C1 was made a trimmer capacitor. Nominal value shown for C1 matched the 5 Kohm input resistance to within 1% of desired value.

Total attenuation above resonance (into a resistive 50 Ohm load) is 82 db at 21.45 MHz, 134 db at 71.5 MHz.

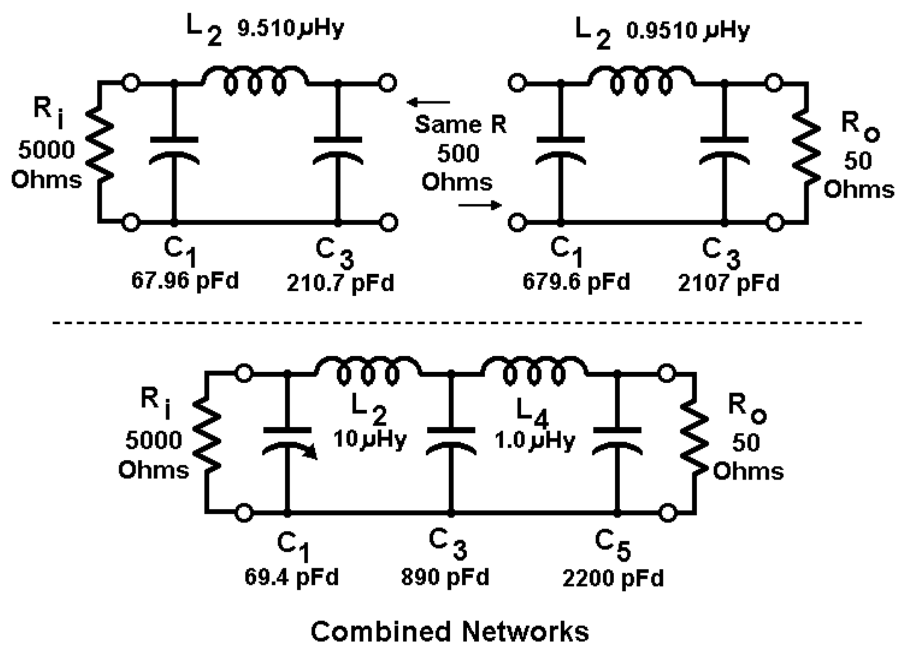

Figure 9-9 Two lowpass pi-networks in cascade to form a single network. Fixed values picked for all but C1 (890 pFd is 560 in parallel with 330 pFd). C1 is a tuning trimmer.

Appendix 9-1

Derivation of the Network Component Formulas

Asymmetric Networks, Figures 9-1 through 9-4

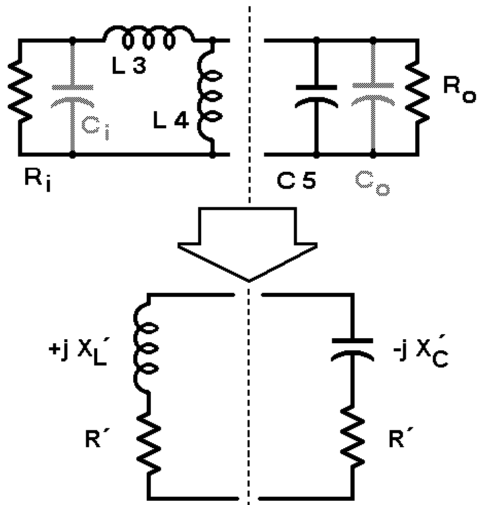

Figure 9-10 Transformation of the Figure 9-1 network to equivalent series resonance representation.

These were originally done for incorporation into programmable calculator and computer programs. Using lossless components, the resonance Q was determined as affect by both input and output termination resistances as shown in Figure 9-10 for the network of Figure 9-1.

A convenient dividing line (dotted in the figure) was established, then the parallel circuits on each side converted to series equivalents. Solutions required each R′ to be equal and a conjugate match established for the reactances.

$$|+j\,X_L| = |-j\,X_C| = X$$

$$Q = \frac{X}{2R'} \quad \text{[series resistances additive]}$$

The resulting algebraic solutions are exact and include stray capacitances across each end resistance.

Symmetric Pi-Network Derivation

The pi-network required a slightly different methodology plus some convenience constants:

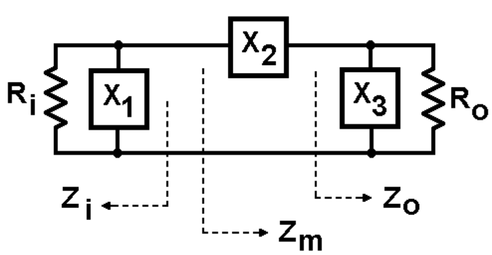

Figure 9-11 Pi-Network model to derive component value formulas.

$$k = \frac{R_i}{R_o} \qquad R_i \rangle R_o$$

$$m = \frac{X_1}{X_3} \quad \text{[both reactances same sign]}$$

Impedances are then expressed according to the diagram.

$$Z_o = \left(\frac{R_o X_3^2}{R_o^2 + X_3^2}\right) - j\left(\frac{R_o^2 X_3}{R_o^2 + X_3^2}\right)$$

$$Z_m = \left(\frac{R_o X_3^2}{R_o^2 + X_3^2}\right) + j\left[\frac{X_2 (R_o^2 + X_3^2) - R_o^2 X_3}{R_o^2 + X_3^2}\right]$$

$$Z_i = \left(\frac{R_i X_1^2}{R_i^2 + X_1^2}\right) - j\left(\frac{R_i^2 X_1}{R_i^2 + X_1^2}\right) = \left(\frac{k R_o m^2 X_3^2}{k^2 R_o^2 + m^2 X_3^2}\right) - j\left(\frac{k^2 R_o^2 m X_3}{k^2 R_o^2 + m^2 X_3^2}\right)$$

To match Z_i with Z_m, begin with equating the Real parts of each:

$$\left(\frac{k R_o m^2 X_3^2}{k^2 R_o^2 + m^2 X_3^2}\right) = \left(\frac{R_o X_3^2}{R_o^2 + X_3^2}\right) \quad \text{then solve for } X_3\text{:}$$

$$X_3^2 = \frac{k R_o^2 \left(k - m^2\right)}{m^2 (k-1)} \quad \text{and by m relationship} \quad X_1^2 = \frac{k R_o^2 \left(k - m^2\right)}{(k-1)}$$

For a conjugate match, equate the Imaginary parts (sign on the right reverses):

$$\frac{X_2 (R_o^2 + X_3^2) - R_o^2 X_3}{R_o^2 + X_3^2} = \frac{k^2 R_o^2 m X_3}{k^2 R_o^2 + m^2 X_3^2} \quad \text{which becomes:}$$

$$R_o^2 X_3 \left[k^2 R_o^2 + k^2 m\left(R_o^2 + X_3^2\right) + m^2 X_3^2\right] = X_2\left[\left(k^2 R_o^2 + m^2 X_3^2\right)\left(R_o^2 + X_3^2\right)\right]$$

$$R_o^2 X_3\left(k^2 R_o^2 + m^2 X_3^2\right) + k^2 m R_o^2 X_3\left(R_o^2 + X_3^2\right) = X_2\left(R_o^2 + X_3^2\right)\left(k^2 R_o^2 + m^2 X_3^2\right)$$

now $R_o^2 + X_3^2 = R_o^2 + \left[\dfrac{k R_o^2\left(k - m^2\right)}{m^2 (k-1)}\right] = \dfrac{R_o^2\left(k^2 - m^2\right)}{m^2 (k-1)}$ is substituted:

$$R_o{}^2X_3\left[\frac{k^2R_o{}^2\left(k^2-m^2\right)}{m\,(k-1)}\right]=\left(k^2R_o{}^2+m^2X_3{}^2\right)\left[\frac{X_2R_o{}^2\left(k^2-m^2\right)-R_o{}^2X_3m^2(k-1)}{m^2(k-1)}\right]$$

multiplying both sides by $\frac{m^2(k-1)}{R_o{}^2}$:

$$k^2m\,R_o{}^2X_3\left(k^2-m^2\right)=\left(k^2R_o{}^2+m^2X_3{}^2\right)\left[X_2\left(k^2-m^2\right)-m^2X_3R_o{}^2(k-1)\right]$$

then substituting $X_3{}^2=\frac{k\,R_o{}^2\left(k-m^2\right)}{m^2(k-1)}$:

$$k^2m\left(k^2-m^2\right)R_o{}^2X_3=\left[\frac{k^2R_o{}^2(k-1)+k\,R_o{}^2\left(k-m^2\right)}{k-1}\right]\left[X_2\left(k^2-m^2\right)-X_3m^2(k-1)\right]$$

and dividing both sides by $\left(k\cdot R_o{}^2\right)$:

$$mX_3k\left(k^2-m^2\right)=\left[\frac{k^2-m^2}{k-1}\right]\left[X_2\left(k^2-m^2\right)-X_3m^2(k-1)\right]$$

divide both sides by $\left(k^2-m^2\right)$ and re-arrange:

$$X_3\left(k^2m-km+km^2-m^2\right)=X_2\left(k^2-m^2\right)$$ expanding some terms:

$$X_3m(k+m)(k-1)=X_2(k+m)(k-m)$$ which reduces to:

$$X_3\,m\,(k-1)\;=\;X_2(k-m)$$ substituting $X_3=\left(\frac{R_o}{m}\right)\sqrt{\frac{k\left(k-m^2\right)}{k-1}}$:

$$X_2=\frac{R_o\sqrt{k\left(k-m^2\right)(k-1)}}{k-m}\qquad X_1=R_o\sqrt{\frac{k\left(k-m^2\right)}{k-1}}\qquad X_3=\frac{X_1}{m}$$

The Q at resonance may be found by (X_2 / R) as shown in Figure 9-7. R in the right-hand transformation is the series-equivalent sums of the two R-primes in the middle. Note that the reactance could also be the series-equivalent capacitive reactance magnitudes divided by R since, at the operating frequency capacitive and inductive reactances magnitudes are equal.

Appendix 9-2

A Method of Calculating Frequency Response of Ladder Networks

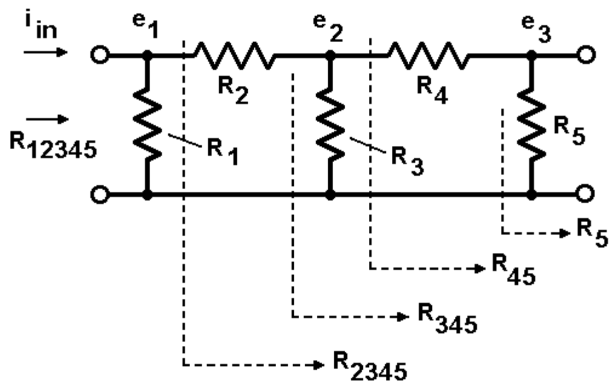

Figure 9-12 Generic resistor ladder arrangement to illustrate calculation method.

A *ladder* network is an arrangement of alternating shunt and series components. One such arrangement is shown in Figure 9-12 using resistors. The resistors could be replaced by impedances in an actual application.

The ladder arrangement is found in all of the Chapter 9 matching networks. It is convenient to have some frequency response analysis tool to determine the relative response of a network away from resonance frequency. The response of a ladder network may be analyzed as a succession of voltage dividers:

$$e_3 = e_2\left(\frac{R_5}{R_{45}}\right) \quad \text{where } R_{45} = R_4 + R_5 \qquad e_2 = e_1\left(\frac{R_{345}}{R_2 + R_{345}}\right) \qquad R_{345} = \frac{R_3 R_{45}}{R_3 + R_{45}}$$

$$\text{If } i_{in} = 1.0 \text{ then } e_1 = R_{12345} = \frac{R_1 R_{2345}}{R_1 + R_{2345}} \quad \text{where } R_{2345} = R_2 + R_{345}$$

This can also be stated in terms of both resistance and conductance as:

$$e_3 = e_2 R_5 G_{45} \quad \text{where } G_{45} = \frac{1}{R_4 + R_5}$$

$$e_2 = e_1 R_{345} G_{2345} \quad \text{where } R_{345} = \frac{1}{G_3 + G_{45}} \text{ and } G_{2345} = \frac{1}{R_2 + R_{345}} \text{ and if } i_{in} = 1$$

$$e_1 = R_{12345} = \frac{1}{G_1 + G_{2345}} \qquad \text{then, by substitution:}$$

$$e_3 = e_1 R_5 G_{45} R_{345} G_{2345} = R_5 G_{45} R_{345} G_{2345} R_{12345} = e_{out} \text{ with } i_{in} = 1$$

The output voltage value is then a relative one. Since every frequency will have the input current at unity, output voltage is then a function of all branches of the ladder network at that frequency.

This yields an algorithm ideally suited to a small programmable calculator. All the series branches would be precalculated as impedances, all the shunt branches would be precalculated as admittances at each frequency. Calculation would work from the output end and progress towards the input end. A pair of storage registers would hold a temporary complex number. At start the temporary storage would hold a complex unity of (1 + j 0). The iterations would then be:

1. Invert the rearmost shunt admittance to an impedance, multiply the temporary storage by that impedance.
2. Add the next-forward series impedance to the impedance found by inverting the branch admittance.
3. Divide the temporary storage by the impedance sum found in step (2). This completes one voltage divider.
4. Invert the impedance sum found in step (2).
5. Add the admittance of the next-forward branch to the admittance found in step (4).
6. Invert this admittance to an impedance and multiply the temporary storage by the admittance sum found in step (5).

7a. If this is not the branch closest to the input, repeat steps (2) through (6) for the next series and shunt pair.

7b. If this is the most forward shunt branch, display it as the input impedance.

8. Recall the temporary storage complex number as the relative network output voltage at that frequency.

Ending step (8) could also include division of the output voltage by another temporary which is a reference voltage applicable to all analysis frequencies. The result of the division could be converted to a decibel value.

References for Chapter 9

[20] HP-67 calculator programs 3973D, 3975D, 3976D, 4361D, 4362D, 4364D submitted to and accepted by the Hewlett-Packard Company User's Library, 1000 N.E. Circle Boulevard, Corvallis, Oregon 97330 during 1978-1979. The program documentation contain most of the formulas found in this chapter. Unfortunately, this extensive library of calculator programs was discontinued by the early 1990s with no further support from Hewlett-Packard. Many of the programs live on in private collections as well as internal corporate program collections such as at Rockwell International.

[21] *Admittance, Impedance, and Circuit Analysis,* Ham Radio magazine, August 1977, by the author. A general tutorial on the subject.

[22] *Calculator-Aided Circuit Analysis,* Ham Radio magazine, October 1977, by the author. This outlined the ladder network analysis method given in Appendix 9-2.

[23] *Pi Network Design*, Ham Radio magazine, March 1978, by the author.

[24] *How to Design Matching Networks*, Ham Radio magazine, April 1978, by the author. Covers the general area of matching impedances.

[25] *Quick and Simple Antenna Match*, Ham Radio magazine, January 1981, by the author. Easy methods of antenna matching, primarily by the 2-component networks of Chapter 8.

Ham Radio magazine and its parent publisher, Communications Technology Inc., were sold to CQ Communications in 1990 and ***HR*** ceased publication after 22 years. Their entire 22-year history of articles is available on three CD-ROMs available from CQ or the American Radio Relay League (ARRL) for the 2007-year price of $150.

[26] *Design and Match RF Amplifiers*, Electronic Design magazine, May 10, 1969, by Gunnar Richwell. This has two QUIKTRAN (similar to FORTRAN format) program listings, RFAMP and ZMATCH. The latter program finds a maximum of 28 combinations of 3 components for interstage matching.

Chapter 10

Basic L-C Filters

Passive filters of inductors and capacitors may be designed to select whole *groups* of frequencies in any wide to narrow frequency span. There are four basic types: Lowpass, Highpass, Bandpass, and Band-stop. All begin with the *lowpass prototype.* Lowpass prototype component values are presented in tables for 1 Hz, 1 Ohm terminating resistance. Design involves *scaling* the lowpass prototype table values for frequency and resistance. Elliptic lowpass filters are special cases of the Chebyshev.

General Passive Filters

There are four basic passive filter structures: *Lowpass, Highpass, Bandpass,* and *Bandstop* or Band-reject. The *lowpass* filter will pass all frequencies below a -3 db cutoff frequency, attenuating everything above that cutoff frequency. The *highpass* filter does just the opposite, passing all frequencies above the cutoff while attenuating those below. The *bandpass* filter will pass frequencies within a given band while attenuating all those above and below the band. The *bandstop* filter is the opposite of the bandpass, attenuating frequencies within a specified band while passing all those above and below the band.

The *passband* of a filter is that frequency region where very little attenuation takes place. The *stopband* is the frequency region where relatively more attenuation is supposed to happen. The frequency of transition between passband and stopband is the *cutoff* frequency, generally regarded as being -3 db in amplitude response relative to maximum output in the passband.

In *modern network theory* the *lowpass prototype* structure and values are used for lowpass filters and, with simple transformation, the start of highpass, bandpass, and bandstop design.[1]

Frequency Domain Response

Amplitude and *group delay* versus frequency describe passive filter characteristics both in the passband and stopband. *Group delay* is the small increment of phase change over a small increment of frequency change. It is an actual signal delay through the filter. Response types consist of:

Bessel: Linear phase change or constant time delay through the passband. Amplitude

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[1] *Modern network theory* in filters began just after the end of World War 2 of 1945 so it is hardly in the modern category. Those that came before were, among several types, the *m-derived* and *constant-k*. Whole books can, and have been written about all of them in detail. These chapters on passive filters will concentrate on design of realizeable *modern network theory* types.

response varies from 0 db to -3 db through passband and stopband attenuation is relatively low. This is an ideal filter to pass complex waveforms or wide-band modulations with minimum distortion.

Butterworth: Sometimes called *maximally flat* (amplitude response), a Butterworth response is characterized by very little amplitude change in the passband with a slight increase of time delay near cutoff frequency.

Chebyshev: Passband amplitude varies over a specified peak-to-peak amplitude of 0 db to a specified minus-db *ripple* value. The ripple nulls versus frequency correspond to the number of filter sections. Chebyshev stopband attenuation is greater than Butterworth designs. Attenuation is greater for higher passband ripple. Time delay is relatively high near cutoff frequency.

Cauer: Also called *Elliptic-Function* or just *elliptic*, these also have ripple in the passband plus one or more very great attenuation nulls in the stopband. The transition from cutoff frequency to maximum attenuation is the most abrupt of the four types. Time delay is as great as the Chebyshev near cutoff frequency. Elliptic filters add resonant *trap* circuits that correspond to the stopband attenuation null frequency location.

Except for the elliptic filter, all response types have the same basic physical structure of alternating series-shunt-series-shunt ladder structures. Their responses are due to the relative inductor and capacitor values. Passband and stopband characteristics are covered in more detail under specific filter types.

The Lowpass Prototype

Figure 10-1 shows the basic structure of the ***lowpass prototype*** that is the basis for all passive L-C filters. Intuitively, the lowpass can be seen to pass DC directly. As the frequency increases, the series inductors will have more reactance and the shunt capacitors will have less reactance. The interaction of the relative reactance values results in the sudden cutoff or change in amplitude response from passband to stopband. Each end of any filter is terminated in a resistance. In this chapter all filters will have equal or near-equal end resistances. End resistances are generally called *terminations.*

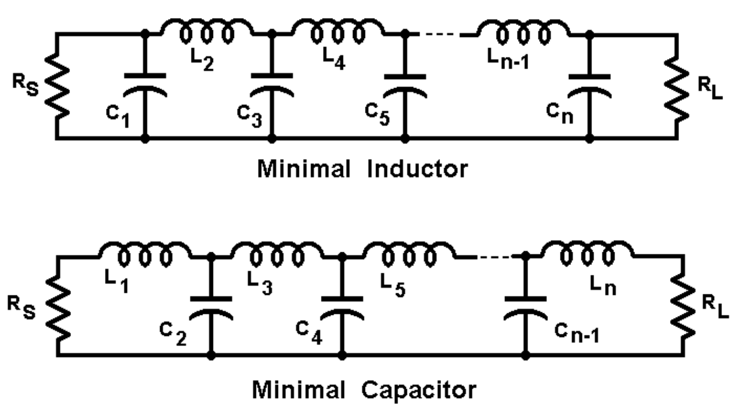

Figure 10-1 The lowpass prototype in two versions. Component subscripts are successive from source to load.

The total number of inductors and capacitors describes the number of *sections* of a passive filter. In Figure 10-1 the **n** subscript refers to the number of sections and the last section. The **n-1** subscript is then next-to-last section.

Note that there are two forms of the lowpass prototype in Figure 10-1. Each acts the same way as the other. The choice is whether or not odd-number filters would have more inductors or more capacitors. Even-number filters would have the same number of each.

Normalized Values

Normalization is a term for a baseline condition where values are related in ratio to the baseline value. To fine a true value from a normalized value, multiply the normalized by the ratio of true to normalization baseline. For a given filter type (Butterworth, Chebyshev, etc.), all reactance values have specific ratios to one another and to the termination resistance values. For all passive filters the *normalization* condition is 1 Hertz radian frequency and 1 Ohm termination resistance in other texts. In here the tables are precalculated for 1 Hz normalized frequency, the 2π of the radian frequency taken care of in table values.

In conventional filter texts:

$$L_n = \frac{R\,k_n}{\omega} \qquad C_n = \frac{k_n}{\omega R} \qquad \text{where:} \qquad (10-1)$$

n = Subscript number for a filter section

k_n = Table value for a particular section number n

L_n, C_n = Inductor or capacitor value at section n, Hy and Fd respectively

R = End resistance / termination in Ohms

ω = Radian frequency = $2\pi f$, where f is frequency in Hz

Tables in here already account for the 2π so the 3-section Butterworth lowpass table would have:

	In This Designbook			In Conventional Texts		
	k1	k2	k3	k1	k2	k3
Value:	0.15915	0.31831	0.15915	1.00000	1.00000	1.00000

The component value scaling would then be:

Component scaling in this Designbook are then:

$$L_n = \frac{R\,k_n}{f_B} \qquad C_n = \frac{k_n}{f_B\ R} \qquad \text{where:} \qquad (10-2)$$

n = Subscript number for a filter section

k_n = Table value for a particular section number n

L_n, C_n = Inductor or capacitor value at section n, Hy and Fd respectively

R = End resistance / termination in Ohms

f_B = Design cutoff frequency in Hz

The example's first section could be a shunt capacitor with the second section having a series inductor. Third section would be a shunt capacitor with same value as the first section Calculations would then be:

$$C_1 = \frac{k_1}{R \cdot f_B} = \frac{0.15915}{50 \cdot 10 \cdot 10^6} = 318.30 \text{ pFd} = C_3 \text{ [since } k_3 = k_1\text{]}$$

$$L_2 = \frac{R \cdot k_2}{f_B} = \frac{50 \cdot 0.31831}{10 \cdot 10^6} = 1.5916 \text{ mHy}$$

If the first section was a series inductor, the second a shunt capacitor, the third a series inductor equal in value to the first, the calculations would be:

$$L_1 = \frac{R \cdot k_1}{f_B} = \frac{50 \cdot 0.15915}{10 \cdot 10^6} = 0.79575 \text{ mHy} = L_3 \text{ [since } k_3 = k_1\text{]}$$

$$C_2 = \frac{k_2}{R \cdot f_B} = \frac{0.31831}{50 \cdot 10 \cdot 10^6} = 636.62 \text{ pFd}$$

What is being calculated is *ideal values*. Since those aren't stock items one has to get as close as possible. For example, with the minimal inductor version, C_1 and C_3 might be selected as 330 pFd (on the high side) while L_2 is 1.5 μHy (on the low side). Will that work? The answer lies in calculation of response over frequency.[2] Input-output response in db would be:

Frequency, MHz	Ideal Values	5% Tolerance Values
10.0	-3.01	-2.66
12.6	-6.97	-6.65
15.9	-12.3	-12.1
20.0	-18.1	-18.0
25.1	-24.0	-24.0
31.6	-30.0	-30.0
39.8	-36.0	-36.0
50.1	-42.0	-42.1
63.1	-48.0	-48.1
79.4	-54.0	-54.1
100	-60.0	-60.1

The input impedance (with input end resistor removed) of the ideal-value version is 50 Ohms through 2 MHz, rising to 100 Ohms at 7 MHz, then 112 Ohms at 10 MHz. The 5-percent tolerance value version is 49.8 Ohms at 1 MHz rising to 50 Ohms at 3 MHz to 89.1 Ohms at 7 MHz and finally 110.6 Ohms at 10 MHz. That's fairly close, but everything would depend on the amount of attenuation in the stopband desired plus the passband response.

Determination of Filter Type and Sections

[2] An obvious tool is CAE or Computer-Assisted Engineering computer programs to do frequency-domain analysis. Lacking that, a scientific calculator can do small filters by the method given in Appendix 9-3.

Figure 10-2 shows the stopband response of Butterworth lowpass filters based on cutoff or bandwidth (f_B) design frequency. Butterworth filters have a slope of 6 db per octave per section attenuation in the stopband.

Butterworth passband response is always -3.01 db at the cutoff frequency, about -1.0 db at 0.707 f_B. Zero attenuation at DC.

The passband response of Chebyshev and Elliptic filters is specified by the amount of *ripple* in db. This ripple value describes the passband response varying from 0 db to the specified minus-db of ripple. The designed-in ripple value has a great effect on the stopband attenuation of Chebyshev filters and the *rate of cutoff* or slope of attenuation above the f_B frequency.

By way of explanation of ripple, Figure 10-3 shows two lowpass filters' passband response for 3-section filters of the same f_B. In grey is the Butterworth response. In black is a 3 db ripple Chebyshev 3-section filter passband for comparison. The Chebyshev response drops down to -3 db at about half the cutoff frequency, then up again to 0 db at about 0.9 f_B, then down to -3 db at exactly f_B. If this were a 1 db ripple Chebyshev or Elliptic, the dip at half cutoff would be -1 db and the level at exactly cutoff frequency would also be -1 db.

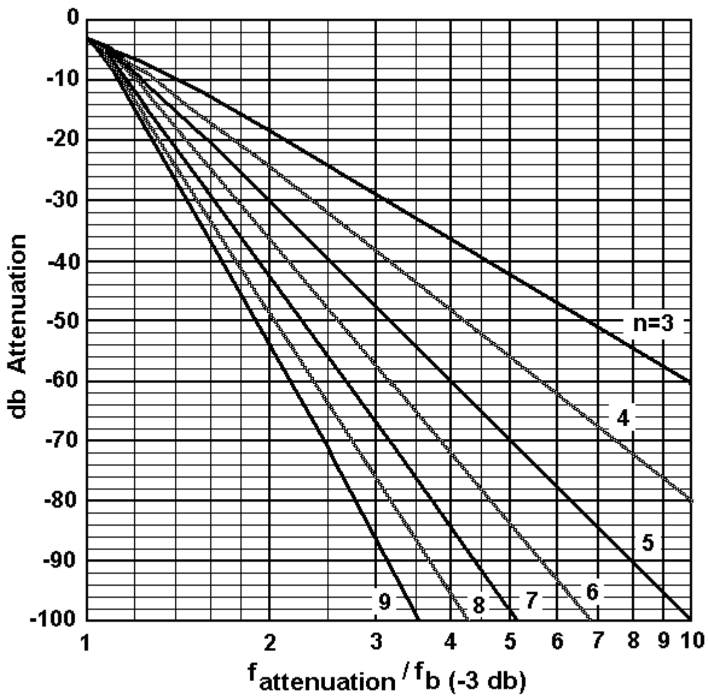

Figure 10-2 Stopband attenuation of Butterworth response filters from 3 to 9 sections relative to cutoff frequency. Slopes are straight-line on log-log graphs from -24 db and greater attenuation.

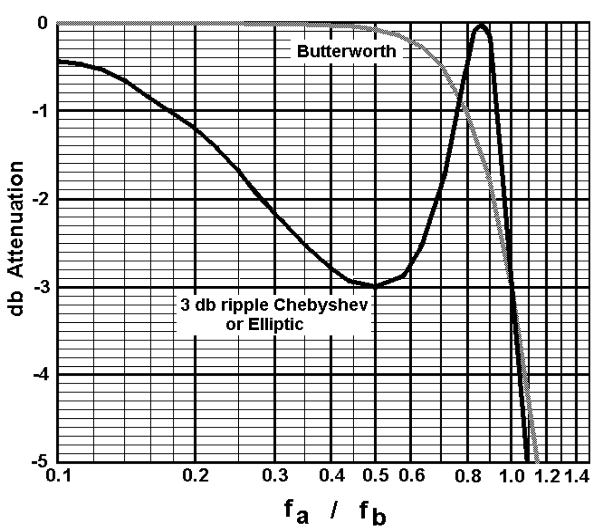

Figure 10-3 Passband response relative to f_B for a 3-section Butterworth lowpass filter and a 3-section Chebyshev filter with 3 db ripple.

Dips to ripple value applies only to odd-section-number Chebyshev filters. Even-section-number Chebyshev filters have *peaks.* These peaks are *above* the DC level reference of 0 db and the output level is always 0 db at f_B for even-number Chebyshevs The DC level of even-number Chebyshev filters versus odd-numbers is raised by the amount shown in Table 10-10.

For large value ripple filter needs, the non-flat response may make a difference in application. Designers must consider the passband response shape.

Figure 10-4 shows a comparison of seven ideal 5-section lowpass filters' stopbands, a Butterworth with six Chebyshev filters ranging from 0.01 db ripple to 3 db ripple. It is obvious that greater allowable passband ripple results in greater stopband attenuation.

If passband ripple is considered a problem for certain frequencies yet a high stopband attenuation is necessary, it is worthwhile considering adding more sections while holding to a lower ripple value.

Stopband attenuation at selected relative-to-f_B frequencies is given in Tables 10-12 to 10-16 for Chebyshev response filters having a ripple range from 0.25 db to 3 db. Attenuation at other stopband frequencies can be extrapolated from table values. Attenuation has a general straight-line slope for all filters when plotted with a logarithmic frequency scale.

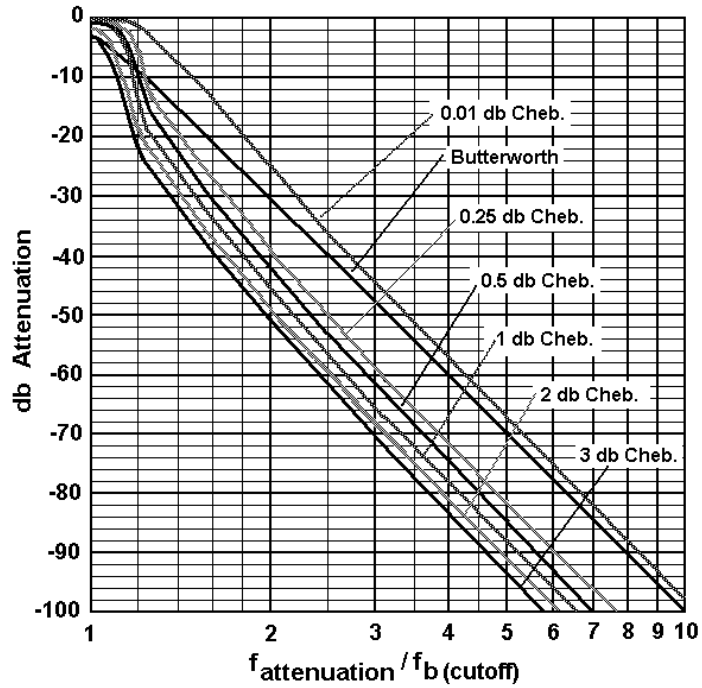

Figure 10-4 Comparison of 5-section lowpass filters.

The number of sections with a given ripple (if Chebyshev) should be selected to allow at least 6 db greater attenuation at the frequency of interest. This provides a safety margin against non-ideal values and a possibility that inductor distributed capacity may affect actual attenuation.

Table 10-1
Normalized Values for Butterworth Response Lowpass Filters

Sections	k1	k2	k3	k4	k5	k6	k7	k8	k9
3	0.1592	0.3183	0.1592	–	–	–	–	–	--
4	0.1218	0.2941	0.2941	0.1218	–	–	–	–	--
5	.09836	0.2575	0.3183	0.2575	.09836	–	–	–	--
6	.08239	0.2251	0.3075	0.3075	0.2251	.08239	–	–	--
7	.07083	0.1985	0.2868	0.3183	0.2868	0.1985	.07083	–	--
8	.06210	0.1768	0.2647	0.3122	0.3122	0.2647	0.1768	.06210	--
9	.05527	0.1592	0.2430	0.2991	0.3183	0.2991	0.2438	0.1592	.05527

Table 10-2

Normalized Values for Chebyshev Response Lowpass Filters
Passband Ripple = 0.01 db

Sections	k1	k2	k3	k4	k5	k6	k7	k8	k9
3	0.1001	0.1544	0.1001	–	–	–	–	–	--
4	0.1135	0.1910	0.2103	0.1031*	–	–	–	–	--
5	0.1204	0.2077	0.2510	0.2077	0.1204	–	–	–	--
6	0.1244	0.2165	0.2689	0.2443	0.2383	0.1130*	–	–	--
7	0.1268	0.2216	0.2782	0.2599	0.2782	0.2216	0.1268	–	--
8	0.1285	0.2249	0.2837	0.2679	0.2949	0.2577	0.2476	0.1167*	--
9	0.1296	0.2271	0.2872	0.2726	0.3033	0.2726	0.2872	0.2271	0.1296

Table 10-3

Normalized Values for Chebyshev Response Lowpass Filters
Passband Ripple = 0.1 db

Sections	k1	k2	k3	k4	k5	k6	k7	k8	k9
3	0.1642	0.1826	0.1642	–	–	–	–	–	--
4	0.1765	0.2079	0.2818	0.1302*	–	–	–	–	--
5	0.1825	0.2182	0.3143	0.2182	0.1825	–	–	–	--
6	0.1859	0.2235	0.3273	0.2415	0.3029	0.1372*	–	–	--
7	0.1880	0.2265	0.3337	0.2504	0.3337	0.2265	0.1880	–	--
8	0.1894	0.2283	0.3374	0.2548	0.3454	0.2489	0.3055	0.1397*	--
9	0.1903	0.2296	0.3397	0.2573	0.3510	0.2573	0.3397	0.2296	0.1903

Table 10-4

Normalized Values for Chebyshev Response Lowpass Filters
Passband Ripple = 0.25 db

Sections	k1	k2	k3	k4	k5	k6	k7	k8	k9
3	0.2074	0.1824	0.2074	–	–	–	–	–	--
4	0.2194	0.2020	0.3272	0.1354*	–	–	–	–	--
5	0.2251	0.2098	0.3567	0.2098	0.2751	–	–	–	--
6	0.2283	0.2136	0.3681	0.2273	0.3460	0.1410*	–	–	--
7	0.2303	0.2158	0.3736	0.2338	0.3736	0.2158	0.2303	–	--
8	0.2316	0.2172	0.3768	0.2369	0.3837	0.2327	0.3518	0.1430*	--
9	0.2324	0.2181	0.3788	0.2387	0.3886	0.2387	0.3788	0.2181	0.2324

Table 10-5
Normalized Values for Chebyshev Response Lowpass Filters
Passband Ripple = 0.5 db

Sections	k1	k2	k3	k4	k5	k6	k7	k8	k9
3	0.2541	0.1745	0.2541	–	–	–	–	–	--
4	0.2658	0.1898	0.3766	0.1340*	–	–	–	–	--
5	0.2715	0.1957	0.4044	0.1957	0.2715	–	–	–	--
6	0.2746	0.1986	0.4148	0.2091	0.3940	0.1384*	–	–	--
7	0.2765	0.2003	0.4199	0.2140	0.4199	0.2003	0.2765	–	--
8	0.2777	0.2013	0.4228	0.2163	0.4292	0.2131	0.3994	0.1400*	--
9	0.2786	0.2020	0.4246	0.2176	0.4335	0.2176	0.4246	0.2020	0.2786

Table 10-6
Normalized Values for Chebyshev Response Lowpass Filters
Passband Ripple = 1.0 db

Sections	k1	k2	k3	k4	k5	k6	k7	k8	k9
3	0.3221	0.1582	0.3221	–	–	–	–	–	--
4	0.3341	0.1694	0.4506	0.1256*	–	–	–	–	--
5	0.3398	0.1737	0.4776	0.1737	0.3398	–	–	–	--
6	0.3429	0.1757	0.4876	0.1833	0.4674	0.1289*	–	–	--
7	0.3448	0.1769	0.4924	0.1868	0.4924	0.1769	0.3448	–	--
8	0.3461	0.1776	0.4951	0.1884	0.5011	0.1861	0.4725	0.1301*	--
9	0.3469	0.1781	0.4968	0.1893	0.5053	0.1893	0.4968	0.1781	0.3469

Table 10-7
Normalized Values for Chebyshev Response Lowpass Filters
Passband Ripple = 2.0 db

Sections	k1	k2	k3	k4	k5	k6	k7	k8	k9
3	0.4314	0.1325	0.4314	–	–	–	–	–	--
4	0.4444	0.1402	0.5740	0.1085*	–	–	–	–	--
5	0.4506	0.1430	0.6020	0.1430	0.4506	–	–	–	--
6	0.4539	0.1444	0.6122	0.1495	0.5913	0.1108*	–	–	--
7	0.4560	0.1452	0.6171	0.1518	0.6171	0.1452	0.4560	–	--
8	0.4573	0.1456	0.6199	0.1529	0.6260	0.1514	0.5965	0.1117*	--
9	0.4582	0.1460	0.6216	0.1535	0.6302	0.1535	0.6216	0.1460	0.4582

Table 10-8
Normalized Values for Chebyshev Response Lowpass Filters
Passband Ripple = 3.0 db

Sections	k1	k2	k3	k4	k5	k6	k7	k8	k9
3	0.5330	0.1133	0.5330	–	–	–	–	–	--
4	0.5473	0.1191	0.6919	.09422*	–	–	–	–	--
5	0.5541	0.1213	0.7222	0.1213	0.5541	–	–	–	--
6	0.5578	0.1223	0.7331	0.1262	0.7105	.09602*	–	–	--
7	0.5600	0.1229	0.7383	0.1279	0.7383	0.1229	0.5600	–	--
8	0.5615	0.1233	0.7413	0.1288	0.7479	0.1276	0.7160	.09665	--
9	0.5624	0.1235	0.7431	0.1292	0.7523	0.1292	0.7431	0.1235	0.5624

Table 10-9
Normalized Values for Bessel (Linear Delay) Response Lowpass Filters

Sections	k1	k2	k3	k4	k5	k6	k7	k8	k9
3	.05370	0.1545	0.3507	–	–	–	–	–	--
4	.03715	0.1070	0.1721	0.3566	–	–	–	–	--
5	.02774	.08072	0.1280	0.1768	0.3594	–	–	–	--
6	.02173	.06369	0.1017	0.1359	0.1771	0.3604	–	–	--
7	.01760	.05187	.08354	0.1117	0.1383	0.1759	0.3606	–	--
8	.01463	.04327	.07017	.09447	0.1162	0.1384	0.1744	0.3606	--
9	.01241	.03681	.06000	.08130	0.1004	0.1179	0.1375	0.1729	0.3605

Table 10-10
Normalized Load End Resistance for Even-Section Chebyshev Filters

Passband Ripple, db	Load-End Resistance	DC shift, db
0.01	1.101	+0.41
0.25	1.620	+1.85
0.5	1.984	+2.48
1	2.660	+3.25
2	4.095	+4.12
3	5.801	+4.64

Table 10-11

Peaks, Dips of Chebyshev and Elliptic Filter Passband Frequency Relative to f_B

	Number of Sections						
	3	**4**	**5**	**6**	**7**	**8**	**9**
Dip:	0.50	0.00	0.31	0.00	0.23	0.00	0.17
	1.00	0.71	0.81	0.50	0.62	0.38	0.50
		1.00	1.00	0.84	0.90	0.71	0.77
				1.00	1.00	0.92	0.94
						1.00	1.00
Peak:	0.00	0.38	0.00	0.26	0.00	0.19	0.00
	0.87	0.94	0.60	0.71	0.43	0.56	0.34
			0.96	0.97	0.78	0.83	0.64
					0.98	0.98	0.86
							0.98

Table 10-12

Stopband Response, db, Butterworth Lowpass Filters, Relative to fb Cutoff Frequency

fa / fb	n=3	n=4	n=5	n=6	n=7	n=8	n=9
1.0	-3.01	-3.01	-3.01	-3.01	-3.01	-3.01	-3.01
1.5	-10.9	-14.3	-17.7	-21.2	-24.7	-28.2	-31.7
2	-18.1	-24.1	-30.1	-36.1	-42.1	-48.2	-54.2
3	-28.6	-38.2	-47.7	-57.3	-66.8	-76.3	-85.9
4	-36.1	-48.2	-60.2	-72.3	-84.3	-96.3	-108.4
5	-41.9	-55.9	-69.9	-83.9	-97.9	-111.8	-125.8

Table 10-13

Stopband Response, db, Chebyshev Lowpass Filters, Relative to fb Cutoff Frequency
Passband Ripple = 0.1 db

fa / fb	n=3	n=4	n=5	n=6	n=7	n=8	n=9
1.0	-0.1	0	-0.1	0	-0.1	0	-0.1
1.5	-4.6	-11.3	-19.5	-27.7	-36.2	-44.4	-52.9
2	-12.2	-23.3	-34.9	-46.2	-57.7	-69.1	-80.6
3	-23.6	-38.8	-54.2	-69.4	-84.8	-100.0	-115.5
4	-31.4	-49.2	-67.3	-85.1	-103.1	-120.9	-139.0
5	-37.4	-57.2	-77.2	-97.0	-117.0	-136.9	-156.9

Table 10-14

Stopband Response, db, Chebyshev Lowpass Filters, Relative to fb Cutoff Frequency
Passband Ripple = 0.25 db

fa / fb	n=3	n=4	n=5	n=6	n=7	n=8	n=9
1.0	-0.25	0	-0.25	0	-0.25	0	-0.25
1.5	-7.6	-15.0	-23.5	-31.6	-40.2	-48.9	-56.9
2	-16.1	-27.2	-38.9	-50.1	-61.8	-73.0	-84.7
3	-27.7	-42.7	-58.3	-73.3	-88.9	-103.9	-119.5
4	-35.5	-53.2	-71.3	-89.0	-107.2	-124.8	-142.0
5	-41.4	-61.1	-81.3	-100.9	-121.1	-140.8	-160.9

Table 10-15

Stopband Response, db, Chebyshev Lowpass Filters, Relative to fb Cutoff Frequency
Passband Ripple = 0.5 db

fa / fb	n=3	n=4	n=5	n=6	n=7	n=8	n=9
1.0	-0.5	0	-0.5	0	-0.5	0	-0.5
1.5	-10.4	-17.9	-26.7	-34.5	-43.4	-51.2	-60.1
2	-19.2	-30.1	-42.0	-53.0	-64.9	-75.9	-87.8
3	-30.6	-45.6	-61.4	-76.2	-92.0	-106.8	-122.6
4	-38.6	-56.0	-74.5	-91.9	-110.3	-127.7	-146.2
5	-44.6	-64.0	-84.4	-103.8	-124.2	-143.6	-164.1

Table 10-16

Stopband Response, db, Chebyshev Lowpass Filters, Relative to fb Cutoff Frequency
Passband Ripple = 1 db

fa / fb	n=3	n=4	n=5	n=6	n=7	n=8	n=9
1.0	-1	0	-1	0	-1	0	-1
1.5	-13.4	-20.6	-29.9	-37.3	-46.6	-54.0	-63.4
2	-22.5	-32.9	-45.3	-55.7	-68.2	-78.6	-91.1
3	-34.1	-48.4	-64.7	-79.0	-95.3	-109.6	-125.9
4	-41.9	-58.8	-77.7	-94.7	-113.6	-130.5	-149.4
5	-47.9	-66.8	-87.7	-106.6	-127.5	-146.4	-167.3

Table 10-17

Stopband Response, db, Chebyshev Lowpass Filters, Relative to fb Cutoff Frequency
Passband Ripple = 2 db

fa / fb	n=3	n=4	n=5	n=6	n=7	n=8	n=9
1.0	-2	0	-2	0	-2	0	-2
1.5	-16.7	-23.1	-33.5	-39.8	-50.2	-56.5	-66.9
2	-26.0	-35.4	-48.8	-58.3	-71.7	-81.2	-94.6
3	-37.6	-50.9	-68.2	-81.5	-98.8	-112.1	-129.5
4	-45.4	-61.3	-81.3	-97.2	-117.1	-133.0	-153.0
5	-51.4	-69.3	-91.2	-109.1	-131.0	-148.9	-170.9

Table 10-18

Stopband Response, db, Chebyshev Lowpass Filters, Relative to fb Cutoff Frequency
Passband Ripple = 3 db

fa / fb	n=3	n=4	n=5	n=6	n=7	n=8	n=9
1.0	-3	0	-3	0	-3	0	-3
1.5	-19.1	-24.4	-35.8	-41.1	-52.5	-57.8	-69.2
2	-28.3	-36.7	-51.2	-59.6	-74.0	-82.5	-96.9
3	-39.9	-52.2	-70.5	-82.8	-101.1	-113.5	-131.8
4	-47.7	-62.7	-83.6	-98.5	-119.4	-134.3	-155.3
5	-53.7	-70.6	-93.5	-110.4	-133.3	-150.3	-173.2

Table 10-19

Stopband Response, db, Elliptic Lowpass Filters, Relative to fb Cutoff Frequency
Passband Ripple = 0.1 db

fa / fb	n=3	n=5	n=7	n=9	
1.0	-0.1	-0.1	-0.1	-0.1	
1.1	-0.5	-2.4	-7.8	-15.5	
1.2	-1.4	-8.2	-19.4	-31.3	
1.3	-2.9	-14.6	-29.1	-43.8	
1.4	-5.0	-20.5	-37.6	-54.7	
1.5	-7.5	-26.1	-45.4	-64.8	
1.6	-10.3	-31.5	-53.0	-74.6	
1.8	-16.5	-42.8	-68.9	-94.6	
2.0	-24.0	-58.9	-93.8	-128.7	← A_{MIN}

Table 10-20

Stopband Response, db, Elliptic Lowpass Filters, Relative to fb Cutoff Frequency
Passband Ripple = 0.25 db

fa / fb	n=3	n=5	n=7	n=9	
1.0	-0.25	-0.25	-0.25	-0.25	
1.1	-1.1	-4.6	-11.4	-19.5	
1.2	-2.9	-11.8	-23.5	-35.4	
1.3	-5.3	-18.5	-33.2	-47.9	
1.4	-8.1	-24.5	-41.7	-58.8	
1.5	-11.1	-30.1	-49.5	-68.9	
1.6	-14.1	-35.5	-57.1	-78.6	
1.8	-20.4	-46.8	-72.9	-99.0	
2.0	-28.1	-63.0	-97.9	-132.8	← A_{MIN}

Table 10-21

Stopband Response, db, Elliptic Lowpass Filters, Relative to fb Cutoff Frequency
Passband Ripple = 0.5 db

fa / fb	n=3	n=5	n=7	n=9	
1.0	-0.5	-0.5	-0.5	-0.5	
1.1	-2.1	-6.9	-14.4	-22.6	
1.2	-4.7	-14.8	-26.6	-38.5	
1.3	-7.7	-21.6	-36.3	-51.0	
1.4	-10.9	-27.7	-44.8	-61.9	
1.5	-14.0	-33.3	-52.6	-72.0	
1.6	-17.2	-38.7	-60.2	-81.7	
1.8	-23.6	-49.9	-76.1	-102.1	
2.0	-31.2	-66.1	-101.0	-135.9	← A_{MIN}

Table 10-22

Stopband Response, db, Elliptic Lowpass Filters, Relative to fb Cutoff Frequency
Passband Ripple = 1 db

fa / fb	n=3	n=5	n=7	n=9	
1.0	-1	-1	-1	-1	
1.1	-3.6	-9.7	-17.6	-25.9	
1.2	-7.1	-18.0	-29.8	-41.8	
1.3	-10.6	-24.9	-39.6	-54.3	
1.4	-14.0	-30.9	-48.1	-65.2	
1.5	-17.2	-36.5	-55.9	-75.3	
1.6	-20.4	-42.0	-63.5	-85.0	
1.8	-26.8	-53.2	-79.3	-105.4	
2.0	-34.5	-69.4	-104.3	-139.2	← A_{MIN}

Table 10-23

Stopband Response, db, Elliptic Lowpass Filters, Relative to fb Cutoff Frequency
Passband Ripple = 2 db

fa / fb	n=3	n=5	n=7	n=9	
1.0	-2	-2	-2	-2	
1.1	-6.0	-12.9	-21.1	-29.4	
1.2	-10.1	-21.5	-33.4	-45.3	
1.3	-13.9	-28.4	-43.1	-57.8	
1.4	-17.4	-34.5	-51.6	-68.7	
1.5	-20.7	-40.1	-59.4	-78.8	
1.6	-23.9	-45.5	-67.0	-88.6	
1.8	-30.4	-56.8	-82.9	-108.9	
2.0	-38.8	-72.9	-107.8	-142.7	← A_{MIN}

Table 10-24

Stopband Response, db, Elliptic Lowpass Filters, Relative to fb Cutoff Frequency
Passband Ripple = 3 db

fa / fb	n=3	n=5	n=7	n=9	
1.0	-3	-3	-3	-3	
1.1	-7.8	-15.1	-23.4	-31.7	
1.2	-12.3	-23.8	-35.7	-47.6	
1.3	-16.2	-30.7	-45.4	-60.1	
1.4	-19.7	-36.8	-53.9	-71.0	
1.5	-23.0	-42.4	-61.8	-81.1	
1.6	-26.2	-47.8	-69.3	-90.9	
1.8	-32.7	-59.1	-85.2	-111.2	
2.0	-40.3	-75.2	-110.1	-145.0	← A_{MIN}

Elliptic (Cauer) Lowpass Filters

The Elliptic filter is an outgrowth of the Chebyshev, adding a capacitor in parallel with each series inductor. This introduces a high series impedance in certain stopband frequencies due to the parallel resonance of L_2 and C_2, L_4 and C_4, and so on. These deep attenuation nulls in the stopband, plus differing values of the shunt-series-shunt C-L-C elements, account for the rapid attenuation from f_B to $2f_B$.

A useful property of Elliptic filters is that the stopband above 2f will always be attenuated by at least **A_{MIN}** db. That stopband minimum attenuation value (or maximum level) is indicated by the little

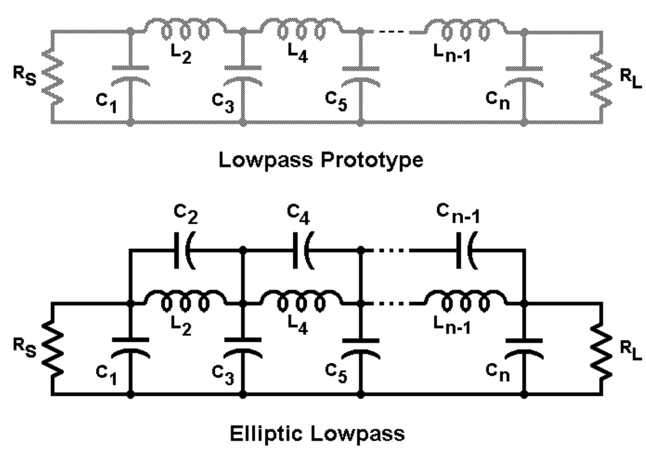

Figure 10-5 Elliptic lowpass (black, bottom) evolution from lowpass prototype (grey, top).

arrow and A_{MIN} on Tables 10-19 through 10-24 at twice the cutoff frequency.

New normalized (to 1 Hz, 1 Ohm end resistance) component values for Elliptic lowpass filters are found in Tables 10-25 through 10-30. Elliptic filters have only odd section numbers. The normalized stopband deep-null frequencies are indicated by f_{22}, f_{44}, f_{66}, and f_{88} on the tables. These correspond to the normalized parallel-resonant frequencies of the L_2 and C_2, L_4 and C_4, L_6 and C_6, L_8 and C_8 pairs. All of the tables' normalized values are converted to unnormalized values through equation (10-2).

Table 10-25

Normalized Values for Elliptic Response Lowpass Filters
Passband Ripple = 0.1 db

Sections	k1	k2	k3	k4	k5	k6	k7	k8	k9
3	0.1425	0.1492	0.1425	–	–	–	–	–	--
5	0.1731	0.2058	0.2855	0.1820	0.1555	–	–	–	--
7	0.1829	0.2196	0.3056	0.2152	0.2954	0.2022	0.1699	–	--
9	0.1871	0.2253	0.3179	0.2293	0.3082	0.2216	0.3054	0.2131	0.1780

	k22	k44	k66	k88	f22	f44	f66	f88
3	.03294	–	–	–	2.270	–	–	--
5	.01165	.03189	–	–	3.251	2.089	–	--
7	.006083	.02816	.02020	–	4.354	2.045	2.490	--
9	.003723	.02144	.02783	.01332	5.496	2.270	2.027	2.987

Table 10-26

Normalized Values for Elliptic Response Lowpass Filters
Passband Ripple = 0.25 db

Sections	k1	k2	k3	k4	k5	k6	k7	k8	k9
3	0.1850	0.1534	0.1850	–	–	–	–	–	--
5	0.2151	0.1983	0.3246	0.1773	0.1969	–	–	–	--
7	0.2248	0.2095	0.3428	0.2017	0.3316	0.1939	0.2112	–	--
9	0.2290	0.2141	0.3549	0.2131	0.3417	0.2061	0.3412	0.2032	0.2194

	k22	k44	k66	k88	f22	f44	f66	f88
3	.03205	–	–	–	2.270	–	–	--
5	.01209	.03273	–	–	3.251	2.089	–	--
7	.006378	.03005	.02106	–	4.354	2.045	2.490	--
9	.003917	.02307	.02993	.01397	5.496	2.270	2.027	2.987

Table 10-27
Normalized Values for Elliptic Response Lowpass Filters
Passband Ripple = 0.5 db

Sections	k1	k2	k3	k4	k5	k6	k7	k8	k9
3	0.2330	0.1491	0.2330	–	–	–	–	–	--
5	0.2605	0.1852	0.3683	0.1668	0.2412	–	–	–	--
7	0.2705	0.1945	0.3857	0.1850	0.3732	0.1806	0.2559	–	--
9	0.2749	0.1984	0.3981	0.1945	0.3816	0.1881	0.3229	0.1886	0.2645

	k22	k44	k66	k88	f22	f44	f66	f88
3	.03297	–	–	–	2.270	–	–	--
5	.01294	.03480	–	–	3.251	2.089	–	--
7	.006869	.03276	.02261	–	4.354	2.045	2.490	--
9	.004228	.02528	.03278	.01506	5.496	2.270	2.027	2.987

Table 10-28
Normalized Values for Elliptic Response Lowpass Filters
Passband Ripple = 1 db

Sections	k1	k2	k3	k4	k5	k6	k7	k8	k9
3	0.2948	0.1367	0.2948	–	–	–	–	–	--
5	0.3272	0.1646	0.4354	0.1489	0.3055	–	–	–	--
7	0.3379	0.1719	0.4527	0.1618	0.4382	0.1601	0.3214	–	--
9	0.3426	0.1750	0.4661	0.1694	0.4451	0.1639	0.4484	0.1666	0.3309

	k22	k44	k66	k88	f22	f44	f66	f88
3	.03595	–	–	–	2.270	–	–	--
5	.01457	.03897	–	–	3.251	2.089	–	--
7	.007725	.03746	.02552	–	4.354	2.045	2.490	--
9	.004793	.02902	.03763	.01704	5.496	2.270	2.027	2.987

Table 10-29
Normalized Values for Elliptic Response Lowpass Filters
Passband Ripple = 2 db

Sections	k1	k2	k3	k4	k5	k6	k7	k8	k9
3	0.3980	0.1155	0.3980	–	–	–	–	–	--
5	0.4351	0.1356	0.5492	0.1232	0.4088	–	–	–	--
7	0.4475	0.1411	0.5678	0.1316	0.5497	0.1317	0.4273	–	--
9	0.4529	0.1434	0.5835	0.1374	0.5556	0.1330	0.5615	0.1367	0.4386

	k22	k44	k66	k88	f22	f44	f66	f88
3	.04256	–	–	–	2.270	–	–	--
5	.01768	.04710	–	–	3.251	2.089	–	--
7	.009469	.04603	.03102	–	4.354	2.045	2.490	--
9	.005848	.03578	.04638	.02076	5.496	2.270	2.027	2.987

Table 10-30
Normalized Values for Elliptic Response Lowpass Filters
Passband Ripple = 3 db

Sections	k1	k2	k3	k4	k5	k6	k7	k8	k9
3	0.4934	.09907	0.4934	–	–	–	–	–	--
5	0.5356	0.1151	0.6590	0.1047	0.5046	–	–	–	--
7	0.5499	0.1195	0.6795	0.1110	0.6579	0.1116	0.5261	–	--
9	0.5561	0.1214	0.6977	0.1157	0.6634	0.1120	0.6715	0.1158	0.5393

	k22	k44	k66	k88	f22	f44	f66	f88
3	.04961	–	–	–	2.270	–	–	--
5	.02084	.05542	–	–	3.251	2.089	–	--
7	.01118	.05458	.03660	–	4.354	2.045	2.490	--
9	.006911	.04249	.05507	.02452	5.496	2.270	2.027	2.987

As an example of an Elliptic filter application, assume there is a need for a 6 MHz lowpass filter having 75 Ohm end terminations with at least 80 db attenuation at 12 MHz and higher. A cautious choice is a seven-section Elliptic with 0.1 db passband ripple. From Table 10-19 the attenuation at $2f_B$ (12 MHz) would be at least 93.8 db. From Table 10-25 the component values would be:

$$C_1 = \frac{k_1}{f_B R} = \frac{0.1829}{6 \cdot 10^6 \cdot 75} = \frac{0.1829}{450 \cdot 10^6} = 406.4 \text{ pFd}$$

$$L_2 = \frac{k_2 R}{f_B} = \frac{0.2196 \cdot 75}{6 \cdot 10^6} = \frac{16.418}{6 \cdot 10^6} = 2.736 \ \mu\text{Hy}$$

$$C_3 = \frac{k_3}{f_B R} = \frac{0.3056}{450 \cdot 10^6} = 679.1 \text{ pFd} \quad C_5 = 656.7 \text{ pFd} \quad C_7 = 377.5 \text{ pFd}$$

$$L_4 = \frac{k_4 R}{f_B} = \frac{0.2152 \cdot 75}{6 \cdot 10^6} = 2.690 \ \mu\text{Hy} \quad L_6 = 2.527 \ \mu\text{Hy}$$

The "added parallel resonating" capacitors are scaled the same way:

$$C_2 = \frac{k_{22}}{f_B R} = \frac{0.006083}{450 \cdot 10^6} = 13.52 \text{ pFd} \quad C_4 = 62.57 \text{ pFd} \quad C_6 = 44.90 \text{ pFd}$$

Parallel resonant frequencies are scaled only by frequency

$$f_{22} = 4.354 \cdot 6 \cdot 10^6 = 26.13 \text{ MHz} \quad f_{44} = 12.27 \text{ MHz} \quad f_{66} = 14.94 \text{ MHz}$$

The usual problem arises here: Exact values don't correspond to stock values.[3] Also, the finite Q of inductors will change the near-cutoff frequency response. Fortunately, the effects are only slight. All three inductors' exact values are close to 2.7 µHy, a 10% tolerance value. C1 and C7 are close to 390 pFd, C3 and C5 are close to 680 pFd. There might or might not be a problem with the "parallel resonant" capacitors so C2 is chosen as 15 pFd, higher than exact but then L2 is slightly lower than exact inductance. C4 is chosen as 68 pFd. C6 is picked to be 39 pFd, lower than exact but L6 is higher than exact.

This filter was analyzed with exact values and infinite Q, exact values with an inductor Q of 30, the ten-percent stock values given, and stock values with inductor Q of 30. As expected, the finite inductor Q caused some distortion of frequency response immediately around the cutoff frequency. This always happens and that includes highpass filters and, especially, bandpass and bandstop filters.

[3] See Appendix 10-1 for a large table that allows many "in between" values from stock tolerance parts.

	Example Filter Response, db			
Frequency, MHz	**Exact**	**Exact, Q=30**	**Stock Value**	**Stock, Q=30**
1.0	-.1	-.2	-.1	-.2
1.5	-.1	-.3	-.1	-.3
2.0	-.1	-.3	-.1	-.3
2.5	-.0	-.3	-.0	-.3
3.0	-.0	-.4	-.0	-.4
3.5	-.1	-.6	-.1	-.6
4.0	-.1	-.7	-.1	-.7
4.5	-.0	-.8	-.0	-.8
5.0	-.0	-1.0	-.1	-1.1
5.5	-.1	-1.4	-.1	-1.5
6.0	-.1	-2.3	-.7	-3.1
6.5	-5.8	-7.7	-8.2	-9.9
7.0	-15.8	-16.7	-17.9	-18.8
7.5	-24.5	-24.9	-26.3	-26.8
8.0	-32.0	-32.3	-33.8	-34.1
8.5	-38.9	-39.1	-40.7	-40.9
9.0	-45.4	-45.6	-47.2	-47.3
9.5	-51.8	-51.9	-53.8	-53.8
10.0	-58.1	-58.1	-60.4	-50.5
10.5	-64.7	-64.7	-67.6	-67.7
11.0	-71.8	-71.8	-76.2	-76.1
11.5	-80.4	-80.1	-89.9	-88.0
12.0	-93.8	-91.8	-93.7	-91.6

Note: The "-.0" db values are the result of rounding off and refer to a finite value between zero and one-tenth db.

Lowpass and highpass filters are rather forgiving of not-quite-exact values. What is important is to maintain the relative ratios of the reactances within the filter.

Appendix 10-1

Obtaining Component Values Other Than Stock Tolerance Values

Common passive electronic components are stocked as ±10% or ±5% values. While these are specified guaranteed within their tolerance, values are often within ±3% of nominal for the 10%, within ±2% for the 5% types. The sequence of values for 10% tolerance parts is:

10 12 15 18 22 27 33 39 47 56 68 82 100

The 5% tolerance values include the above plus the following:

11 13 16 20 24 30 36 43 51 62 75 91

While that is convenient for manufacturers and distributors, small designs by hobbyists are limited in the number of parts to stock in the home workshop. By holding to ten-percent nominal values, there are a number of values in between nominal that can be achieved with just two components. Two capacitors in parallel or two resistors or inductors in series will enable many of these "in between" values.

The following table is a matrix of two ten-percent nominal value components to reach as many values as possible in a 20 to 200 value range. Add the column and line values to obtain the total in the column at left.

	Add column and line to get Total											
Total	**10**	**12**	**15**	**18**	**22**	**27**	**33**	**39**	**47**	**56**	**68**	**82**
20	10											
22		10										
24		12										
25	15											
27		15										
28				10								
30		18	15	12								
32	22											
34		22										
36				18								
37	27		22									
39			27									
40				22								
42			27									
43	33											
44					22							
45		33		27								
48			33									
49	39				27							

Total	**10**	**12**	**15**	**18**	**22**	**27**	**33**	**39**	**47**	**56**	**68**	**82**
51		39		33								
54			39			27						
55					33							
57	47			39								
59		47										
60						33						
61					39							
62			47									
65				47								
66	56					39	33					
68		56										
69					47							
71			56									
72							39					
74				56		47						
78	68				56			39				
80		68					47					
83			68			56						
86				68				47				
89							56					
90					68							
92	82											
94		82										
95						68		56				
97			82									
100				82								
101							68					
103									56			
104					82							
107								68				
109						82						
112		100								56		
115			100				82		68			
118				100								
121								82				
122					100							
124										68		
127						100						
129									82			
130	120											
132		120										
133							100					
135			120									
Total	**10**	**12**	**15**	**18**	**22**	**27**	**33**	**39**	**47**	**56**	**68**	**82**

136											68	
138				120						82		
139								100				
142					120							
147						120			100			
153												
156										100		
159								120				
160	150											
162		150										
164												82
165			150									
167									120			
168				150							100	
172					150							
176										120		
177						150						
182												100
183							150					
188											120	
189								150				
190	180											
192		180										
195			180									
197									150			
198				180								
Total	**10**	**12**	**15**	**18**	**22**	**27**	**33**	**39**	**47**	**56**	**68**	**82**

Appendix 10-2

Normalized Lowpass Filter Tables in Other References

Passive L-C filters are not limited to the types given in this or the subsequent chapter. A great number of tables and types are available which allow unequal end terminations, Elliptic/Cauer filters of many characteristics, various forms of Bessel or linear-phase filters. Passive L-C filter texts are whole book subjects in themselves. Such texts are useful for specialized applications.

The methodology of information presented in other references begins with the lowpass prototype tables normalized to 1 Hz *radian frequency* and 1 Ohm termination. If there are unequal terminations then one value is specified as the normalized value; the other a multiple of that normalized value. Scaling those table values for a real filter is done by equation (10-1).[4]

Calculations from any tables usually result in unrealizeable component values. The purpose of any passive L-C filter calculation set is to set the filter response characteristics: Flat passband, passband ripple with moderate to sharp cut-off slope to stopband, or a specific minimum attenuation in the stopband. *Exact* values are much less important than the *relative values* among components, therefore 2, 5, or even 10 percent tolerance values can be used in most hobbyist filter designs. The only way to make sure that certain component values will work is to test them in a computer analysis program before building or even acquiring parts.

Component losses will occur in real-world parts, particularly in inductors. Inductor winding resistance affects below-LF range filters, core material and shape factors influence inductor Q at above-LF. While some filter types include component losses in tables these tables' results must still be examined by analysis.[5]

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[4] Tables in this chapter and chapter 11 have the 2pi constant already included to reduce calculation and possible errors. Scaling to real values is then more direct by frequency and termination resistance. Other texts seem held to radian frequency which always requires that 2pi constant handling to calculate real values. That is the main reason for including both equation (10-1) and (10-2).

[5] References [30] and [31] do take into account losses within a filter but their use is laborious and response analysis is still required to be sure all the calculations are correct.

References for Chapter 10 through 12

[27] *LCie4*, a personal computer program by the author. *LCie4* is used as the basis for all normalized filter values in Chapters 10, 11, and 12, plus many of the analyses of filters. *LCie4* is freeware, handles both synthesis and analysis of lowpass, highpass, bandpass, and bandstop filters having sections from 3 to 13. It includes separate Q settings of inductors and capacitors, modification of individual component values, and monte carlo analysis of random tolerance changes. Frequency response analysis is done by the ladder method given in Appendix 9-2 and reference [21]. *LCie4* synthesis is based on reference [28].

[28] Chebyshev and Elliptic function computer program algorithms by David C. Greene that appeared in RF Design magazine for March, 1989, pp. 43-45 and the RF Design 1990 Directory Issue, pp. 70-80. While these do not seem to correspond with many normalized filter tables published elsewhere, the algorithms are correct and result in viable filters.

[29] *Electronic Filter Design Handbook* by Arthur B. Williams, McGraw-Hill Book Company, 1981. This is a comprehensive passive and active filter design text for those who wish to know more detail on filters' synthesis as well as transient, step-function response.

[30] *A Handbook of Electrical Filters* by Donald R. J. White, published by White Electro-magnetics, Rockville, MD, 1963. This has many tables of normalized lowpass values plus many forms of conversion of lowpass prototype to narrow bandwidth bandpass filters.

[31] *Computer-Prepared Tables Enable Design of Ultra-Flat Networks* by Phillip R. Geffe, Electronic Design magazine, August 31, 1960. Geffe's work here is in new table values for Butterworth response filters that include inductor losses. The same work as appeared in other publications, including some Hewlett-Packard programmable calculator programs.

[32] *Pseudo-Exact Band-Pass Filter Design Saves Time* by Robert L. Sleven, 7 parts, Microwaves magazine, August 1968 through May 1969. While these concentrate on bandpass designs, they are based on lowpass prototypes and have a good methodology for filter design with particular attention to the microwave region. The *pseudo-exact* values are perhaps best described as *in-between* values of the mathematical Butterworth, Chebyshev, or elliptic response values.

[33] *Handbook of Filter Synthesis,* Anatol I. Zverev, Wiley-Interscience. The 1967 edition has been replaced by the 2005 edition which cost between $85 and $90 for soft-cover in 2007; hard-cover was $290. A large collection of tables and charts at 576 pp, it is considered to be the best such handbook by many professional designers. ISBN: 9780471749424 for soft-cover. A bit expensive for most hobbyists, the 1967 edition should be available in technical libraries.

Chapter 11

Bandpass, Highpass, Bandstop L-C Filters

A bandpass filter has a passband centered on a geometric center frequency with stopbands on either side. The lowpass prototype establishes the values of components to form the passband bandwidth. Each branch has an added reactance to resonate that branch at the geometric center frequency. A highpass filter is the inverse of a lowpass filter in component placement and in normalized values and in frequency response. A bandstop filter is the inverse of the bandpass filter, using the highpass values to set bandstop bandwidth plus resonators at the geometric bandstop center frequency.

Bandpass Filters

Creation of a bandpass filter from a lowpass prototype begins by calculating a lowpass filter whose f_B (cutoff frequency) is the same bandwidth in Hertz as the desired bandpass filter bandwidth. The next step is to *add* reactances to resonate at the bandpass *geometric center* frequency.

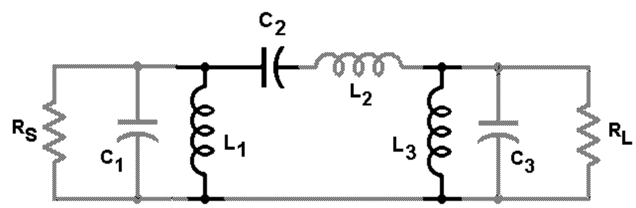

Figure 11-1 A 3-section lowpass filter evolved to a 3-section bandpass filter. Original lowpass components are shown in grey, added resonating reactances are shown in black.

Figure 11-1 shows the added reactances and their placement in a 3-section lowpass filter. Each series branch is series-resonated at the bandpass geometric center frequency while each shunt branch is parallel-resonated at the same frequency. A bandpass filter has twice the number of components of a lowpass filter. The center frequency must be the *geometric center*, not the arithmetic center (half the distance between passband low and high frequencies). Geometric center frequency is calculated by:

$$f_0 = \sqrt{f_L \cdot f_H} = \frac{\sqrt{4f_C^{\,2} - f_B^{\,2}}}{2} \tag{11-1}$$

f_0 = Geometric center frequency f_L = Lower bandpass frequency limit

f_H = Higher bandpass frequency limit f_B = Bandpass bandwidth = $f_H - f_L$

f_C = Arithmetic center frequency = $f_L + (f_B / 2) = f_H - (f_B / 2)$

In short a bandpass filter is created by using the **n** components of an **n**-section lowpass filter whose bandwidth equals the bandpass width, then *adding* **n** more components to resonate each branch at the geometric bandpass center. It is as simple as that.

As an example, assume a bandpass filter is desired to pass 4.5 MHz to 10.5 MHz with 300 Ohm terminations. A 5-section Butterworth lowpass prototype is chosen as the basic arrangement. The bandpass bandwidth is exactly 6.0 MHz. The geometric center frequency is 6.87386 MHz or, rounded off to 4 digits, 6.874 MHz (arithmetic center would be 7.50 MHz). From Table 10-1:

$$k1 = 0.09836 = k5 \qquad k2 = 0.2575 = k4 \qquad k3 = 0.3183$$

Assuming minimal - inductor lowpass configuration with the first shunt branch a capacitor and using (10 - 2) to scale the components with $f_B = 6$ MHz, R = 300 Ohms:

$$C1 = C5 = \frac{0.09836}{6\cdot 10^6 \cdot 300} = 54.64\text{ pFd} \qquad C3 = 176.8\text{ pFd}$$

$$L2 = L4 = \frac{0.2575\cdot 300}{6\cdot 10^6} = 12.88\ \mu\text{Hy}$$

This yields the basic lowpass filter for a bandwidth of 6 MHz with 300 Ohm terminations. The next step is to find the added components to resonate each branch at 6.874 MHz:

$$\omega = 2\pi\cdot 6.874\cdot 10^6 = 43.19\cdot 10^6 \qquad \text{Since } L = \frac{1}{\omega^2\ C}:$$

$$L1 = L5 = \frac{1}{\omega^2\ C1} = \frac{1}{1.865\cdot 10^{15}\cdot 54.64\cdot 10^{-12}} = 9.811\ \mu\text{Hy}$$

$$C2 = C4 = \frac{1}{\omega^2\ L2} = \frac{1}{1.865\cdot 10^{15}\cdot 12.88\cdot 10^{-6}} = 41.63\text{ pFd}$$

$$L3 = \frac{1}{\omega^2\ C3} = 3.031\ \mu\text{Hy}$$

These are all realizable values. If the same bandwidth and terminations are kept but the bandpass is moved to 10 to 16 MHz, the resonating values begin to become unwieldy.

For 10 to 16 MHz, the bandwidth is the same, therefore the lowpass components are the same.

$$f_0 = \sqrt{10\cdot 16}\cdot 10^6 = 12.65\cdot 10^6 \qquad \omega = 79.48\cdot 10^6$$

$$L1 = L5 = \frac{1}{\omega^2\cdot C1} = \frac{1}{6.317\cdot 10^{15}\cdot 54.64\cdot 10^{-12}} = 2.897\ \mu\text{Hy}$$

$$C2 = C4 = \frac{1}{6.317\cdot 10^{15}\cdot 12.88\cdot 10^{-6}} = 12.29\text{ pFd} \qquad L3 = 0.8952\ \mu\text{Hy}$$

The value of L3 is still realizable at nearly 0.9 µHy, down from 3 µHy, but C2 and C4 at about 12 pFd show a drastic drop from about 41 pFd in the previous bandpass filter.

An important thing to keep in mind is that, in keeping the same passband bandwidth, the resonating values change by the *square of the center frequency change.* The original lowpass component values did not change, therefore the resonating component values must change by the square.

Percentage Bandwidth

Bandpass filters of inductors and capacitors have realizable values if the *percentage bandwidth* is above about 10 to 15 percent for the geometry shown in Figure 11-1. Percentage bandwidth is defined by the passband bandwidth divided by the geometric center frequency. In the first example given, the percentage bandwidth would be (6/6.874)x100 or about 87 percent. In the second example it would be (6/12.65)x100 or about 47 percent. The Q of the passive components is the chief cause of this limitation.[1]

4.5 to 10.5 MHz Filter			10 to 16 MHz Filter		
f, MHz	**∞ Q**	**Q = 50**	**f, MHz**	**∞ Q**	**Q = 50**
2	-55.7	-55.9	6	-53.7	-53.9
3	-32.7	-33.1	8	-30.1	-30.6
3.5	-22.2	-22.7	9	-16.6	-17.5
4	-11.8	-12.5	9.5	-9.3	-10.7
4.5	-3.0	-4.1	10	-3.0	-5.0
5	-0.2	-1.2	10.5	-0.2	-2.3
5.5	-0.0	-0.8	11	0	-1.6
6	0	-0.7	11.5	0	-1.4
7	0	-0.7	12	0	-1.4
8	0	-0.8	13	0	-1.3

4.5 to 10.5 MHz Filter			10 to 16 MHz Filter		
f, MHz	**∞ Q**	**Q = 50**	**f, MHz**	**∞ Q**	**Q = 50**
9	0	-0.9	14	0	-1.4
9.5	-0.3	-1.3	14.5	0	-1.6
10	-1.1	-2.3	15	-0.2	-1.9
10.5	-3.0	-4.2	15.5	-0.9	-2.9
11	-6.1	-7.1	16	-3.0	-5.0
11.5	-9.6	-10.4	16.5	-6.5	-8.2
12	-13.1	-13.7	17	-10.6	-11.8
14	-24.8	-25.1	17.5	-14.6	-15.5
16	-33.7	-33.9	18	-18.2	-18.9
18	-40.9	-41.0	20	-30.1	-30.5
20	-46.8	-46.9	24	-46.1	-46.2

Note that the passband, especially the "corners" of the passband, are the most affected by real Q of components. The passband is still relatively flat with lossy components, just reduced versus lossless component level. The frequency response in db of the two examples is tabulated below, the infinite Q column indicating ideal, lossless components. The Q = 50 column denotes capacitor Q at 500 and

[1] Some texts on filters refer to the passband center divided by bandwidth as Q. While that's okay in a very broad sense, to the circuit designer that losses in L and C components have individual Qs which do frequency-domain distortion on the response of filters. The filter's overall "Q" remains about the same but the frequency response *shape* is altered by component Q. Saying *percentange bandwidth* instead of (overall) Q is more definitive for filters.

inductor Q at 50.

Note also that the passband center of the lossy-component higher frequency filter has a lower level compared to the lossy-component lower frequency filter. This is due primarily from the series inductors. Since those are the same value for both filters, the series reactance (and thus resistive part due to Q loss) would almost double with the higher-frequency filter.

Stopband attenuation is relatively unchanged at frequencies far from center frequency, lossless case versus lossy components.

Elliptic Bandpass Filters from Elliptic Lowpass Filters

Elliptic bandpass filters are based on the elliptic lowpass prototype. As with other lowpass-to-bandpass conversions, the lowpass shunt capacitors are resonated at passband geometric center frequency with added parallel inductors (L1 across C1, L3 across C3 in the figure).

The normal conversion of the lowpass parallel-resonant pair in the series branch would have the capacitor (C2 in the figure) with a parallel-resonant inductor at center frequency while the inductor (L2) would have a series-resonant capacitor. This would create a rather odd-looking (but functional) condition of a series-resonant pair in parallel with a parallel-resonant pair for the bandpass filter series branch conversion. This conversion results in highly-interactive components that are usually difficult to trim.

Rather than continue with the parallel connection of a series-resonant with a parallel-resonant, that circuit can be replaced by two parallel-resonant pairs in series as shown in Figure 11-2. These are the C2a-L2a and C2b-L2b pairs. Before continuing, it is worth thinking about elliptic lowpass filters and their use of parallel-resonant pairs in the series lowpass branches. The null in the stopband results from parallel resonance; below that resonance the parallel-resonant pair appears inductive to form the non-elliptic filter's C-L-C structure.

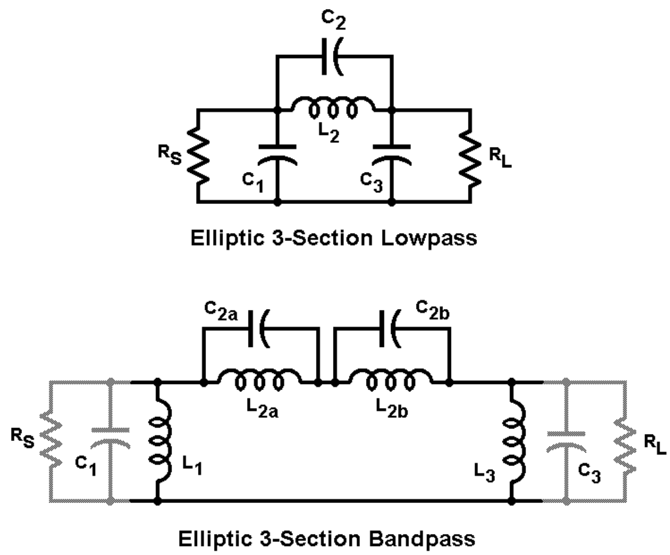

Figure 11-2 Evolution of a 3-section elliptic lowpass to a 3-section elliptic bandpass filter. Bandpass filter components shown in grey do not change value between lowpass and bandpass.

For a bandpass filter there are *two* stopbands, one below the passband and one above it. To maintain the sharp transition characteristic from passband to stopband of the elliptic function, there must be two parallel-resonant pairs. One parallel-resonant pair establishes the null in the lower stopband, the other the null in the upper stopband. Values of the four components in the elliptic bandpass series branch are calculated from the original elliptic lowpass single parallel-resonant pair through:

L_S = elliptic lowpass single parallel - resonant inductor, Hy (11-2)

C_S = elliptic lowpass single parallel - resonant capacitor, Fd

f_0 = geometric center frequency of passband, Hz

$\omega_0 = 2\pi f_0$ Temporary constant calculations are

$$M = 2\omega_0^2 L_S C_S \qquad K = \frac{M+1+\sqrt{2M+1}}{M} \qquad P = K+1$$

Then the bandpass parallel - resonant pairs, in Hy and Fd, are:

$$C_a = \frac{P\,C_S}{K} \qquad C_b = K\,C_a \qquad L_a = \frac{1}{\omega_0^2\ P\,C_S} \qquad L_b = K\,L_a$$

Individual pairs' parallel - resonant frequency relationships are:

$$f_a = f_0\sqrt{K} \qquad f_b = \frac{f_0}{\sqrt{K}}$$

The resulting bandpass component values and their resonant frequencies (based on center frequency) may not appear to have any direct or proportional relationship to the lowpass prototype values.

As an example, assume an elliptic bandpass filter has a 20 to 26 MHz passband and uses 5 sections with 0.5 db passband ripple and 73 Ohm terminations. The passband is 6 MHz. From Table 10-27 the normalized values for an elliptic lowpass are:

k1 = 0.2605 k2 = 0.1852 k3 = 0.3683 k4 = 0.1668 k5 = 0.2412

k22 = 0.01294 k44 = 0.03480 f22 = 3.251 f44 = 2.089

Using (10-2) to scale the normalized values for 73 Ohms and 6 MHz cutoff, the lowpass prototype values become:

C1 = 594.8 pFd C2 = 29.54 pFd L2 = 2.254 μHy f22 = 19.51 MHz

C3 = 840.5 pFd C4 = 79.46 pFd L4 = 2.029 μHy f44 = 12.54 MHz C5 = 550.6 pFd

The geometric center frequency of the bandpass filter is 22.8035 MHz. The three added resonating inductors for the shunt branches are, using (11-1):

L1 = 81.90 nHy L3 = 57.93 nHy L5 = 88.47 nHy

Note that these are in nanoHenries, rather small values.

Each of the two series branches must be considered separately. In the first series branch the lowpass prototype values are C2 at 29.54 pFd and L2 at 2.254 μHy. From (11-2) the temporary constants are:

M = 2.7337 K = 2.2961 P = 3.2961

From those the component values become:

C2a = 42.41 pFd   L2a = 0.5003 μHy   f2a = 35.55 MHz

C2b = 97.37 pFd   L2b = 1.1487 μHy   f2b = 15.05 MHz

For the second series branch the lowpass prototype values are C4 at 70.46 pFd and L4 at 2.029 μHy. Using (11-2) the calculations result in:

M = 6.6183   K = 1.7212   P = 2.7212
C4a = 125.6 pFd   L4a = 0.2253 μHy   f4a = 29.92 MHz
C4b = 216.2 pFd   L4b = 0.3878 μHy   f4b = 17.38 MHz

Examining the Elliptic Bandpass Example in Detail

As usual with L-C filters, calculations result in some odd values. Hobbyists need to be concerned with realistic part values without adding too many trimmer components. Hobbyists are also generally limited in the accuracy of determination of component values. Add to all of this the unknown factor of filter insertion loss due to inductor and capacitor Q.

For Q values one can assume a mica dielectric capacitor Q of about 1000 (fixed or trimmer types). For smaller inductor values such as in the example, a Q of 80 is reasonable for air-core or toroidal type. The accuracy of determination of values is somewhere within ±5%. How far within that five-percent value depends on the type, kind, and calibration of hobbyist test equipment. Using toroidal core manufacturer's data for turns versus inductance, careful construction can result in an actual toroid inductance within five percent. The same is true for careful construction of air-core, free-standing inductors. Fixed capacitors can be regarded to be within their manufacturer's five-percent specification values. The five-percent value variability is a reasonable *worst-case* tolerance.[2]

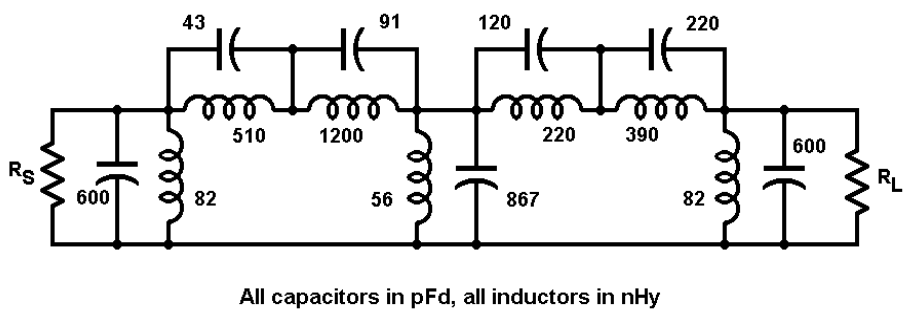

Figure 11-3 Example 5-section elliptic BP filter with all fixed values.

The bandpass filter example has about a 26 percent bandwidth and might be realized using all fixed parts. If so, a question is the effect on passband and stopband response under worst-case tolerances of those fixed parts? The comparison fixed-value model is shown in Figure 11-3. Five

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[2] *Worst-case* is a long-standing buzzword for extremes in tolerances resulting in the worst-possible condition away from the desired condition.

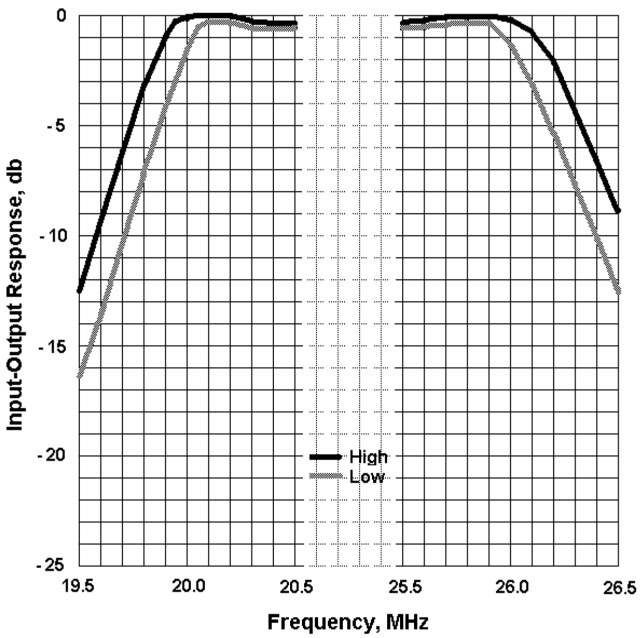


Figure 11-4 Passband edge response, exact values within ±1% random variation for 3000 analyses.

percent tolerance components are assumed. Note: The 600 pFd value is the parallel of a 270 and 330 pFd; the 867 pFd value is the parallel of an 820 and 47 pFd. Termination resistors are 73 Ohms.

Before going further, there must be some basic examination on the necessary tolerance of the component values. This can start with assuming exact values from calculations. Those values are then varied randomly within a selected plus-minus percentage limit. Doing repeated frequency domain analyses, each with random component variation, then accumulating highest, lowest, and mean response at each analysis frequency yields a worst-case boundary. Figure 11-4 is a result of exact-value components varied ±1% for 3000 analyses.

To interpret the figure accurately, a final filter response would be plotted somewhere between the two graph limits. The lower edge of the passband could be close to the grey low-limit while the higher passband edge could be close to the black high-limit curve, or vice-versa.

In Figure 11-4 the passband is within 1 db over about 5.7 MHz but at the 20 and 26 MHz edges it is down about 2 db. Figure 11-5 shows the result of ±2% random variation. The worst-case boundaries have increased but the passband width has shrunk only slightly.

What can be seen in both Figures 11-4 and 11-5 is that it is possible to have some slight shifting of the overall passband slightly lower or higher than originally intended. The actual passband shape for any particular tolerance could "slide" up or down in frequency within the boundary limits.

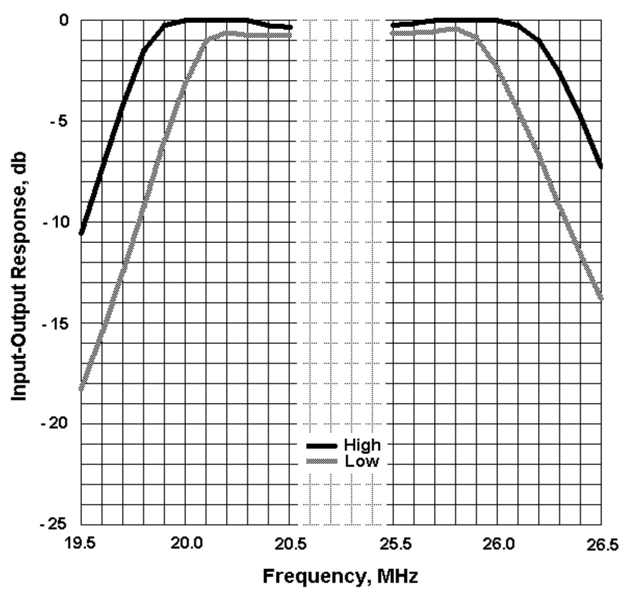


Figure 11-5 Passband edge response with exact values varied randomly ±2% for 3000 analyses.

If the random variation of components becomes 5 percent as in Figure 11-6, the passband response shape may be even more distorted. Figure 11-6 includes the mean response (thin black line between boundaries). This mean curve is closer to the exact, zero-tolerance filter response. There is no guarantee that the passband ripple is low since that ripple can be between the high and low boundaries.

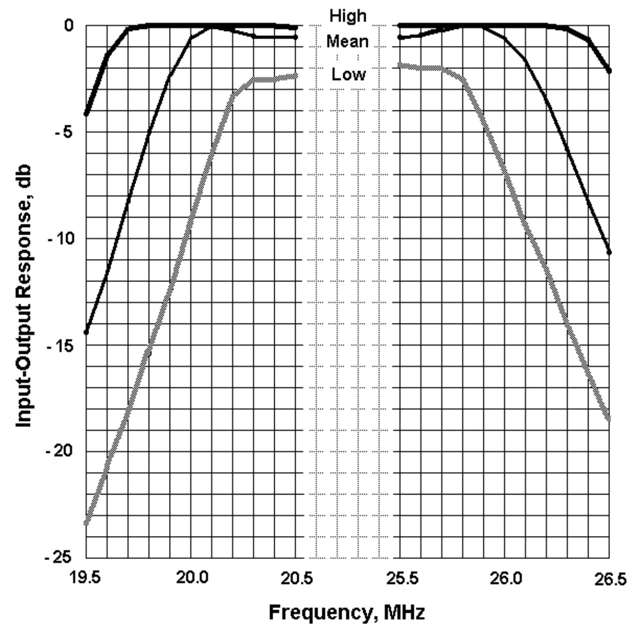


Figure 11-6 Passband edge response, exact values varied randomly ±5% for 3000 analyses.

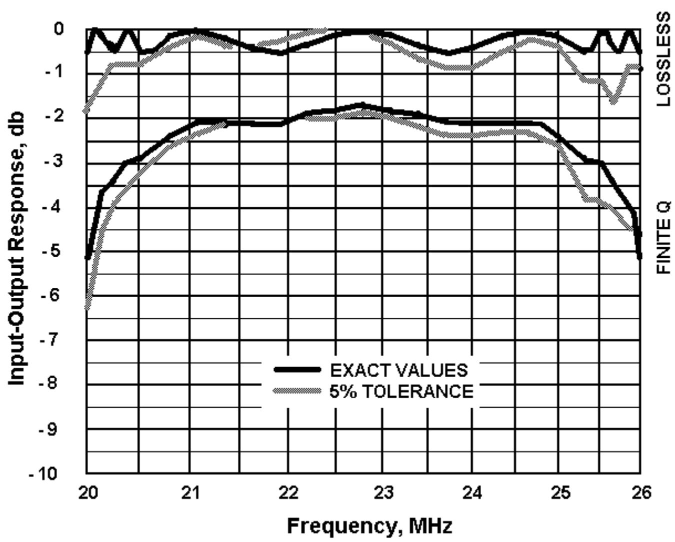


Figure 11-7 Comparison of passband response of exact-value components versus fixed-value parts available in 5% tolerance values. Lower curve pair has capacitive Q of 1000, inductive Q of 80.

Is there a change in the stopbands' attenuation with varying component tolerances? This depends on the definition of attenuation. The A_{MIN} of the lowpass prototype remains at about 66 db and that transfers to the bandpass filter. The null frequencies have changed slightly and that will change the response shape of the edges, the transition area between passband and stopband sometimes referred to as the *skirts*.

One cannot guarantee skirt response to be exact unless component tolerances are around one percent or smaller. A_{MIN} will remain fairly consistent as the ultimate attenuation of an elliptic filter. Chebyshev and Butterworth bandpass filters have continuous skirt curves. Stopband attenuation cannot be as easily guaranteed without analysis of response. Designers should be aware of that as it applies to block and subsystem design.

Figure 11-7 shows the exact-value response of the example with lossless parts at the top black line. The five passband ripple peaks are clearly visible. Note: Elliptic and Chebyshev bandpass filters have peaks equal to the number of sections. The lower black curve is the same exact-value filter with capacitor Qs of 1000 and inductor Qs of 80.

Real-world component losses will always distort passband response. The general effect is to put the most attenuation from finite Q at the passband edges, the least attenuation at passband geometric center. In this example the loss at center frequency was 1.8 db while the extreme passband edges had 4.6 db.

The upper grey line of Figure 11-7 represents the filter of Figure 11-3 using nominal 5% tolerance stock values. Ripple within the passband can range between 0 and -1.8 db even with lossless components. The odd response shape at the high edge of the passband is due to the choice of stock values; those cause a disruption of the numerical relationship necessary to maintain the specified ripple. The lower grey curve in Figure 11-7 shows a greater rounding-off of response due to component losses but the overall additional attenuation is approximately the same as in the exact-value lossless versus finite-Q comparison.

Comparison of Elliptic to Chebyshev and Butterworth

Selection of the type of filter response depends on the need for passband smoothness (in amplitude) versus the skirt response or, rather, skirt attenuation. Butterworth and Chebyshev fixed-value (to 5% stock values) filters were designed for the same bandwidth and end impedance as the elliptic filter example. These were analyzed by the same criteria: ±5% random variation for 3000 analyses. Capacitive Q was assumed 1000 and inductive Q assumed to be 80. Schematic diagram shown in Figure 11-8 with amplitude response in Figure 11-9.

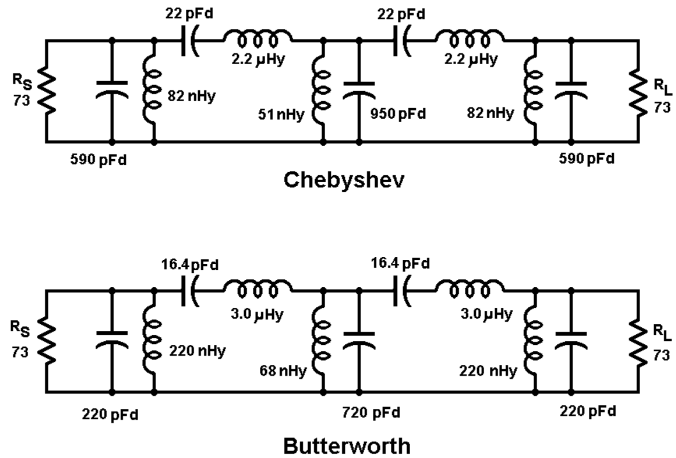


Figure 11-8 Comparison filter schematics.

The curves in Figure 11-9 are the mean of 3000 analyses. Extremes of amplitude at the skirt lower than -20 db has about a ±2 db variation for all three filter types. Far-from-passband response of the Butterworth and Chebyshev types continue falling. The elliptic filter skirt response goes through two deep nulls and then rises to the dotted line level is a bit more than -66 db marked as A_{MIN}. While the very-far-from-passband response never goes above that A_{MIN} level, the Butterworth and Chebyshev types show a greater attenuation there than the elliptic.

The elliptic filter has superior skirt response in the close-to-passband frequency region. The -60db level bandwidth of the elliptic is about 12 MHz, the Chebyshev about 18 MHz, while the Butterworth is about 24 MHz. A common filter skirt response descriptor, particularly with narrowband crystal bandpass filters, is the *60/6 db* ratio that describes the slopes of the skirt in bandwidth values. For the three examples those would be about 2:1 for the elliptic, 3:1 for the Chebyshev, and 4:1 for the Butterworth.

Stopband response is not the only criterion for selection of filter response type. The passband responses shown in Figure 11-10 may be more important in the ultimate application.

Figure 11-10 can be compared with Figure 11-7 for passband response of all three types. The *High* and *Low* curves refer to the extremes of response with random tolerance variations in analysis.

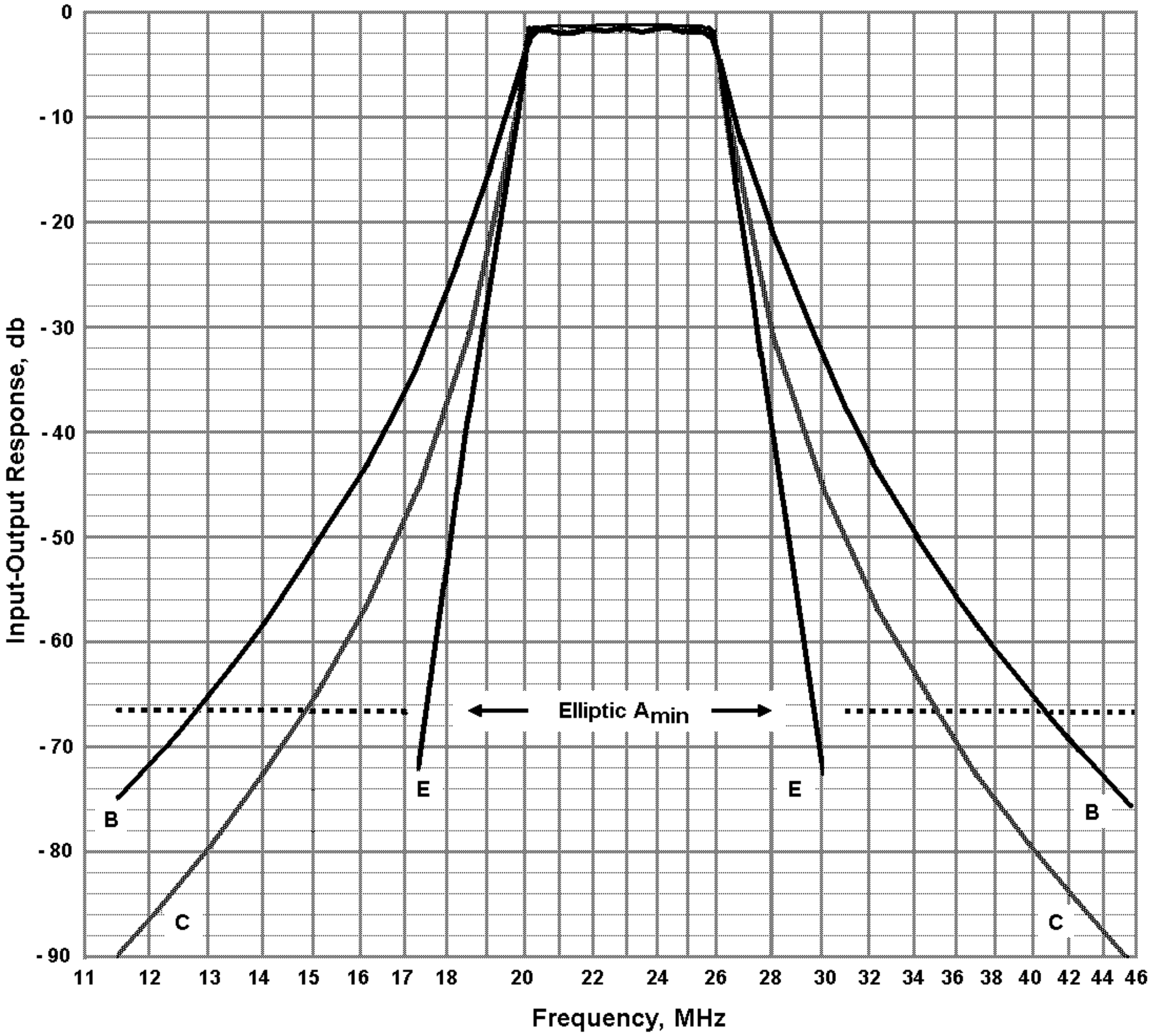


Figure 11-9 Comparison of filter types. Curve B is Butterworth, curve C is Chebyshev, and curve E is Elliptic. Dotted line marked *Amin* is maximum attenuation level.

While the elliptic lossless passband response is almost a carbon-copy of the Chebyshev passband, component tolerances tend to skew the curves to favor the Chebyshev for smoothness. Note: The elliptic has more components therefore is likely to show more response variation.

Butterworth passband response is the most benign of all three insofar as tolerance variations. With finite component Qs the passband curve drops about 1.5 db nearly equally anywhere in the passband.

Each filter type can be designed and made with stock tolerance values. Deciding which type to use will depend on the circuit block or system.

Highpass Filters

Figure 11-11 shows the two possible forms of a highpass filter for odd numbers of sections. Even section numbers would have the same number of inductors and capacitors. In comparing this with Figure 10-1, the inductors and capacitors have "changed places." As with the lowpass, intuition says that series capacitors will have less reactance magnitude with increasing frequency while shunt inductors have more, therefore the filter will pass high frequencies while blocking low frequencies.

Most texts will state that normalized lowpass prototype value tables are inverted to obtain highpass values. In here, highpass value tables are already inverted and normalized to 1 Hertz frequency (not radian frequency) and 1 Ohm equal end termination.

Frequency response of highpass filters is the mirror image of lowpass filters, pivoting around
cutoff frequency, f_B. The passband is above cutoff frequency while the stopband is below. is still the cutoff frequency but the passband is

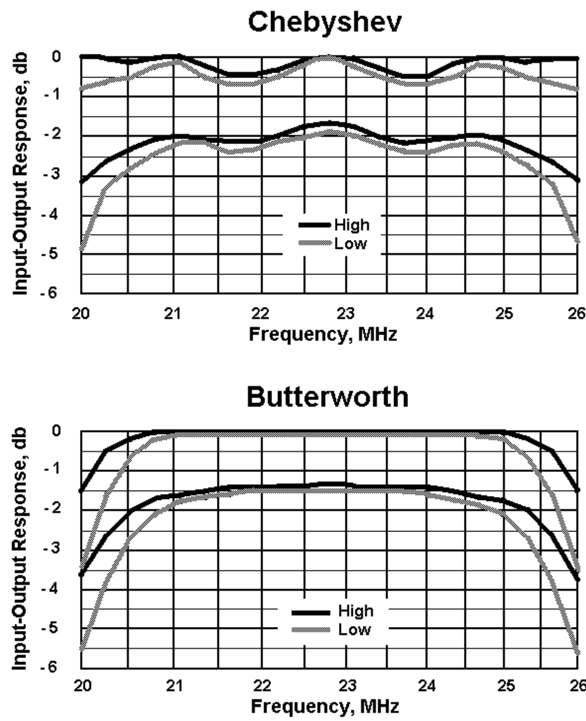


Figure 11-10 Passband response of the two comparison filters, lossless and finite-Q.

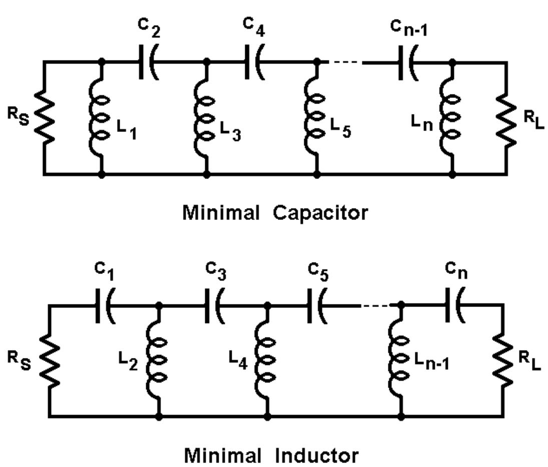


Figure 11-11 Prototype highpass filters.

now above f_B while the stopband is below f_B. All of the passband and stopband response and attenuation curves for lowpass filters in Chapter 10 apply here except that the relative frequency scales are f_B / f_A instead of f_A / f_B.

Chebyshev and elliptic filter ripple still appears in the passband. However, the highpass passband is above cutoff frequency and the peak and ripple frequencies are inverted above cutoff.

In the following normalized highpass filter value tables, use equation (10-2) to scale values for the cutoff frequency and end resistance.

Table 11-1

Normalized Values for Butterworth Response Highpass Filters

Sections	k1	k2	k3	k4	k5	k6	k7	k8	k9
3	0.1592	.07958	0.1592	–	–	–	–	–	--
4	0.2080	.08613	.08613	0.2080	–	–	–	–	--
5	0.2575	.09836	.07958	.09836	0.2575	–	–	–	--
6	0.3075	0.1125	.08239	.08239	0.1125	0.3075	–	–	--
7	0.3576	0.1276	.08832	.07958	.08832	0.1276	0.3576	–	--
8	0.4079	0.1432	.09571	.08114	.08114	.09571	0.1432	0.4079	--
9	0.4583	0.1592	0.1042	.08469	.07958	.08469	0.1042	0.1592	0.4583

Table 11-2

Normalized Values for Chebyshev Response Highpass Filters
Passband Ripple = 0.01 db

Sections	k1	k2	k3	k4	k5	k6	k7	k8	k9
3	0.2530	0.1640	0.2530	–	–	–	–	–	--
4	0.2233	0.1326	0.1205	0.2458*	–	–	–	–	--
5	0.2104	0.1220	0.1009	0.1220	0.2104	–	–	–	--
6	0.2037	0.1170	.09419	0.1037	0.1063	0.2242*	–	–	--
7	0.1997	0.1143	.09104	.09745	.09104	0.1143	0.1997	–	--
8	0.1972	0.1126	.08929	.09455	.08589	.09829	0.1023	0.2170*	--
9	0.1954	0.1115	.08821	.09293	.08351	.09293	.08821	0.1115	0.1954

Table 11-3

Normalized Values for Chebyshev Response Highpass Filters
Passband Ripple = 0.1 db

Sections	k1	k2	k3	k4	k5	k6	k7	k8	k9
3	0.1543	0.1387	0.1543	–	–	–	–	–	--
4	0.1435	0.1218	.08990	0.1938*	–	–	–	–	--
5	0.1388	0.1161	.08059	0.1161	0.1388	–	–	–	--
6	0.1363	0.1134	.07740	0.1049	.08364	0.1656*	–	–	--
7	0.1347	0.1119	.07591	0.1012	.07591	0.1119	0.1347	–	--
8	0.1338	0.1109	.07508	.09941	.07334	0.1018	.08292	0.1813*	--
9	0.1331	0.1103	.07456	.09844	.07217	.09844	.07456	0.1103	0.1331

Table 11-4

Normalized Values for Chebyshev Response Highpass Filters
Passband Ripple = 0.25 db

Sections	k1	k2	k3	k4	k5	k6	k7	k8	k9
3	0.1221	0.1388	0.1221	–	–	–	–	–	--
4	0.1155	0.1254	.07742	0.1870*	–	–	–	–	--
5	0.1125	0.1208	.07101	0.1208	0.1125	–	–	–	--
6	0.1109	0.1186	.06882	0.1115	.07322	0.1797*	–	–	--
7	0.1100	0.1174	.06780	0.1084	.06780	0.1174	0.1100	–	--
8	0.1094	0.1166	.06722	0.1069	.06601	0.1089	.07201	0.1772*	--
9	0.1090	0.1161	.06687	0.1061	.06519	0.1061	.06687	0.1161	0.1090

Table 11-5

Normalized Values for Chebyshev Response Highpass Filters
Passband Ripple = 0.5 db

Sections	k1	k2	k3	k4	k5	k6	k7	k8	k9
3	.09970	0.1451	.09970	–	–	–	–	–	--
4	.09528	0.1335	.06726	0.1890*	–	–	–	–	--
5	.09330	0.1294	.06264	0.1294	.09330	–	–	–	--
6	.09224	0.1275	.06106	0.1212	.06428	0.1830*	–	–	--
7	.09161	0.1265	.06032	0.1184	.06032	0.1265	.09161	–	--
8	.09120	0.1258	.05991	0.1171	.05902	0.1189	.06343	0.1810*	--
9	.09092	0.1254	.05966	0.1164	.05843	0.1164	.05966	0.1254	.09092

Table 11-6

Normalized Values for Chebyshev Response Highpass Filters
Passband Ripple = 1.0 db

Sections	k1	k2	k3	k4	k5	k6	k7	k8	k9
3	0.3221	0.1582	0.3221	–	–	–	–	–	--
4	0.3341	0.1694	0.4506	0.1256*	–	–	–	–	--
5	0.3398	0.1737	0.4776	0.1737	0.3398	–	–	–	--
6	0.3429	0.1757	0.4876	0.1833	0.4674	0.1289*	–	–	--
7	0.3448	0.1769	0.4924	0.1868	0.4924	0.1769	0.3448	–	--
8	0.3461	0.1776	0.4951	0.1884	0.5011	0.1861	0.4725	0.1301*	--
9	0.3469	0.1781	0.4968	0.1893	0.5053	0.1893	0.4968	0.1781	0.3469

Table 11-7

Normalized Values for Chebyshev Response Highpass Filters
Passband Ripple = 2.0 db

Sections	k1	k2	k3	k4	k5	k6	k7	k8	k9
3	0.4314	0.1325	0.4314	–	–	–	–	–	--
4	0.4444	0.1402	0.5740	0.1085*	–	–	–	–	--
5	0.4506	0.1430	0.6020	0.1430	0.4506	–	–	–	--
6	0.4539	0.1444	0.6122	0.1495	0.5913	0.1108*	–	–	--
7	0.4560	0.1452	0.6171	0.1518	0.6171	0.1452	0.4560	–	--
8	0.4573	0.1456	0.6199	0.1529	0.6260	0.1514	0.5965	0.1117*	--
9	0.4582	0.1460	0.6216	0.1535	0.6302	0.1535	0.6216	0.1460	0.4582

Table 11-8

Normalized Values for Chebyshev Response Highpass Filters
Passband Ripple = 3.0 db

Sections	k1	k2	k3	k4	k5	k6	k7	k8	k9
3	0.5330	0.1133	0.5330	–	–	–	–	–	--
4	0.5473	0.1191	0.6919	.09422*	–	–	–	–	--
5	0.5541	0.1213	0.7222	0.1213	0.5541	–	–	–	--
6	0.5578	0.1223	0.7331	0.1262	0.7105	.09602*	–	–	--
7	0.5600	0.1229	0.7383	0.1279	0.7383	0.1229	0.5600	–	--
8	0.5615	0.1233	0.7413	0.1288	0.7479	0.1276	0.7160	.09665	--
9	0.5624	0.1235	0.7431	0.1292	0.7523	0.1292	0.7431	0.1235	0.5624

As an example assume a highpass filter with 100 Ohm end terminations and an f_B of 10 MHz, 5 sections with minimal inductance configuration. Let it be a Chebyshev response with 0.1 db ripple in the passband. Table 11-3 has the normalized values and equation (10-2) will do the scaling:

$k1 = k5 = 0.1388 \quad k2 = k4 = 0.1161 \quad k3 = 0.08059$

Since this is minimal inductance, odd sections, the first component is a series capacitor. Using equation (10 - 2):

$$C_1 = \frac{k1}{f_B\, R} = \frac{0.1388}{10 \cdot 10^6 \cdot 100} = 138.8 \text{ pFd} = C_5$$

$$L_2 = \frac{k2\, R}{f_B} = \frac{0.1161 \cdot 100}{10 \cdot 10^6} = 1.161\ \mu\text{Hy} = L_4$$

$$C_3 = \frac{k3}{f_B\, R} = \frac{0.08059}{10 \cdot 10^6 \cdot 100} = 80.59 \text{ pFd}$$

Elliptic Highpass Filters

The Elliptic highpass filter is made by adding series capacitors to the shunt inductors as illustrated in Figure 11-2. The individual series-resonant pairs create nulls in the stopband in the same way that Elliptic lowpass filters use parallel-resonant pairs in the lowpass series branches.

Frequency response of Elliptic highpass filters is the same as Elliptic lowpass if all frequencies are inverted around f_B.

Normalized values for Elliptic highpass filters can be taken from Tables 11-9 through 11-14. Use equation (10-2) to scale table values. All of the series-resonant pair capacitors must be calculated from the double-subscript values; i.e., k22 for C2, k44 for C4. Double-subscript frequencies (f22) refer to the series-resonance relative to f_B .

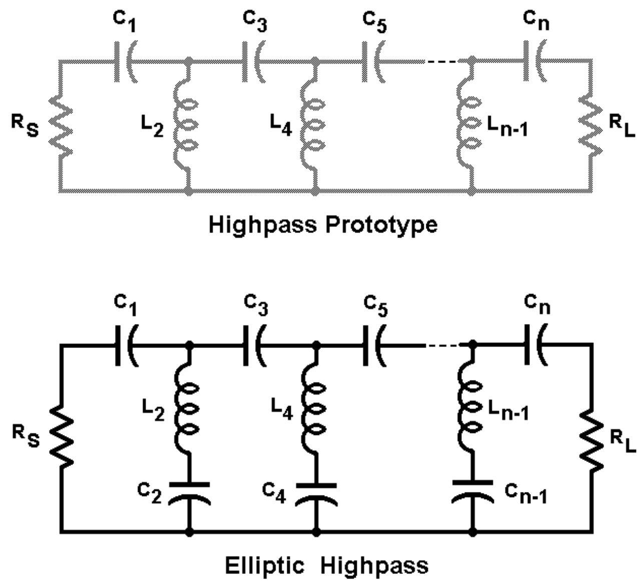


Figure 11-12 Evolution of elliptic highpass from highpass prototype.

Table 11-9
Normalized Values for Elliptic Response Highpass Filters
Passband Ripple = 0.1 db

Sections	k1	k2	k3	k4	k5	k6	k7	k8	k9
3	0.1777	0.1698	0.1777	–	–	–	–	–	--
5	0.1463	0.1231	.08872	0.1392	0.1629	–	–	–	--
7	0.1385	0.1153	.08288	0.1177	.08572	0.1253	0.1491	–	--
9	0.1354	0.1124	.07967	0.1105	.08219	0.1143	.08294	0.1189	0.1423

Sections	k22	k44	k66	k88	f22	f44	f66	f88
3	0.7990	–	–	–	0.4405	–	–	--
5	2.175	0.7943	–	–	0.3076	0.4786	–	--
7	4.164	0.8996	1.254	–	0.2296	0.4891	0.4016	--
9	6.805	1.182	0.9102	1.901	0.1820	0.4405	0.4934	0.3348

Table 11-10
Normalized Values for Elliptic Response Highpass Filters
Passband Ripple = 0.25 db

Sections	k1	k2	k3	k4	k5	k6	k7	k8	k9
3	0.1369	0.1651	0.1369	–	–	–	–	–	--
5	0.1178	0.1277	.07805	0.1429	0.1286	–	–	–	--
7	0.1127	0.1209	.07387	0.1256	.07638	0.1306	0.1199	–	--
9	0.1106	0.1182	.07137	0.1204	.07413	0.1229	.08140	0.1247	0.1154

Sections	k22	k44	k66	k88	f22	f44	f66	f88
3	0.7905	–	–	–	0.4405	–	–	--
5	2.096	0.7739	–	–	0.3076	0.4786	–	--
7	3.972	0.8264	1.203	–	0.2296	0.4891	0.4016	--
9	6.467	1.098	0.8465	1.813	0.1820	0.4405	0.4934	0.3348

Table 11-11
Normalized Values for Elliptic Response Highpass Filters
Passband Ripple = 0.5 db

Sections	k1	k2	k3	k4	k5	k6	k7	k8	k9
3	0.1102	0.1699	0.1102	–	–	–	–	–	--
5	.09723	0.1367	.06877	0.1519	0.1050	–	–	–	--
7	.09364	0.1303	.06567	0.1369	.06787	0.1402	.09901	–	--
9	.09216	0.1277	.06362	0.1303	.06638	0.1346	.06616	0.1343	.09576

Sections	k22	k44	k66	k88	f22	f44	f66	f88
3	0.7682	–	–	–	0.4405	–	–	--
5	1.958	0.7278	–	–	0.3076	0.4786	–	--
7	3.688	0.7732	1.120	–	0.2296	0.4891	0.4016	--
9	5.991	1.002	0.7728	1.6825	0.1820	0.4405	0.4934	0.3348

Table 11-12

Normalized Values for Elliptic Response Highpass Filters
Passband Ripple = 1 db

Sections	k1	k2	k3	k4	k5	k6	k7	k8	k9
3	.08594	0.1853	.08594	–	–	–	–	–	--
5	.07741	0.1539	.05818	0.1701	.08292	–	–	–	--
7	.07496	0.1474	.05595	0.1566	.05781	0.1583	.07882	–	--
9	.07393	0.1448	.05434	0.1496	.05691	0.1546	.05649	0.1520	.07655

	k22	k44	k66	k88	f22	f44	f66	f88
3	0.7045	–	–	–	0.4405	–	–	--
5	1.7390	0.6500	–	–	0.3076	0.4786	–	--
7	3.279	0.6762	0.9926	–	0.2296	0.4891	0.4016	--
9	5.285	0.8728	0.6732	1.487	0.1820	0.4405	0.4934	0.3348

Table 11-13

Normalized Values for Elliptic Response Highpass Filters
Passband Ripple = 2 db

Sections	k1	k2	k3	k4	k5	k6	k7	k8	k9
3	.06364	0.2193	.06364	–	–	–	–	–	--
5	.05822	0.1868	.04612	0.2056	.06197	–	–	–	--
7	.05661	0.1796	.04461	0.1924	.04608	0.1924	.05928	–	--
9	.05593	0.1766	.04341	0.1844	.04560	0.1905	.04511	0.1853	.05775

	k22	k44	k66	k88	f22	f44	f66	f88
3	0.5951	–	–	–	0.4405	–	–	--
5	1.4331	0.5378	–	–	0.3076	0.4786	–	--
7	2.6751	0.5503	0.8165	–	0.2296	0.4891	0.4016	--
9	4.3314	0.7079	0.5462	1.2200	0.1820	0.4405	0.4934	0.3348

Table 11-14
Normalized Values for Elliptic Response Highpass Filters
Passband Ripple = 3 db

Sections	k1	k2	k3	k4	k5	k6	k7	k8	k9
3	.05134	0.2557	.05134	–	–	–	–	–	--
5	.04729	0.2202	.03844	0.2419	.05020	–	–	–	--
7	.04607	0.2120	.03728	0.2282	.03850	0.2270	.04815	–	--
9	.04555	0.2087	.03631	0.2190	.03818	0.2262	.03772	0.2188	.04697

	k22	k44	k66	k88	f22	f44	f66	f88
3	0.5106	–	–	–	0.4405	–	–	--
5	1.2158	0.4570	–	–	0.3076	0.4786	–	--
7	2.2653	0.4641	0.6920	–	0.2296	0.4891	0.4016	--
9	3.6653	0.5962	0.4600	1.0330	0.1820	0.4405	0.4934	0.3348

Table 11-15
Normalized Load End Resistance for Even-Section Chebyshev Filters

Passband Ripple, db	Load-End Resistance	DC shift, db
0.01	1.101	+0.41
0.25	1.620	+1.85
0.5	1.984	+2.48
1	2.660	+3.25
2	4.095	+4.12
3	5.801	+4.64

Table 11-16
Peaks, Dips of Chebyshev and Elliptic Highpass Filter Passband Frequency Relative to f_B

	Number of Sections						
	3	4	5	6	7	8	9
Dip:	2.0	∞	3.2	∞	4.4	∞	5.9
	1.0	1.4	1.2	2.0	1.6	2.6	2.0
		1.0	1.0	1.2	1.1	1.4	1.3
				1.0	1.0	1.09	1.06
						1.0	1.0
Peak:	∞	2.6	∞	3.9	∞	5.3	∞
	1.15	1.06	1.3	1.4	2.3	1.79	2.9
			1.04	1.03	1.28	1.20	1.6
					1.02	1.02	1.16
							1.02

Bandstop Filters

Bandstop filters do the exact opposite of bandpass filters: They attenuate at their (bandpass equivalent) passband and have minimum attenuation on either side of that bandstop area. Their design begins much the same as a bandpass filter evolves from a lowpass prototype, *except:*

1. Series elements are parallel-resonated at geometric bandstop center.

2. Shunt elements are series-resonated at geometric bandstop center.

The geometric bandstop center frequency is calculated the same as the geometric bandpass center frequency. The bandwidth of the bandstop zone is the same as the passband of the bandpass filter.

A Typical 5-section bandstop filters are shown in Figure 11-13. In both, L_2, L_4, C_1, C_3, C_5 are the values from the lowpass prototype. Where the Elliptic bandpass filter needed *two* parallel-resonant pairs in series arms, the Elliptic bandstop filter needs two series-resonant pairs in the shunt arms.

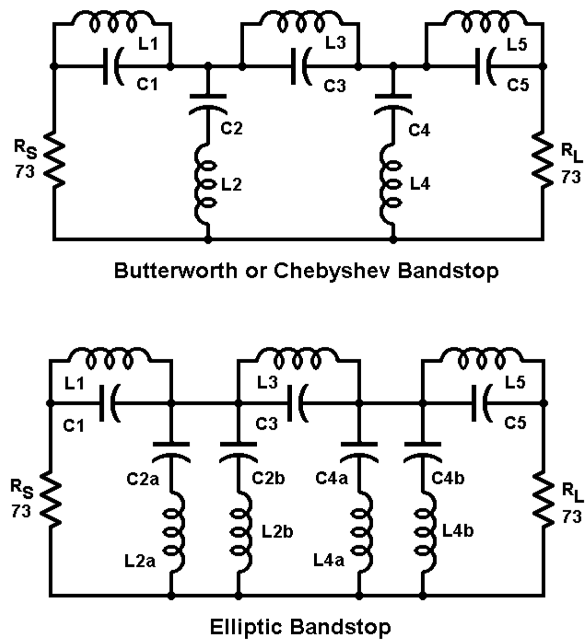


Figure 11-13 Bandstop filters from lowpass prototype

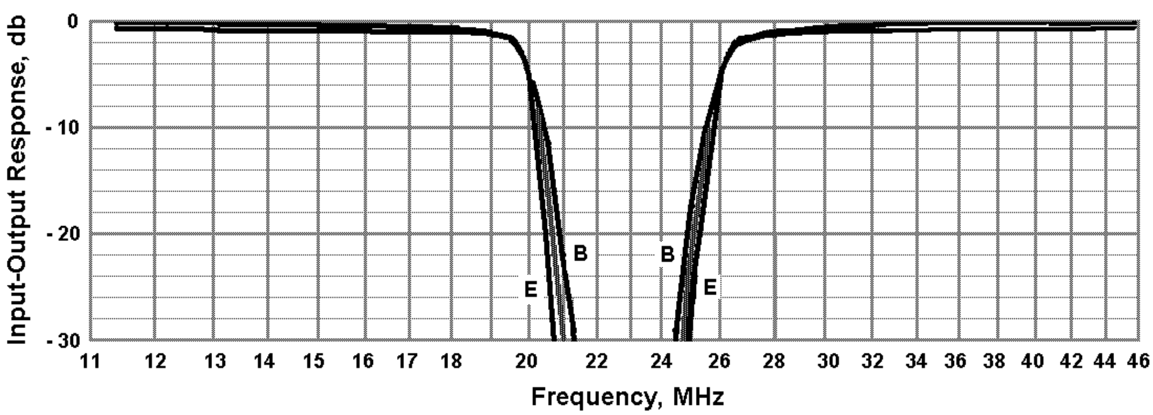


Figure 11-14 Wideband comparison of Butterworth (curve B), Chebyshev (Curve C between B and E), and Elliptic (E) Bandstop filters designed to same specifications.

Practical conditions, particularly Q of inductors, may prohibit successful application of bandstop filters. As an example a 5-section bandstop filter was analyzed for a bandstop of 20 to 26

MHz, 73 Ohm equal end terminations, having capacitive Q of 1000 and inductive Q of 80, Chebyshev and Elliptic ripple of 1 db. The results are shown in Figure 11-14. Insertion loss below 18 MHz and above 30 MHz remained at least 1.5 db, dropping to 1.0 db only below 12 MHz and above 38 MHz.

Given that a bandstop filter is the *inverse* of the bandpass, the above action is reasonable.
The greatest discrepancy is in the stopband, both is response shape and attenuation. This is shown in greater detail in Figure 11-15.

As can be expected intuitively, the Butterworth bandstop curve (B) is smoothest but also has the least attenuation at the bandstop geometric center frequency. The Chebyshev (C) has a sharper curve and more attenuation at band center than Butterworth. Elliptic (E) has the sharpest curves but has some profound ripple at its maximum bandstop attenuation region. That ripple is the same as passband ripple of a bandpass filter designed from Elliptic lowpass prototype data.

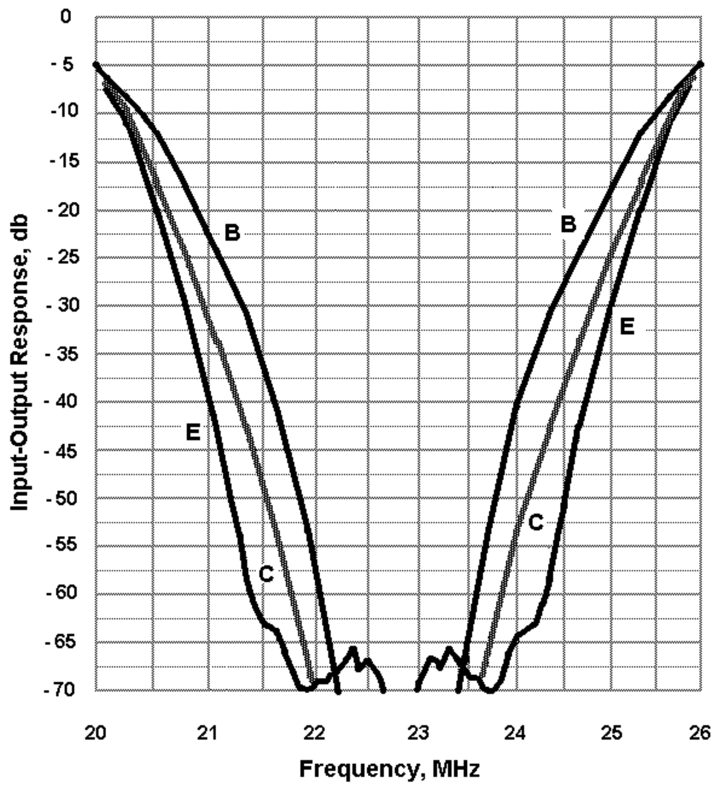


Figure 11-15 Close-in comparison of bandstop curves, B for Butterworth, C for Chebyshev, E for Elliptic.

Don't be alarmed about the attenuation shape of the Elliptic type in Figure 11-15. What was originally desired was a minimum attenuation over a certain frequency band. If 40 db attenuation was that limit, the Elliptic does it from 21.0 to 24.7 MHz. The Chebyshev does that from 21.3 to 24.5 MHz while the Butterworth is only 2.4 MHz wide from 21.6 to 24.0 MHz.

A greater nuisance in bandstop filters is the insertion loss. While the bandstop width could be taken equal to *-3db* points, the actual bandstop width of the example was about -6 db. Normally that would not be much of a problem with bandpass filters since one can always add gain to make up for the loss. With bandstop filters the additional finite-Q loss adds up near the bandstop frequency region. The frequency response curvature can't be corrected by additional gain.

Do not expect a sharp bandstop curvature with narrow bandstop percentages (bandstop width divided by center frequency). At roughly 10 percent bandstop width, the stopband attenuation decreases with decreasing width percentage.

As a general rule-of-thumb using practical L-C components, the bandstop region begins at 6 db down; more sections will increase maximum bandstop attenuation but also increase insertion loss slightly; sharpness of bandstop attenuation curvature is best with Elliptic, medium with Chebyshev, least with Butterworth; Butterworth response values are affected the least by component tolerances.

Chapter 12

Resonator Bandpass Filters and IF Transformers

Symmetric resonator bandpass filters are a transformation of lowpass-prototype bandpass filters covered in the previous chapter, ideal for practical applications and can be aligned in-circuit with simple shorting resistors. Intermediate Frequency Transformers are a special case of magnetically-coupled two-resonator arrangements.

General

Resonator bandpass filters are only *resonant* at the geometric center frequency under special conditions as will be described subsequently. This category of bandpass filters is *symmetric* as to component values and features the *same value* of one shunt element (depending on the coupling type) for ease of design. The resonator bandpass filters described here have *no magnetic coupling*; all coupling of shunt branches is done by specific capacitive or inductive components. This makes construction using low-external-magnetic-field toroidal inductors more compact than with cylindrical form inductors.

Magnetic coupling by lines of force between resonators is a difficult, time-consuming task for most hobbyists. Those are common in vacuum tube architecture superheterodyne receivers and were available from manufacturers, ready-built and shielded, from the 1930s through the 1980s. Some of those are described at the end of this chapter.

Basic Resonator BPF Topologies

Figure 12-1 illustrates the two basic topologies of resonator BPFs. The capacitively-coupled type has greater attenuation below the passband than above. The inductively-coupled type has greater attenuation above the passband than below it. It is possible to achieve near-even attenuation above and below the passband with odd-numbered resonator types. That is described a bit later.

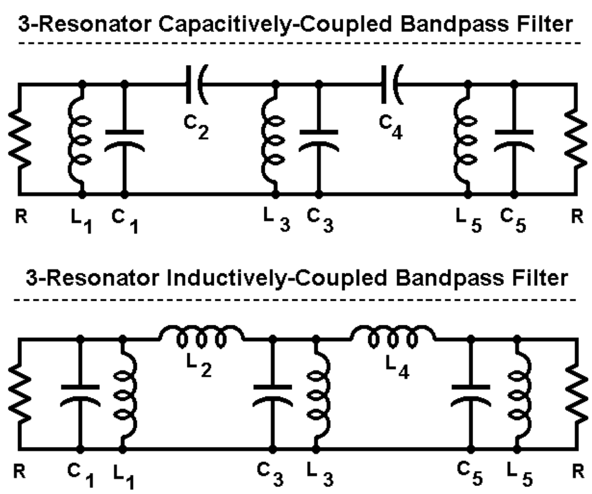


Figure 12-1 The two basic resonator BPF topologies, capacitive and inductive.

The *resonator* name refers to the *appearance* of parallel-resonant L-C circuits such as the L_1-C_1, L_3-C_3, and L_5-C_5 pairs in Figure 1. In the capacitively-coupled version, all shunt inductors are identical (L_1, L_3, L_5 etc). In the inductively-coupled type all shunt capacitors are identical (C_1, C_3, C_5, etc.). To better understand the resonant part of the name, consider the isolation of sections of Figure 12-1 as shown in Figure 12-2. In Figure 12-2 the adjacent series

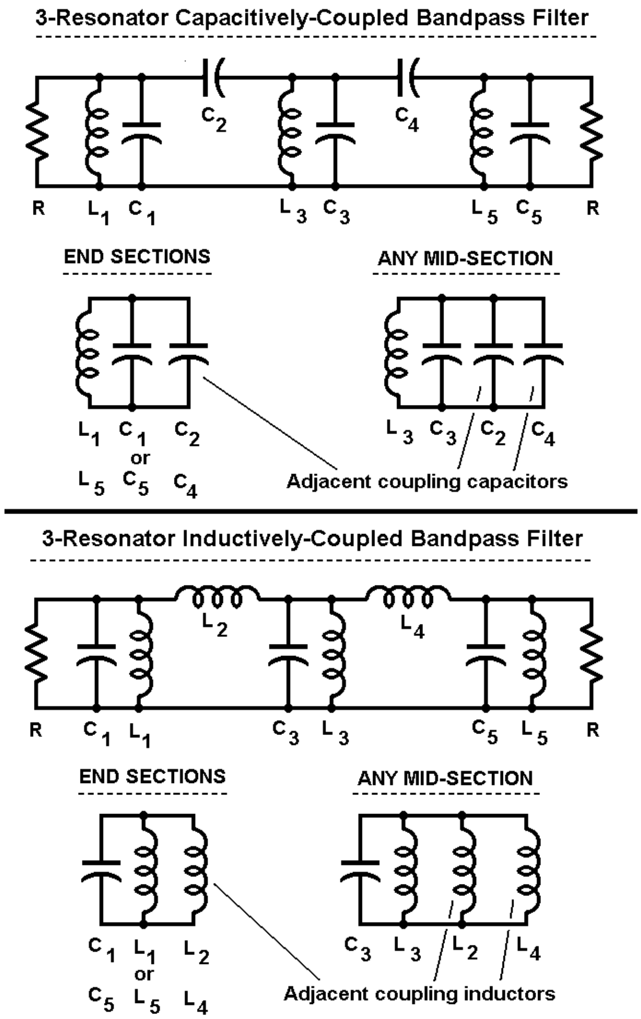


Figure 12-2 Resonances at center frequency for Ends and Middle sections. Note that all mid-sections have two adjacent coupling components in parallel; ends have one.

branches paralleling the shunt capacitance or inductance.

Resonances of shunt branches are always at the geometric center frequency, f_O, and that is found from:

$$f_O = \sqrt{f_{LOW} \cdot f_{HIGH}} \qquad \text{Where:}$$

f_{LOW}, f_{HIGH} are the Low and High limits of passband, respectively.
All frequencies in same units.

See also equation set (11-1) on the first page of Chapter 11. Geometric center frequency will be slightly lower than the arithmetic center frequency.

Design must start by selecting an end terminating resistance for the source end (assumed to be left-hand end of filter), R in Ohms. For all odd-number-of-resonators filters the source and load terminating resistors are the same. Load-end terminating resistance will be higher than source-end and normalized to it from table date for all even-number-of-resonator filters of the Chebyshev response only; Butterworth response filters are symmetric for any number of resonators.

Tables in this chapter give the normalized values for the identical shunt branches (inductors in capacitively-coupled BPF or capacitors in inductively-coupled BPF) and the single components of the series branches. Component values for capacitively-coupled filters are calculated from normalized table values by:

For identical shunt inductances L_N in Hy, and series capacitors C_N in Fd:

$$L_N = \frac{K_L \cdot R \cdot P}{f_O} \quad \text{and} \quad C_N = \frac{K_{CN}}{R \cdot f_O} \qquad (12\text{-}1)$$

Where:

f_O = Geometric center frequency, Hz

P = Fractional bandwidth = $(f_{BANDWIDTH} / f_O) = (f_B / f_O)$

K_L = Normalized table values for shunt inductance values

K_{CN} = Normalized table values for series capacitance values; will vary depending on location within filter.

With L_R calculated, the required resonating capacity at f_O is found from:

$$C_R = \frac{1}{\varpi_O^2 \cdot L_N} \quad \text{with } \varpi_O = 2\pi \cdot f_O \qquad (12\text{-}2)$$

For final component values:

$C_N = C_R - C_{(N+1)}$ [source end section] and

$C_N = C_R - C_{(N-1)}$ [load end section] and

$C_N = C_R - C_{(N-1)} - C_{(N+1)}$ [mid-sections]

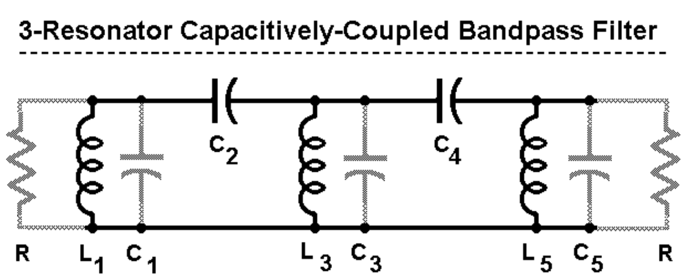


Figure 12-3 Filter components calculated from tables shown dark. Known or to be calculated shown in grey.

Note that each shunt capacitor is the required geometric center frequency resonating value minus each of the adjacent series capacitances. Middle-filter resonators always have *two* series capacitances to subtract. This is always true even if the same series capacitances are subtracted from two shunt branch capacitances.

Figure 12-3 is a reminder of which components are calculated on the basis of normalized table values. Those are shown in black. Use equation set (12-1) with them. Shunt capacitors shown in grey are dependent on shunt inductor resonance requirements and adjacent series capacitor values. Use equation set (12-2) for those.

Table 12-1 Normalized Inductor Values for K_L Capacitive-Coupling BPF

Response	2-Resonator	3-Resonator	4-Resonator
Butterworth	0.11254	0.15916	0.20794
0.1 db Chebyshev	0.18879	0.18879	0.14354
0.25 db Chebyshev	0.14298	0.12211	0.11548
0.5 db Chebyshev	0.11345	0.099704	0.095284
1.0 db Chebyshev	0.087354	0.078650	0.075822
2.0 db Chebyshev	0.063966	0.058714	0.056994
3.0 db Chebyshev	0.051320	0.047526	0.046280

Response	5-Resonator	6-Resonator	7-Resonator
Butterworth	0.25752	0.30746	0.35763
0.1 db Chebyshev	0.13878	0.13625	0.13474
0.25 db Chebyshev	0.11252	0.11094	0.11000
0.5 db Chebyshev	0.093304	0.092244	0.091610
1.0 db Chebyshev	0.074550	0.073868	0.073460
2.0 db Chebyshev	0.056218	0.055802	0.055552
3.0 db Chebyshev	0.045718	0.045414	0.045234

Note: The *n db Chebyshev* response refers to the ripple in passband response; i.e., the variation in amplitude in decibels within that passband.

Table 12-2 Normalized Capacitor Values for K_{CN}, Capacitive-Coupling BPF

	2-Resonator	3-Resonator	4-Resonator	
Response	K-C2	K-C2, K-C4	K-C2, K-C6	K-C4
Butterworth	0.11254	0.15916	0.10243	0.065924
0.1 db Chebyshev	0.18879	0.18879	0.14664	0.11605
0.25 db Chebyshev	0.14298	0.12211	0.16584	0.13579
0.5 db Chebyshev	0.11345	0.099704	0.18835	0.15825
1.0 db Chebyshev	0.087354	0.078650	0.22350	0.19244
2.0 db Chebyshev	0.063966	0.058714	0.28342	0.24940
3.0 db Chebyshev	0.051320	0.047526	0.34118	0.30345

	5-Resonator		6-Resonator		
Response	K-C2, K-C8	K-C4,K-C6	K-C2, K-C10	K-C4, K-s8	K-C6
Butter.	0.098363	0.054679	0.096289	0.049843	0.042645
0.1 db Cheb.	0.14555	0.11091	0.14517	0.10942	0.10526
0.25 db Cheb.	0.16487	0.13097	0.16454	0.12959	0.12564
0.5 db Cheb.	0.18745	0.15359	0.18714	0.15226	0.14840
1.0 db Cheb.	0.22262	0.18777	0.22233	0.18645	0.18256
2.0 db Cheb.	0.28251	0.24440	0.28221	0.24300	0.23881
3.0 db Cheb.	0.34020	0.29799	0.33987	0.29645	0.29185

	7-Resonator		
Response	K-C2, K-C12	K-C4, K-C10	K-C6, K-C8
Butterworth	0.095080	0.047252	0.037311
0.1 db Chebyshev	0.14501	0.10884	0.10350
0.25 db Chebyshev	0.16440	0.12906	0.12400
0.5 db Chebyshev	0.18701	0.15176	0.14682
1.0 db Chebyshev	0.22220	0.18595	0.18097
2.0 db Chebyshev	0.28208	0.24247	0.23712
3.0 db Chebyshev	0.33973	0.29587	0.29000

As an example, a 500 KHz bandpass filter is desired at a geometric center frequency of 10 MHz and with termination resistances of 3 KOhms. Fractional bandwidth P is then 0.05. A smooth Butterworth response is chosen. For the capacitively-coupled type, K_L will be 0.25752 from Table 12-1. K_{S2} and K_{S8} will be 0.098363, K_{S4} and K_{S6} will be 0.054679 from Table 12-2. Calculations are then:

$$L_1 = L_3 = L_5 = L_7 = L_9 = \frac{R\,P\,K_L}{f_O} = \frac{3\cdot 10^3\cdot 0.05\cdot 0.25752}{10\cdot 10^6} = 3.8628\cdot 10^{-6}$$

$$C_2 = C_8 = \frac{K_{C2}}{R\cdot f_O} = \frac{0.098363}{3\cdot 10^3\cdot 10\cdot 10^6} = 3.27877\cdot 10^{-12} \approx 3.3\text{ pFd}$$

$$C_4 = C_6 = \frac{K_{C4}}{R\cdot f_O} = \frac{0.054679}{3\cdot 10^{10}} = 1..82263\cdot 10^{-12} \approx 1.8\text{ pFd}$$

Note: Resist the urge to use approximate values at this stage.

The shunt capacitances are first found by calculating the resonating capacity for L using (12-2) and then appropriately subtracting the adjacent series capacitances:

$$\varpi_O{}^2 = (2\pi\cdot f_O)^2 = (6.2832\cdot 10\cdot 10^6)^2 = 3.9478\cdot 10^{15}$$

$$C_R = \frac{1}{\varpi_O{}^2\cdot L_R} = \frac{1}{3.9478\cdot 10^{15}\cdot 3.8628\cdot 10^{-6}} = \frac{1}{1.52497\cdot 10^9} = 65.575\cdot 10^{-12}$$

$$C_1 = C_9 = C_R - C_2 = 65.575\cdot 10^{-12} - 3.27887\cdot 10^{-12} = 62.296\cdot 10^{-12}$$

$$C_3 = C_7 = C_R - C_2 - C_4 = 65.575\cdot 10^{-12} - 3.27887\cdot 10^{-12} - 1.82263\cdot 10^{-12} = 60.474\cdot 10^{-12}$$

$$C_5 = C_R - C_4 - C_6 = 65.575\cdot 10^{-12} - 1.82263\cdot 10^{-12} - 1.82263\cdot 10^{-12} = 61.930\cdot 10^{-12}$$

Note that all the calculations result in non-standard part values. Hold off on rejection of method until shown the results of using standard-tolerance part values and alignment.[1]

Inductively-Coupled Resonator Bandpass Filter

For the inductively-coupled bandpass filter, those components calculated from normalized value tables are shown dark in Figure 12-4. Shunt inductor values are calculated as a second step once the series inductors are known in value.

[1] More on alignment methods in a few pages. It is easier than with previous bandpass filters.

Table 12-3 Normalized Inductor Values for K_C, Inductive-Coupling BPF

Response	2-Resonator	3-Resonator	4-Resonator
Butterworth	0.22508	0.15915	0.12181
0.1 db Chebyshev	0.13417	0.16418	0.17647
0.25 db Chebyshev	0.17717	0.20744	0.21935
0.5 db Chebyshev	0.22328	0.25406	0.26584
1.0 db Chebyshev	0.28997	0.32206	0.33407
2.0 db Chebyshev	0.39600	0.43142	0.44443
3.0 db Chebyshev	0.49358	0.53297	0.54732

Response	5-Resonator	6-Resonator	7-Resonator
Butterworth	0.098363	0.082385	0.070831
0.1 db Chebyshev	0.18252	0.18591	0.18799
0.25 db Chebyshev	0.22511	0.22831	0.23027
0.5 db Chebyshev	0.27148	0.27460	0.27650
1.0 db Chebyshev	0.33978	0.34291	0.34482
2.0 db Chebyshev	0.45057	0.45393	0.45597
3.0 db Chebyshev	0.55406	0.55776	0.55999

Note: The *n db Chebyshev* response refers to amplitude ripple in passband response.

Table 12-4 Normalized Capacitor Values for K_{LN}, Inductive-Coupling BPF

	2-Resonator	3-Resonator	4-Resonator	
Response	K-L2	K-L2, K-L4	K-L2, K-L6	K-L4
Butterworth	0.15915	0.22508	0.24729	0.38423
0.1 db Chebyshev	0.13671	0.16785	0.17274	0.21827
0.25 db Chebyshev	0.12506	0.14925	0.15274	0.18654
0.5 db Chebyshev	0.11299	0.13192	0.13448	0.16006
1.0 db Chebyshev	0.097589	0.11155	0.11334	0.13162
2.0 db Chebyshev	0.078645	0.088210	0.089374	0.10157
3.0 db Chebyshev	0.066035	0.073372	0.074244	0.083473

	5-Resonator		6-Resonator		
Response	K-L2, K-L8	K-L4,K-L6	K-L2, K-L10	K-L4, K-L8	K-L6
Butter.	0.25752	0.46325	0.26307	0.50820	0.59397
0.1 db Cheb.	0.17403	0.22838	0.17448	0.23150	0.24064
0.25 db Cheb.	0.15363	0.19340	0.15395	0.19546	0.20161
0.5 db Cheb.	0.13513	0.16492	0.13535	0.16636	0.17069
1.0 db Cheb.	0.11378	0.13490	0.11393	0.13585	0.13875
2.0 db Cheb.	0.089660	0.10364	0.089758	0.10424	0.10607
3.0 db Cheb.	0.074457	0.085005	0.074530	0.085444	0.086793

	7-Resonator		
Response	K-L2, K-L12	K-L4, K-L10	K-L6, K-L8
Butterworth	0.26641	0.53607	0.67890
0.1 db Chebyshev	0.17468	0.23272	0.24473
0.25 db Chebyshev	0.15408	0.19626	0.20427
0.5 db Chebyshev	0.13545	0.16691	0.17253
1.0 db Chebyshev	0.11400	0.13622	0.13997
2.0 db Chebyshev	0.089799	0.10447	0.10682
3.0 db Chebyshev	0.074560	0.085612	0.087347

For identical shunt capacitances C_N in Fd, series inductances L_N in Hy:

$$C_N = \frac{K_C}{R \cdot P \cdot f_O} \quad \text{and} \quad L_N = \frac{R \cdot K_{LN}}{f_O} \qquad (12-3)$$

Where: R, P, f_O are as in equation sets (12 - 1), (12 - 3)

K_C is from Table 12 - 3; K_{LN} from Table 12 - 4

With C_N calculated, the required resonating capacity at f_O is found from:

$$L_R = \frac{1}{\varpi_O^{\,2} \cdot C_N} \quad \text{with } \varpi_O = 2\pi \cdot f_O \qquad (12-4)$$

For final component values:

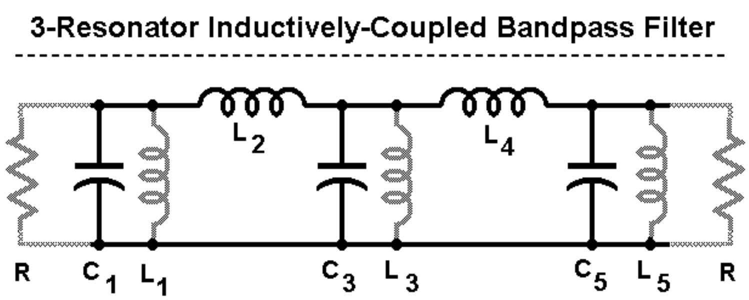
$$L_N = \frac{1}{\left(\frac{1}{L_R}\right) - \left(\frac{1}{L_2}\right)} \quad \text{[source end section] and}$$

$$L_N = \frac{1}{\left(\frac{1}{L_R}\right) - \left(\frac{1}{L_{(N-1)}}\right)} \quad \text{[load end section] and}$$

$$L_N = \frac{1}{\left(\frac{1}{L_R}\right) - \left(\frac{1}{L_{(N-1)}}\right) - \left(\frac{1}{L_{(N+1)}}\right)} \quad \text{[mid - sections]}$$

What equation set (12-4) does is the second step in finding the shunt inductors, using the resonating inductance L_R, taking its reciprocal, then subtracting the reciprocals of each adjacent series inductor, finally taking the reciprocal of that result for the final shunt inductor value. Note that capacitors can

Figure 12-4 Branch components calculated via tables shown dark, others calculated in a second step shown in grey.



simply add or subtract but inductors must use the reciprocal of the sum/difference of reciprocals.[2]

For an example, use the same 5-section Butterworth response filter as before, with termination resistors of 3 KOhms and center frequency of 10 MHz, bandwidth of 500 KHz. Table 12-3 yields $K_{CN} = 0.098363$ and Table 12-4 has $K_{L2} = 0.25752$ and $K_{L4} = 0.46325$. P = 0.05 so equation set (12-3) is used:

$$C_N = \frac{K_C}{R \cdot P \cdot f_O} = \frac{0.098363}{3 \cdot 10^3 \cdot 0.05 \cdot 1 \cdot 10^7} = \frac{0.098363}{1.5 \cdot 10^9} = 65.575 \cdot 10^{-12} = 65.575 \text{ pFd}$$

All the odd-subscripted capacitors (shunt connections) will be identical to that value.

$$L_2 = L_8 = \frac{K_{L2} \cdot R}{f_O} = \frac{0.25752 \cdot 3000}{10 \cdot 10^6} = \frac{772.56}{10 \cdot 10^6} = 77.256 \cdot 10^{-6} = 77.256 \mu\text{Hy}$$

$$L_4 = L_6 = \frac{K_{L4} \cdot R}{f_O} = \frac{0.46325 \cdot 3000}{10 \cdot 10^6} = \frac{1389.75}{10 \cdot 10^6} = 138.98 \ \mu\text{Hy}$$

Since this is a symmetrical arrangement of components the even-subscripted series inductors will be in pairs. To find the final value of the shunt inductors, use equation set (12-4):

$$L_R = \frac{1}{\varpi_O{}^2 \cdot C_N} = \frac{1}{3.94784 \cdot 10^{15} \cdot 65.535 \cdot 10^{-12}} = \frac{1}{2.58722 \cdot 10^5} = 3.86516 \cdot 10^{-6}$$

to make things easier, take the reciprocals of inductances now:

$$\frac{1}{L_R} = 258.88 \cdot 10^3 \qquad \frac{1}{L_2} = 12.944 \cdot 10^3 \qquad \frac{1}{L_4} = 7.1953 \cdot 10^3$$

$$L_1 = L_9 = \frac{1}{\left(\frac{1}{L_R}\right) - \left(\frac{1}{L_2}\right)} = \frac{1}{(258.88 - 12.944) \cdot 10^3} = \frac{1}{245.94 \cdot 10^3} = 4.0661 \cdot 10^{-6}$$

$$L_3 = L_7 = \frac{1}{(258.88 - 12.944 - 7.1953) \cdot 10^3} = \frac{1}{238.74 \cdot 10^3} = 4.1886 \cdot 10^{-6}$$

$$L_5 = \frac{1}{(258.88 - 7.1953 - 7.1953) \cdot 10^3} = \frac{1}{244.49 \cdot 10^3} = 4.0901 \cdot 10^{-6}$$

---

[2] This method of calculation of paralleled inductors (or resistors) is made much easier by using a pocket calculator with a reciprocal function key

Low-Impedance Coupling Versus *High-Impedance*

The series branch values in both examples are becoming impractical for construction but there is a way around this by using a *Pi-to-Tee* network equivalent to restore the practical values. For lack of a more descriptive term, all of the discussion so far has been with the *high-impedance* type of coupling of shunt branches. To do this advantage is taken of the shunt branch resonance at center frequency to transform the capacitors of the capacitively-coupled version or the inductors of the inductively-coupled version. This is shown in Figure 12-6 following for a capacitively-coupled filter.

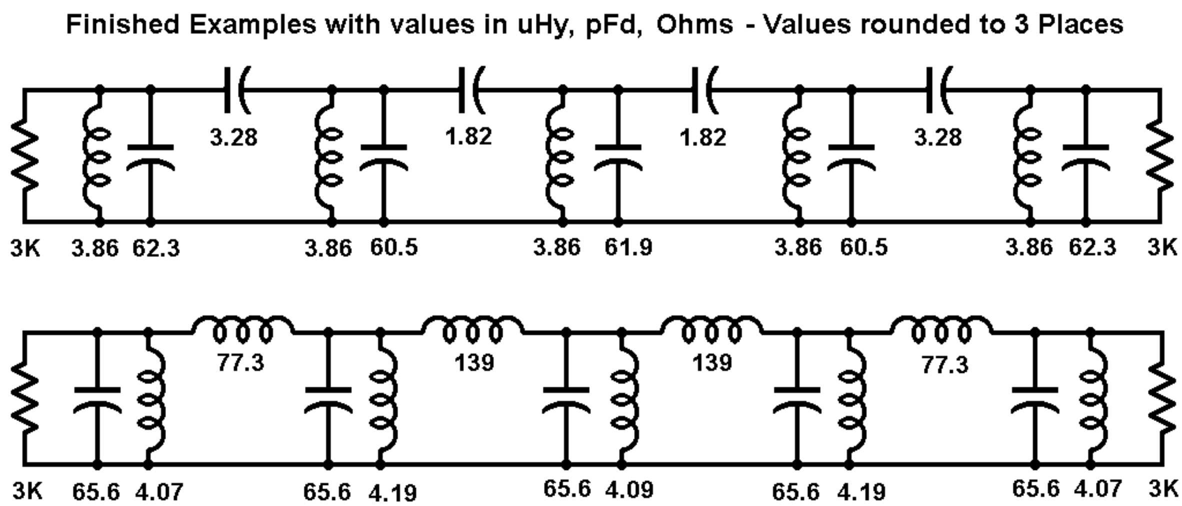


Figure 12-5 Finished examples with 3-place values. Note that series branch values are becoming impractical due to small values as in capacitively-coupled version) or may have distributed capacity in series inductors that could affect frequency response.

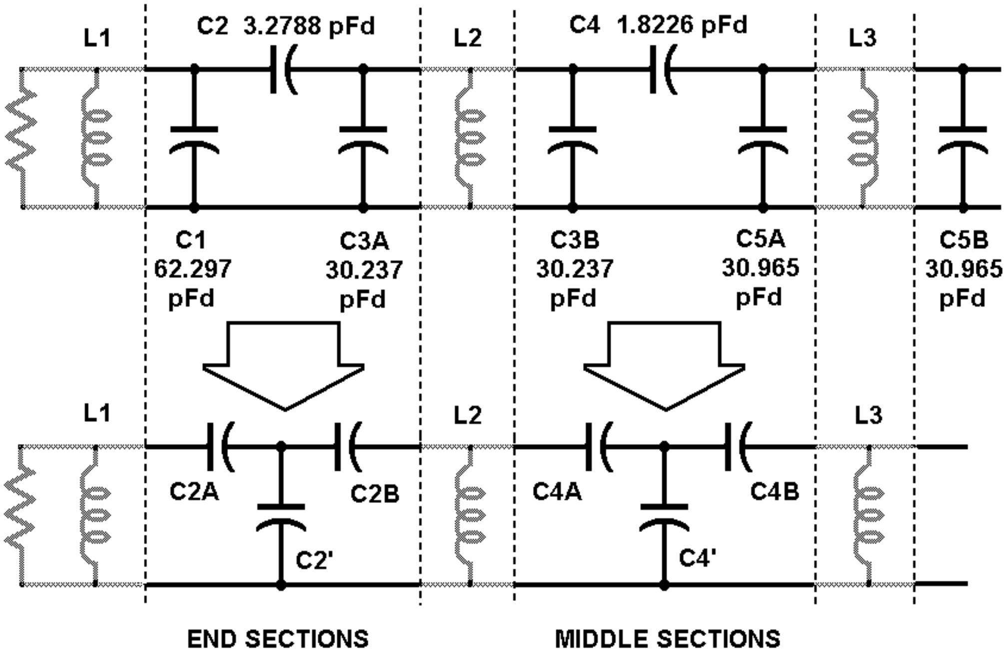


Figure 12-6 Transform of Pi-section of capacitors to a Tee-section in order to increase the resulting shunt capacitor branch values.

To begin the transformation, take all the middle-section shunt capacitor values and divide them by 2. All shunt capacitors directly at ends are not altered. In Figure 12-6 the previous 10 MHz BPF (capacitively-coupled) is shown with C1 = 62.297 pFd (not altered) but C3 (60.474 pFd) is split into C3A and C3B at 30.237 pFd each. C5 at 61.930 pFd is split into C5A and C5B, each at 30.965 pFd. This does not alter the filter since C3A + C3B = C3 and so forth. The division by 2 is a convenience in calculation.[3]

$$
\begin{aligned}
Num &= C_1 \cdot C_2 + C_1 \cdot C_{3A} + C_2 \cdot C_{3A} \quad \text{[end section]} \\
&= C_{3B} \cdot C_4 + C_{3B} \cdot C_{5A} + C_4 \cdot C_{5A} \quad \text{[mid sections]} \qquad (12\text{-}5)
\end{aligned}
$$

$$
C_{2A} = \frac{Num}{C_{3A}} \qquad C_{2B} = \frac{Num}{C_1} \qquad C_2' = \frac{Num}{C_2} \quad \text{[end section]}
$$

$$
C_{4A} = \frac{Num}{C_{5A}} \qquad C_{4B} = \frac{Num}{C_{3B}} \qquad C_4' = \frac{Num}{C_4} \quad \text{[mid sections]}
$$

The use of *Num* is a convenience since it is repeated in all numerators of the Tee-section calculation equations. C_2 -prime and C_4 -prime are used to differentiate the Tee form from the Pi form of the same subscript number. This is not a full and complete set of equations for all resonator bandpass filters. It is meant to show that each group of three capacitors in a Pi-network can be transformed in an equivalent Tee-network. See Appendix 12-1 for a mathematical relationship of it. C_{3A} and C_{3B} are each half the capacitance of C_3 ; C_{5A} is half the value of C_5 and so forth down the line. Using

[3] Filter and network purists would point out that the shunt branch capacitances could be split into values that could find a different Tee-section shunt and so forth. That really isn't needed here. Manual calculation is arduous enough without introducing minutiae that clog thinking and add to the design work. The only rule is that the split capacitor values add up the *same* single shunt branch value as was calculated for the High-impedance filter values.

the previous example of the capacitively-coupled 5-resonator bandpass filter, the calculations for the *low-impedance coupling* Tee-section values are:

$$C_1 = 62.297 \quad C_2 = 3.2788 \quad C_{3A} = 30.237 \quad \text{[all are in same pFd units]}$$

$$Num = 62.297 \cdot 3.2788 + 62.297 \cdot 30.237 + 3.2788 \cdot 30.237 =$$

$$204.2594 + 1883.6744 + 99.14108 = 2187.075$$

$$C_{2A} = \frac{Num}{C_{3A}} = \frac{2187.075}{30.237} = 72.331 \quad C_{2B} = \frac{Num}{C_1} = \frac{2187.075}{62.297} = 35.107$$

$$C_2^{'} = \frac{Num}{C_2} = \frac{2187.075}{3.2788} = 667.04 \quad \text{For the next Pi-to-Tee transform:}$$

$$C_{3B} = 30.237 \quad C_4 = 1.8226 \quad C_{5A} = 30.965$$

$$Num = 30.237 \cdot 1.8226 + 30.237 \cdot 30.965 + 1.8226 \cdot 30.965 =$$

$$55.110 + 936.289 + 56.437 = 1047.835$$

$$C_{4A} = \frac{Num}{C_{5A}} = \frac{1047.835}{30.965} = 33.839 \quad C_{4B} = \frac{Num}{C_{3B}} = \frac{1047.835}{30.237} = 34.654$$

$$C_4^{'} = \frac{Num}{C_4} = \frac{1047.835}{1.8226} = 574.91$$

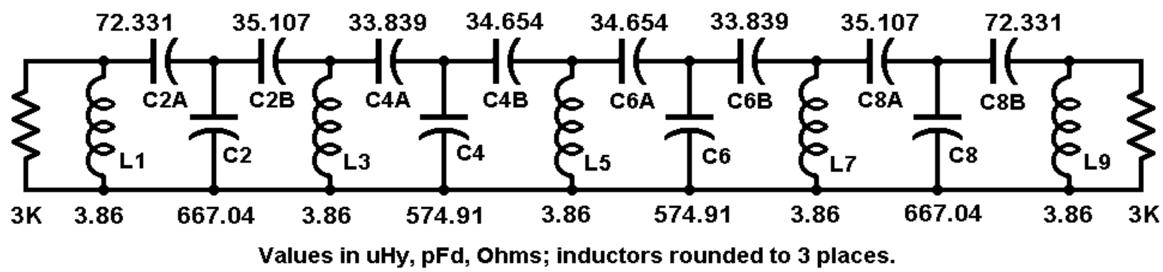


Figure 12-7 The capacitively-coupled bandpass filter of the example transformed from high-impedance coupling to low-impedance by methods given in the text.

F

Figure 12-7 may be numerically correct but the values are, once again, made from *unobtainium*. To be practical, the capacitors can all be fixed values. For example, C2B through C8A are all very close to 33 pFd, a standard 10% tolerance value. C4 and C5 are close to 560 pFd with C2 and C8 close to 680 pFd, those four close to 10% tolerance values. The inductors can be made trimmable so one can take advantage of inductors' values as being resonant at center frequency when in parallel with just the adjacent capacitances. Figure 12-8 shows the result.

For L1 the only adjacent capacitances are C2A in series with C2 and C2B in parallel. That will be 62.079 pFd and L1 will be resonant with that at 10 MHz if its value is 4.0803 µHy. L3 has two sets of adjacent capacitances: C2||C2A in series with C2B, C4||C4B in series with C4A. That will be 31.6056 pFd || 32.2604 pFd or 62.8660 pFd. L3 must be 4.0293 µHy for it to be resonant at 10 MHz. Similarly, L5 has two adjacent pairs: C4A||C4 in series with C4B and C6A in series with C6||C6B. That yields 62.5208 pFd and L5 must be 4.0515 µHy for resonance at 10 MHz. Since these are symmetrical designs, L7 = L3 and L9 = L1.

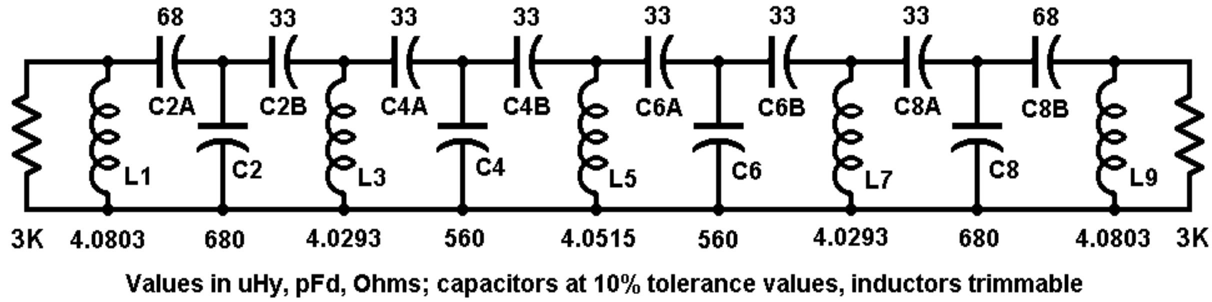


Figure 12-8 BPF of Figure 12-7 with practical fixed-capacitor values.

Will the values of Figure 12-8 result in a BPF with very similar characteristics? Yes, it will by all analyses and similar practical experiences. To do so the inductors ***must be trimmable*** for resonance at the geometric center frequency. That can be done by simply shunting all adjacent inductors with 1/5th to 1/10th of the terminating resistor values, then peaking the unshunted inductor at center frequency. The only drawback to this method is that shunting will result in a large insertion loss of 10 to 15 db per shunted section, all additive.

The difference in passband and stopband response is hardly noticeable, thanks to the ideal values being so close to the fixed values of 33, 68, 680, and 560 pFd. If practical fixed capacitor values are greater than about 5% of ideal, it is a matter of analyzing the frequency response of those first using a good simulation-analysis program.

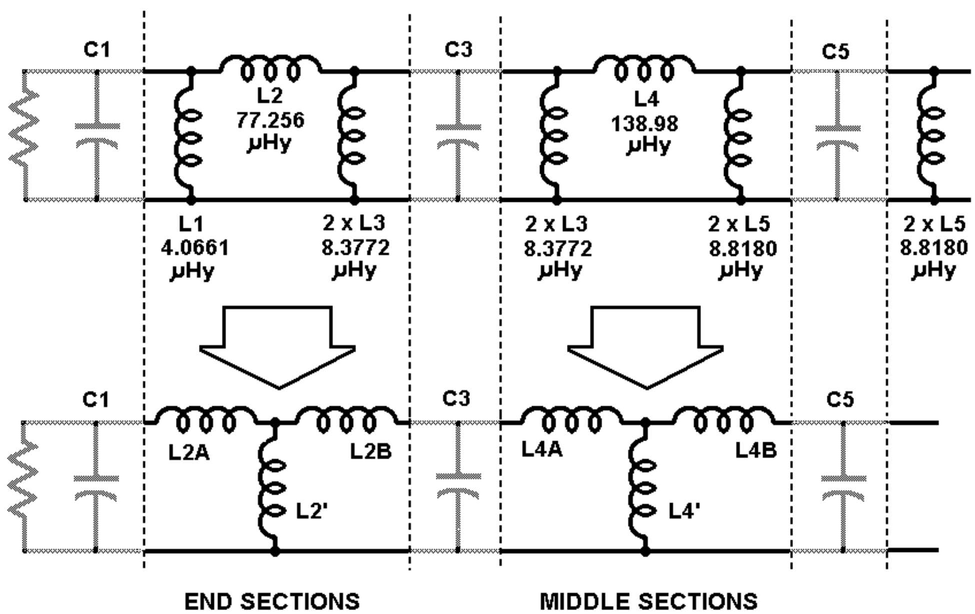


Figure 12-9 A transform from high-impedance to low-impedance coupling sub-sections. The adjacent shunt inductors must double in value for the trans-form to start.

Changing inductively-coupled resonator BPFs is a bit easier mathematically. Note that all the shunt inductors, except for the ends, must double in value to begin the transformation. Since the reactance/impedance of inductors does not require reciprocals, equation set (5-5) can be used directly, substituting inductors for resistances. The previous 10 MHz BPF is used following. The example value for L3 was 4.1886 µHy, L5 was 4.0901 µHy. Shunt capacitor values aren't affected by the transformation.

From (5-5): For the end sections:

$$Den = L_1 + L_2 + L_3 = 4.0661 + 77.256 + 8.3772 = 89.6993$$

Note that all values can be direct as μHy with all results in μHy

$$L_{2A} = \frac{L_1 \cdot L_2}{Den} = \frac{4.0661 \cdot 77.256}{89.6993} = \frac{314.130}{89.6993} = 3.5020$$

$$L_{2B} = \frac{L_2 \cdot L_3}{Den} = \frac{77.256 \cdot 8.3772}{89.6993} = \frac{647.189}{89.6993} = 7.2151$$

$$L_2{}' = \frac{L_1 \cdot L_3}{Den} = \frac{4.0661 \cdot 8.3772}{89.6993} = \frac{34.0625}{89.6993} = 0.37974$$

For the middle sections:

$$Den = L_3 + L_4 + L_5 = 8.3772 + 138.98 + 8.1802 = 155.537$$

$$L_{4A} = \frac{L_3 \cdot L_4}{Den} = \frac{8.3772 \cdot 138.98}{155.537} = \frac{1164.263}{155.537} = 7.4854$$

$$L_{4B} = \frac{L_4 \cdot L_5}{Den} = \frac{138.98 \cdot 8.1802}{155.537} = \frac{1136.884}{155.537} = 7.3094$$

$$L_4{}' = \frac{L_3 \cdot L_5}{Den} = \frac{8.3772 \cdot 8.1802}{155.537} = \frac{68.52717}{155.537} = 0.44058$$

The transformed shunt inductors have drastically reduced in value while the transformed series inductors remained fairly close to original values. That's the trade-off which must be considered, although for this example the transformed shunt inductors are still practical, can be made.

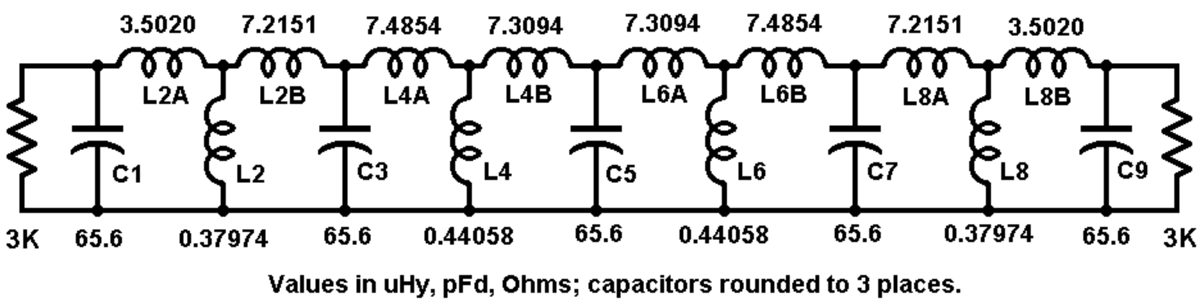


Figure 12-10 Inductively-coupled example Resonator BPF transformed to Low-Z coupling

Ideal values of Figure 12-10 inductors could be changed to fixed, practical 5% tolerance values with hardly any difference in bandpass or bandstop insertion loss. Shunt capacitors would then be trimmable to the geometric center frequency. A slight problem is that 7.5 µHy inductors might be difficult to obtain; 6.8 µHy inductors would be more common. Shunt inductors (L2, L4, L6, L8) would probably have to be made out of solid wire on air-core forms or self-supporting. The value of those shunt inductors would affect the passband width and a slight variation on passband shape. There would be a greater change in Chebyshev response BPFs, particularly with higher ripple in the passband. New nominal values for shunt capacitors would have to be recalculated once fixed inductor values were selected.

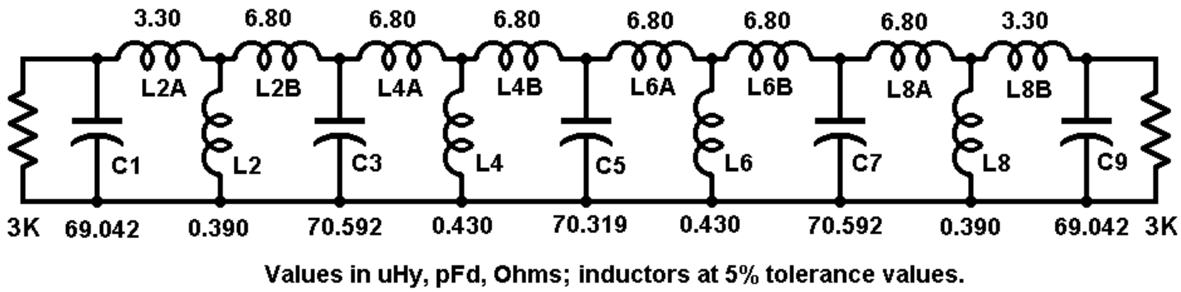


Figure 12-11 Example BPF of Figure 12-10 with inductors changed to 5% tolerance values for a practical working filter. Capacitor values are nominal and would be set by a trimmer capacitor at geometric center frequency, adjacent capacitors shunted.

The example shown in Figure 12-11 had a passband of about 550 KHz at -3 db points instead of 500 KHz as with the ideal-value filter. Otherwise the passband followed the ideal Butterworth response with little change in far-from-passband frequency attenuation. Note: The analysis was first performed with 7.5 μHy inductors in place of the 6.8 μHy values which corresponded very close to ideal-value frequency response. Usually a slightly-wider passband is more acceptable in application than one that is slightly narrower.

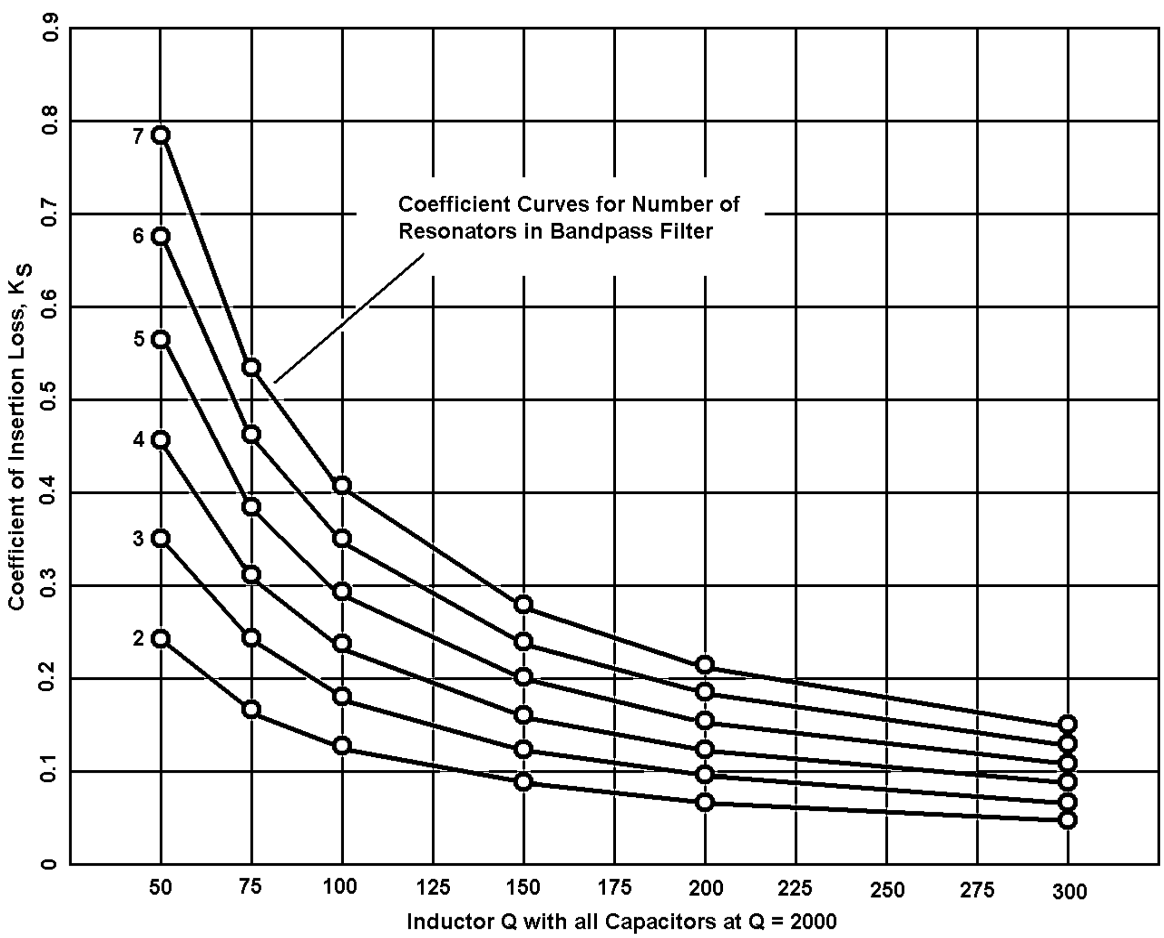


Figure 12-12 Coefficient chart to determine Butterworth Filter Insertion Loss.

Insertion Loss of Butterworth Response with Practical Q

Practical Qs for capacitors and inductors will cause both a passband shape distortion and loss of output versus input. The higher Q values the better; shape goes more towards ideal and insertion loss will be less. As a general rule of thumb, capacitor Q can be taken as 2000 with inductors varying in Q from 50 to 300, depending on form type and core material. The curves of Figure 12-12 are for coefficient K to determine the insertion loss in db for Butterworth resonator BPFs with sections from 2 to 7. That insertion loss is approximately derived from the following simple equation:

$$\text{Loss in decibels} = \frac{K_S}{P} \qquad \text{Where:}$$

K_S is the coefficient from Figure 12-12

$$P \text{ is the fractional bandwidth } = \frac{\text{Passband Width}}{\text{Geometric Center Frequency}}$$

K_S must use the curve for the number of sections.

As an example, use the 5-section filter described before, using inductor Qs of 150. The K coefficient would be 0.20 and P would be 0.05. The approximate loss for an ideal-value filter would be 0.20/0.05 = 4.0 db. Note: This would apply to both capacitively-coupled or inductively-coupled types, high- or low-impedance coupling. If the component values depart more than ± 5% from ideal there might be some change in the insertion loss but the above approximation is fairly accurate for any percentage-bandwidth from 5 to 50%.

Gain of a Stage With a BPF

Voltage gain of a linear amplifier is its transconductance (in mhos) times the input impedance magnitude (in Ohms) of the source end of the bandpass filter. Insertion loss in db is then subtracted from that for the total gain to the input of the next stage.

Input impedance will appear to vary greatly from geometric center frequency's value. This is normal. The overall amplitude versus frequency response will still follow the Butterworth response shape[4] derived from Butterworth coefficients in the tables. Geometric center frequency impedance is taken as the baseline for stage gain. For correct resonator alignment that impedance is resistive.

Butterworth response resonator BPFs' center frequency impedance magnitude will be within 1% of its source-end terminating resistance for 7, 6, and 5 sections for inductive Qs down to 50. With 4 resonator sections the input impedance magnitude is -2 % at inductive Q of 50 and percentage bandwidth of 5%, within 1% for higher Q_L at wider bandwidth. With 3 resonator sections and 5% bandwidth it is -6% at Q_L of 50, -2.4% at Q_L of 75, -1.2 % at Q_L of 100; at 10% bandwidth the input impedance is down 1.1% with Q_L of 50 but less than 1% error at higher Qs. Input impedance magnitude change is greatest with 2-section resonators: -17.5 % at 5 % bandwidth and Q_L of 50, -10% at Q_L of 75, -6.4% at Q_L of 100, -2.1% at Q_L of 200; at 10% bandwidth it is -

[4] There is a lot of interaction of impedances within any filter and the total of that interaction yields the amplitude versus frequency response shape at its output.

6.2% at Q_L of 50, -3.3% at Q_L of 75, -1.9% at Q_L of 150.

Frequency Response Within the Passband

An ideal Butterworth response shape of amplitude versus frequency will generally be very flat in the middle when high-Q components are used. As the Qs are reduced the shape loses its flatness and becomes more rounded. The -3 db bandwidth of Butterworth response bandpass filters will still remain constant with percentage bandwidths down to 5 % and inductor Qs as low as 50.

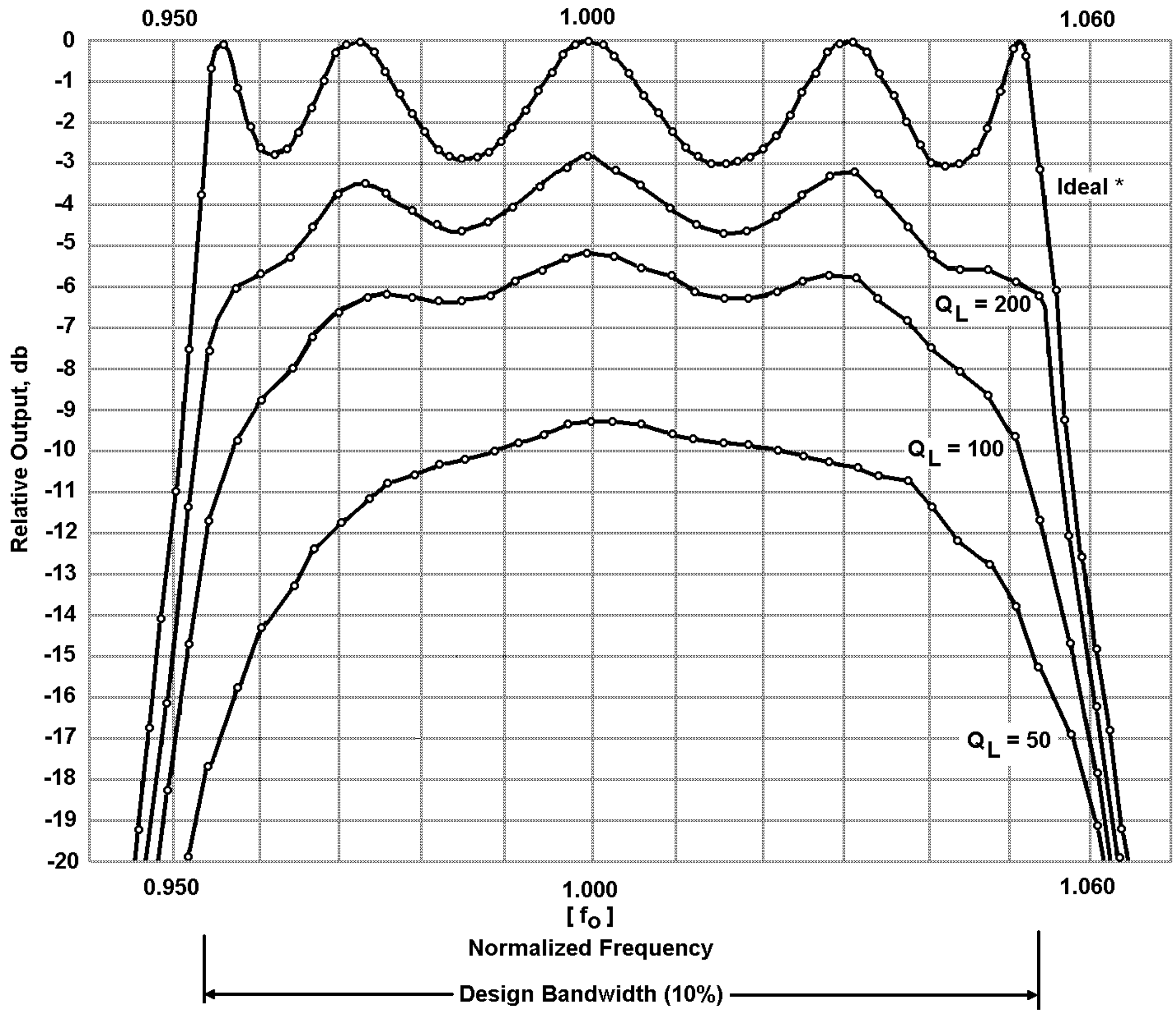


Figure 12-13 Response of 3 db Ripple Chebyshev 5-Section Resonator bandpass filter, capacitively-coupled, 10-percent bandwidth, relative to inductor Q with capacitor Qs equal to 2000. ****Ideal* **refers to all C and L component Qs equal to 10,000. Frequency scale is normalized to geometric center frequency.**

Chebyshev response shapes are more distorted and depend on the allowed passband amplitude ripple in db as well as the number of resonator sections. See Figure 12-13 as an example. That Figure is an extreme case of Chebyshev response with a 3 db peak-to-peak passband ripple. The number of peaks in the passband will correspond to the number of sections/resonators in the BPF, equal to the number of sections in the conventional BPFs described in Chapter 11. The passband ripple is observable only in the ideal situation with component Qs that are perfect, no losses. Once component losses are present, the *ripple* becomes less and the passband shape becomes more rounded. The passband -3 db bandwidth will also shrink slightly, even with high Qs; a ten-percent bandwidth Butterworth response would hold its passband nearly equal down to inductor Qs of 50.

Note that the design bandwidth seems to be centered above the geometric center frequency. This is not in error for two reasons: The frequency scale is logarithmic (though difficult to tell with this small percentage bandwidth); the low-frequency end is slightly above what it would be with an arithmetic center frequency.[5] See equation set (11-1) for the relationships of band edges and arithmetic versus geometric center frequency.

For narrower percentage passbands, the inductor Qs required would be inversely proportional to passband compared with Figure 12-13; a 5% bandwidth would require approximately double the inductor Qs for the same response shape. For wider percentage bandwidths the inductor Qs could be lower for those same shapes. That would include the readily-apparent increase in insertion loss with lower Qs.

Stopband Attenuation Comparisons of Chebyshev Versus Butterworth

Like the Butterworth, Chebyshev response attenuation at frequencies far from center frequency will be relatively independent of Q values. Chebyshev filters have greater stopband attenuation than Butterworth, as indicated in Figure 11-9. Stopband attenuation is proportional to the design ripple values.

Tables 12-5 through 12-8 are a guide to selecting the percentage bandwidth of 5-resonator filters of the capacitively-coupled kind. Frequency is normalized to unity for the geometric center frequency. To determine stopband attenuation, divide the determined frequency by center frequency and interpolate the tables values at the *normalized frequency* row. Percentage bandwidths are in the progression 5, 10, 20, and 40%. That yields and approximation of the BPF but the close-to-passband response will be changeable by the component Q used. Ideally a computer program will do all the mathematics required in a second or two.[6] Similar tables for 2-Resonator magnetically-coupled filters called *IF Transformers* are found a little later in this chapter. Those may be helpful in selecting the number of resonator sections such as 3- and 4-resonator types (by interpolation).

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[5] Figure 12-13 was plotted from tabulated decibel response of computer simulations which have built-in modeling of capacitor and inductor Qs (frequency-independent loss for each type). The *Ideal* shape had the default Q values of 10,000 for both capacitors and inductors, equal to those made from unobtainium. The computer program was the author's own LCie4 filter synthesis-analysis tool using double-precision floating-point variables. Output tabulations are rounded to 5 places for frequency, to hundredths of a db for amplitude response.

[6] The author's *LCie4* is freeware and has been available over the Internet for several years. Another free program on filter synthesis is available at the website of *Almost All Digital Electronics* at www.aade.com since 2005. The final component values in the filter should be checked for frequency response in an analysis program such as one of the SPICE variants. *LTSpice* is a fully functional SPICE program available at the website of Linear Technology, the semiconductor manufacturer.

While it would be nice to have tables or graphs of many more resonator types, numbers, response curves, it must be realized that such are only *guidelines* to *begin* a filter design. Use of practical component values with realistic Qs will result in deviation from ideal textbook curves or tables. Those can be modeled and simulated on analysis programs with good accuracy before committing a design to hardware.

Table 12-5
5 Percent Bandwidth 5-Resonator Bandpass Filter Stopband, db

Normalized Frequency	Butterworth	Chebyshev Response at db Ripple 0.25 db	0.5 db	1.0 db
0.90000	-66.5	-77.6	-80.7	-84.0
0.90475	-64.0	-75.1	-78.2	-81.5
0.90953	-61.5	-72.5	-75.6	-78.9
0.91414	-58.9	-69.7	-72.9	-76.1
0.91917	-56.1	-66.8	-69.9	-73.2
0.92402	-53.1	-63.7	-66.8	-70.1
0.92890	-49.9	-60.4	-63.5	-66.7
0.93381	-46.5	-56.8	-59.9	-63.1
0.93874	-42.9	-52.8	-55.9	-59.2
0.94370	-39.0	-48.5	-51.6	-54.9
0.94868	-34.7	-43.8	-46.9	-50.1
0.95369	-29.9	-38.4	-41.5	-44.7
0.95873	-24.7	-32.1	-35.2	-38.4
0.96380	-18.8	-24.6	-27.6	-30.9
0.96889	-12.1	-15.0	-18.0	-21.1
0.97400	-5.2	-2.8	-4.5	-6.8
1.0000	------ Geometric Center Frequency -------			
1.0321	-9.1	-10.7	-13.7	-17.0
1.0376	-15.2	-20.5	-23.6	-27.0
1.0430	-20.2	-28.0	-31.1	-34.4
1.0486	-25.7	-34.1	-37.2	-40.5
1.0541	-30.1	-39.2	-42.4	-45.7
1.0597	-34.1	-43.8	-46.9	-50.2
1.0653	-37.7	-47.8	-50.9	-54.2
1.0709	-41.0	-51.4	-54.5	-57.8
1.0765	-44.1	-54.7	-57.8	-61.1
1.0822	-46.9	-57.7	-60.8	-64.1
1.0879	-49.6	-60.5	-63.6	-66.9
1.0937	-52.0	-63.1	-66.2	-69.5
1.0995	-54.4	-65.5	-68.6	-71.9
1.1053	-56.5	-67.7	-70.9	-74.2
1.1111	-58.6	-69.9	-73.0	-76.3

Choosing Between Butterworth and Chebyshev Response

That choice will be governed by system design criteria and practicability of components. It narrows down to considering passband shape and insertion loss versus stopband attenuation. The latter is usually governed, in turn, by having a minimum attenuation at certain frequency areas away from the desired passband.

For the most even passband insertion loss, it is difficult to beat a Butterworth. It has the least disturbance from practical component tolerance deviations. In general, using a Butterworth response requires the designer to remember that the passband edges are already *rolling off* before reaching those edges; it is prudent to widen the design passband slightly to make sure passband response is within desired limits.

Table 12-6
10 Percent Bandwidth 5-Resonator Bandpass Filter Stopband, db

Normalized Frequency	Butterworth	Chebyshev Response at db Ripple 0.25 db	0.5 db	1.0 db
0.70711	-97.2	-108.7	-111.9	-115.1
0.71947	-94.3	-105.8	-108.9	-112.2
0.73205	-91.3	-102.8	-105.9	-109.1
0.74484	-88.2	-99.6	-102.7	-106.0
0.75786	-84.9	-96.3	-99.4	-102.6
0.77111	-81.4	-92.8	-95.9	-99.1
0.78459	-77.8	-89.0	-92.2	-95.4
0.79830	-74.0	-85.1	-88.2	-91.5
0.81225	-69.9	-80.9	-84.0	-87.3
0.82645	-65.5	-76.4	-79.5	-82.8
0.84090	-60.7	-71.5	-74.6	-77.8
0.85560	-55.6	-66.1	-69.2	-72.4
0.87055	-49.9	-60.1	-63.2	-66.4
0.88577	-43.5	-53.3	-56.4	-59.6
0.90125	-36.3	-45.3	-48.4	-51.6
0.91700	-27.9	-35.6	-38.7	-41.9
0.93303	-17.7	-22.8	-25.8	-29.0
0.94934	-5.8	-3.6	-5.5	-7.9
1.0000	------ Geometric Center Frequency -------			
1.0718	-11.0	-14.5	-17.7	-21.1
1.0905	-19.8	-27.3	-30.5	-33.9
1.1096	-27.2	-36.3	-39.5	-42.9
1.1280	-33.4	-43.4	-46.6	-49.9
1.1487	-38.7	-49.2	-52.4	-55.8
1.1688	-43.4	-54.2	-57.4	-60.7
1.1892	-47.4	-58.5	-61.7	-65.0
1.2100	-51.1	-62.3	-65.5	-68.8
1.2311	-54.3	-65.7	-68.9	-72.2
1.2527	-57.3	-68.8	-71.9	-75.2
1.2746	-60.0	-71.5	-74.7	-78.0
1.2968	-62.5	-74.1	-77.2	-80.5
1.3195	-64.8	-76.4	-79.6	-82.9
1.3426	-66.9	-78.3	-81.7	-85.0
1.3660	-68.9	-80.6	-83.7	-87.0
1.3699	-70.7	-82.4	-85.6	-88.9
1.4142	-72.5	-84.2	-87.4	-90.6

Chebyshev response has greater stopband attenuation than Butterworth, increasing with the allowed design ripple of passband response. Chebyshev filters are also the most affected by inductor Q and the practical passband edges may not be as sharp as in the ideal filter curves. See Figure 12-13. Tolerance variations of fixed components may result in greater amplitude ripple in the passband than the ideal response. The number of resonator sections has an influence on all choices. Fortunately, the hobbyist is not constrained by having to choose the most economical design for maximum product profit. The only constraint is insertion loss.

Table 12-7
20 Percent Bandwidth 5-Resonator Bandpass Filter Stopband, db

Normalized Frequency	Butterworth	Chebyshev Response at db Ripple 0.25 db	0.5 db	1.0 db
0.70711	-67.7	-78.4	-81.5	-84.7
0.71947	-64.9	-75.5	-78.5	-81.8
0.73205	-61.9	-72.4	-75.4	-78.7
0.74484	-58.8	-69.1	-72.2	-75.4
0.75786	-55.6	-65.7	-68.8	-72.0
0.77111	-52.2	-62.1	-65.2	-68.4
0.78459	-48.6	-58.3	-61.3	-64.5
0.79830	-44.8	-54.2	-57.2	-60.4
0.81225	-40.8	-49.8	-52.8	-55.6
0.82645	-36.5	-45.0	-48.0	-51.1
0.84090	-31.8	-39.6	-42.6	-45.8
0.85560	-26.7	-33.6	-36.6	-39.7
0.87055	-21.1	-26.6	-29.5	-32.6
0.88577	-14.8	-18.1	-20.9	-23.9
0.90125	-8.0	-7.1	-9.5	-12.2
1.0000	------ Geometric Center Frequency -------			
1.1290	-4.2	-1.9	-3.7	-6.3
1.1487	-7.8	-9.7	-13.0	-16.5
1.1688	-11.8	-16.9	-20.4	-23.9
1.1892	-15.6	-22.7	-26.1	-29.6
1.2100	-19.2	-27.6	-30.9	-34.4
1.2311	-22.5	-31.7	-35.0	-38.4
1.2527	-25.5	-35.3	-38.6	-42.0
1.2746	-28.3	-38.5	-41.8	-45.2
1.2968	-30.8	-41.3	-44.6	-48.0
1.3195	-33.1	-43.9	-47.2	-50.6
1.3426	-35.3	-46.3	-49.6	-53.0
1.3660	-37.3	-48.5	-51.8	-55.1
1.3699	-39.2	-50.5	-53.8	-57.1
1.4142	-41.0	-52.4	-55.6	-59.0

Initial alignment of the resonator bandpass filter is the easiest of all bandpass types under about an octave of passband. That can be done in-circuit, individually for each resonator, just by swamping all other resonators with a low-value resistor. That will assure achieving a passband response closest to the ideal.

Differences in Stopband Attenuation Versus Coupling Type

Capacitively-coupled resonator bandpass filters will always have greater stopband attenuation below the passband than above; Inductively-coupled filters are opposite. The two types have stopband response which are effectively *mirror* images of one another

Table 12-8
40 Percent Bandwidth 5-Resonator Bandpass Filter Stopband, db

Normalized Frequency	Butterworth	Chebyshev Response at db Ripple 0.25 db	0.5 db	1.0 db
0.50000	-82.6	-93.3	-96.4	-99.6
0.51763	-79.0	-89.6	-92.6	-95.8
0.53589	-75.2	-85.7	-88.7	-91.9
0.55478	-71.3	-81.7	-84.7	-87.9
0.57435	-67.4	-77.5	-80.5	-83.7
0.59460	-63.2	-73.2	-76.2	-79.3
0.61557	-59.0	-68.6	-71.6	-74.8
0.63728	-54.5	-63.8	-66.8	-70.0
0.65975	-49.8	-58.8	-61.8	-64.9
0.68302	-44.8	-53.4	-56.3	-59.4
0.70711	-39.6	-47.5	-50.4	-53.5
0.73204	-33.9	-41.0	-43.9	-46.9
0.75786	-27.7	-33.7	-36.5	-39.5
0.78458	-20.8	-25.1	-27.8	-30.8
0.81225	-13.0	-14.3	-16.9	-19.6
0.84090	-4.8	-1.7	-2.6	-3.7
1.0000	------ Geometric Center Frequency -------			
1.3660	-5.0	-6.2	-9.8	-13.7
1.4142	-7.4	-12.3	-16.2	-20.1
1.4641	-10.0	-17.3	-21.2	-25.0
1.5157	-12.6	-21.5	-25.3	-29.0
1.5692	-15.1	-25.0	-28.7	-32.4
1.6245	-17.4	-28.1	-31.7	-35.4
1.6818	-19.5	-30.7	-34.3	-38.0
1.7411	-21.5	-33.1	-36.7	-40.3
1.8025	-23.3	-35.2	-38.8	-42.4
1.8661	-25.0	-37.1	-40.7	-44.3
1.9319	-26.5	-38.9	-42.4	-46.0
2.0000	-28.0	-40.5	-44.0	-47.5

Achieving Near-Symmetrical Stopband Response

This is quite simple with *odd* numbered resonator bandpass filters. Those have an even number of coupling (series) reactances. One simply inverts the sign of the coupling reactance for half of those; i.e., change coupling capacitors to inductors such that they have *the same reactance at the geometric center frequency.* To maintain resonance of the shunt sections, either the shunt capacitor or shunt inductor will have to be changed to accommodate.

For an example, take the first one on pages 12-4, 12-5, a capacitively-coupled 5-section resonator with a 5% bandwidth. The square of the center radian frequency is $3.9478 \bullet 10^{15}$, all inductors are 3.8628 μHy, and C_R is 68.854 pFd. C_2 and C_8 were already calculated as 3.27877 pFd, C_4 and C_6 were 1.8226 pFd. C_2 and C_8 can be replaced by inductors whose value will be:

$$L_2 = L_8 = \frac{1}{\varpi_O^{\,2} \cdot C_2} = \frac{1}{3.9478 \cdot 10^{15} \cdot 3.2788 \cdot 10^{-12}} = 77.3563 \cdot 10^{-6}$$

[L_2 will have the same reactance magnitude as C_2 at 10 MHz]

As a short - cut, the final value of C_1 and C_9 will be C_R plus C_2, not C_R minus C_2 as in the original example:

$$C_1 = C_9 = C_R + C_2 = 65.575 \cdot 10^{-12} + 3.2788 \cdot 10^{-12} = 68.854 \text{ pFd}$$

$$C_3 = C_7 = C_R + C_2 - C_4 = (65.575 + 3.2788 - 1.8226) \cdot 10^{-12} = 67.031 \text{ pFd}$$

The above is not *pure* or true formal mathematics. Changing the sign of C_2 in the calculation of new C_1 and C value will have the same action as changing the series branch reactance sign at center frequency.[7]

Stopband Below Passband		Stopband Above Passband	
Frequency, MHz	Output, db	Output, db	Frequency, MHz
9.00000	-64.09	-64.31	11.1111
9.04754	-61.85	-62.08	11.0527
9.09533	-59.49	-59.73	10.9947
9.14337	-57.06	-57.24	10.9369
9.19166	-54.35	-54.60	10.8794
9.24021	-51.53	-51.79	10.8223
9.28902	-48.53	-48.79	10.7654
9.33808	-45.30	-45.56	10.7088
9.38740	-41.81	-44.07	10.6526
9.43699	-38.02	-38.27	10.5966
9.53694	-33.87	-34.10	10.5409
9.58732	-29.30	-29.48	10.4855
9.63795	-24.21	-24.29	10.4304
9.68886	-18.49	-18.39	10.3756
9.74004	-12.09	-11.64	10.3211
9.79148	-5.51	-4.49	10.2669

The above is from a separate analysis using logarithmic frequency increments; frequencies above passband are in reverse order to show the closeness to response curve symmetry.

Stopband attenuation symmetry is only required if there is a need for attenuation on both sides of the passband. One of the practical problems with narrow percentage bandwidths is the small value of series capacitors and large values of series inductors. Large series inductors may exhibit internal distributed capacity which will be equivalent to a parallel capacitor across the inductor. Small series capacitors are simply difficult to get in stock sizes; one may have to make them out of two-sided PCB stock.[8]

---

[7] It works in this particular case. One can go the long route and calculate the effect of changing a series branch to an inductance on adjacent shunt capacitor values but that is rather wasted effort. The results will be the same as the *short-cut* method.

[8] Appendix 6-1 and equation set (6-17).

Intermediate Frequency Transformers or *IFTs*

General

IFTs became a *component* of superheterodyne receivers in the vacuum tube era. They have always been little magnetically-coupled two-resonator bandpass filters regardless of their familiar name. Since they are magnetically-coupled they offered good isolation between the +100 to +250 VDC plate circuit connections of the driving stage and the essentially-ground-potential grid input of the following stage. See Figure 12-14 for an example of a typical *permeability-tuned* IFT.[9] They were designed and manufactured in small metal shield cans about the size of octal to miniature glass vacuum tube physical occupancy in a typical metal chassis.

The amount of magnetic coupling between the two resonant circuits in the IFTs was all-important to its in-circuit bandwidth. Since the IF amplification of a typical superheterodyne receiver is fixed, the amount of actual tuning is limited and they are *aligned* once so that they could compensate for the slight variation in tube and circuit capacity resulting from tube replacement. In a few cases IFTs used a combination of magnetic coupling and fixed capacitance coupling, particularly in FM *discriminator* and *ratio detector* demodulator circuits.[10]

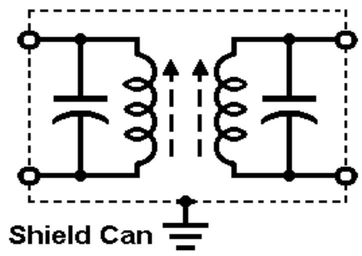

Figure 12-14 A typical IFT with adjustable inductors; variable capacitors with fixed inductors were used in older versions.

In a few pre-WWII receiver designs and a few post-war *transistorized* superhets, their IFTs were really single resonant transformers with either the primary or secondary winding untuned; untuned or tuned dependent on the amount of magnetic coupling between the two windings. In general, IFTs were always dual resonant circuits which acted like two-resonator bandpass filters.

Commercial IFTs Available Off-The-Shelf

Most IFTs used and made during the vacuum tube era were centered around 455 KHz. That frequency became a standard for nearly every early designer.[11] Other frequencies were 262 KHz and 1500 KHz with 10.7 MHz being the standard for FM BC receivers. 4.5 MHz was added for

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[9] *Permeability tuning* was a sort of buzzword in marketing in the 1950s through the 1980s, always referring to the variability of inductors. Since the word *variable* had become associated with variable or tuning capacitors, including trimmer capacitors for alignment, the word permeability (referring to the change of magnetic field around an inductor by changing its core material) was picked up and became component jargon.

[10] See Chapter XFM for more details on special IFTs for such demodulator applications.

[11] The standard was adopted ex officio and the three-number *455* became a synonym for an IFT. The why of such adoption is obscure (and irrelevant) but may be rooted in the technology of the 1930s and the difficulty of making economical components for the AM BC receiver market.

NTSC TV *intercarrier sound* FM audio circuitry.[12] IF stage gains were rather fixed by the availability of IF Transformer components from specialty inductor manufacturers such as the J. W. Miller Company of Los Angeles, CA. The following data is from a small 1965 Miller listing that was packed inside every boxed IFT of theirs[13]:

Center Frequency	-3 db Bandwidth	Voltage Gain	Equivalent |Z|
455 KHz (Input)	16 KHz	36 to 37 db	144K to 160K
455 KHz (Output)	21 KHz	44 to 45 db	55K to 56K
262 KHz (Input)	9 KHz	38.5 db	189K
262 KHz (Output)	10 KHz	45.7 db	58K
1.5 MHz (Input)	51 KHz	34.5 db	16K
1.5 MHz (Output)	62 KHz	28.9 db	62K
4.5 MHz	150 KHz	30.1 db	9.7K
10.7 MHz	250 KHz	32.3 db	12.4K
21.25 MHz	350 KHz	31.6 db	11.5K

The difference between *Input* and *Output* types is that the Input was intended as the load for a 6BE6 pentagrid converter stage. As the pentagrid had a typical low conversion transconductance of 450 μmho, its higher impedance magnitude would attempt to make up for lesser gain than the Output types. Output types were shown with a typical 6BA6 pentode having a 4.3 mmho transconductance. *Equivalent Impedance magnitude* (Ohms) column is only approximate and calculated as the stated voltage gain divided by the stated tube's transconductance at operating voltage given. Presumably the stated voltage gain was in-circuit so the |Z| would include plate resistance of the driving stage in parallel with the IFT; no data is available on the value of grid return resistance that would be in shunt with the output. *Equivalent |Z|* can be considered as a single resistance value for estimating voltage gain with other vacuum tube types.

There is a proportionality between center frequency and bandwidth up to the 1.5 MHz models above. That it isn't directly proportional may be due to different wire sizes, core material, coupling distances. All of the above were aligned by a powdered iron outer core accessible through a hole in the top and bottom of the IFT. Those at 4.5 MHz and higher were deliberately made wider in bandwidth to accommodate the FM sound sideband occupancy, both for TV and FM-sound-only broadcast receivers.

---

[12] Based on the separation of video and audio carriers in the NTSC TV system, that being 4.5 MHz. A small sound IF was picked off the video detector at 4.5 MHz and then-new FCC regulations required a minimum level of video carrier amplitude to avoid an annoying buzz if the video white level (minimum carrier) was too white. Prior to the intercarrier sound receiver circuitry, TV receiver IFs were split at the output of the tuner and the sound carrier IF was 21.25 MHz.

[13] Miller's *K-Tran* model line, shield can 0.75 inch square by 2.125 inch height, was intended for use with 7-pin miniature glass envelope vacuum tube circuits. J. W. Miller made other models of larger size for use with octal base tubes but this approximate data table would probably apply equally well to those.

Magnetic Coupling Basics

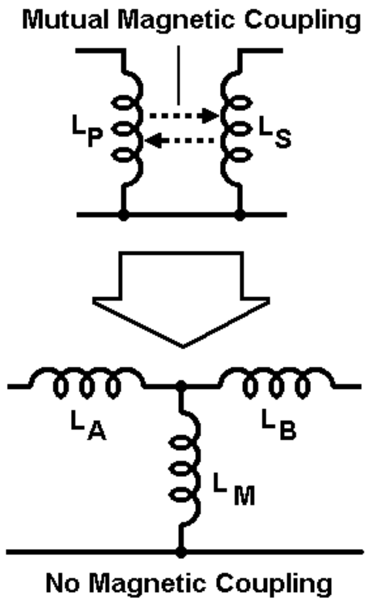

Figure 12-15 Equivalent circuit of two magnetically-coupled inductors.

Nearly all IF Transformers used magnetic coupling between parallel-resonant circuits.[14] The reason was probably economy since winding, assembling coil structures cost more than assembling fixed or variable-trimmable capacitors. Two inductors with magnetic-field coupling have an equivalent circuit depicted in Figure 12-15 as three separate, non-magnetic-field-coupled inductors.

If L_P and L_S in the top of Figure 12-15 are of equal value, then they will appear on the bottom view as shown. A third inductor, L_M, is the equivalent to the magnetic lines-of-force coupling. A problem now is to determine the value of L and that can be found from the ***coefficient of coupling, k***:

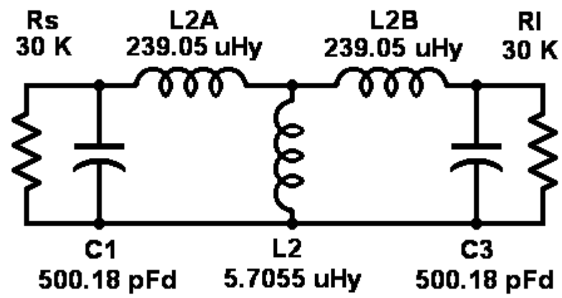

Figure 12-16 Baseline for example IFT to determine the value for L2 and Lm.

$$L_A = L_P - L_M \qquad L_B = L_S - L_M \qquad (12\text{-}6)$$

$$L_M = k\sqrt{L_P^{\,2} \cdot L_S^{\,2}} \qquad \text{(all inductances same units)}$$

$$\text{If } L = L_A = L_B \text{ then:}$$

$$k = \frac{L_M}{L}$$

The usual range of ***k*** will be from about 0.02 to 0.05 and very roughly proportional to the bandwidth divided by center frequency.

Examining the bottom equivalent circuit of Figure 12-15 with the schematic of a two-resonator, low-impedance inductive coupling bandpass filter will show a similarity of inductor configurations. The two-resonator, low-impedance inductive coupling values (Butterworth response) can be used as a baseline. L_M is the center shunt inductor (L2) and L_A, L_B would be equal to L2A, L2B (of the same value) of the resonator BPF.

Using A Two-Resonator BPF as a Baseline for an IFT

[14] Older texts refer to this as *inductive coupling*, a slight misnomer. Capacitive coupling through stray capacity, such as close proximity of connecting wires, is possible but that was never referred to as *capacitive* coupling. Magnetic coupling here refers to lines of magnetic flux coupling two inductors.

As an example, an IFT with 455 KHz geometric center frequency with 15 KHz bandwidth and 30 KOhms terminating resistances with Butterworth response shape was chosen. The schematic is in Figure 12-16. As it is, this BPF would have a percentage bandwidth of 0.032967, rather on the low side for L-C bandpass filters of any type. Coefficient of coupling **k** would be:

$$k = \frac{L_2}{L_{2A}} = \frac{5.7055}{239.05} = 0.023\,867$$

To widen the passband, k would have to increase and thus L2 of Figure 12-16 would have to increase. Response was analyzed with L2 at original value of 5.7055 µHy, then successive increases to 6.3, 6.8, 7.5, and 8.2 µHy, assuming *lossless* reactances (Q > 10,000). Finally, capacitor Q was set to 2000 and inductor Q at 200, more realistic practical values. Figure 12-17 shows the results of this analysis; 6.3 and 7.5 µHy values of L2 omitted for graphics clarity.

As L2 was increased, the passband did widen but a dip in center frequency region started to increase until the dip was -0.5 db with 8.2 µHy. This is normal since the response shape, originally that of a Butterworth, was changing into a low-ripple Chebyshev.

Those were not the only things changing. Input impedance magnitude changes with the changing coupling. Since the driving stage's gain is equal to transconductance times |Z|, a lowering impedance magnitude means less gain.

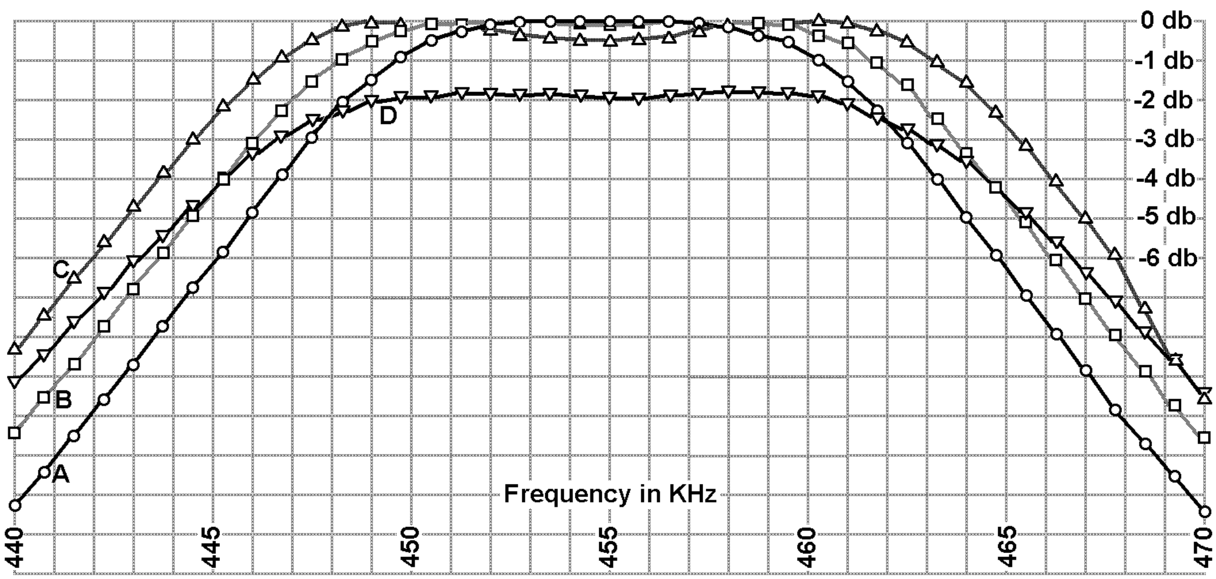

Figure 12-17 Passband characteristics varying L2 in Figure 12-16. Plot A (circle plot points) has L2 = 5.7 µHy; Plot B (square plot points, grey line) has L2 = 6.8 µHy; Plot C (triangle plot points) has L2 = 8.2 µHy; Plot D (inverted triangle plot points) has inductor Q at 200, capacitor Q at 2000 with L2 = 8.2 µHy. Plots A, B, C were all done with all Q values of 10,000.

The following table summarizes the changes for Figure 12-17:

L2, µHy	C1, C3, pFd	K	Zin(without)/Zin(with), Ω
5.7055	500.18	0.023 867	30.00K / 15.00K
6.3	499.02	0.026 354	24.84K / 13.59K
6.8	498.06	0.028 446	21.49K / 12.52K
7.5	496.73	0.031 374	17.86K / 11.20K
8.2	495.41	0.034 302	15.11K / 10.05K
8.2	[with Qc = 2000, Ql = 200]		16.30K / 10.58K

The *(without)/(with)* part of the rightmost column refers to the source resistance being out (disconnected) or in (connected) for the measurement of input impedance. Bandwidth (at -3 db) changed from the original 15 KHz to 21 KHz with L2 = 8.2 µHy. Under the Q conditions stated, the *-1 db* bandwidth with 8.2 µHy was about 15 KHz, the original purpose.[15] That came with a slight (-3 db) reduction in stage gain at center frequency due to differences in $|Z_{IN}|$. Response at far frequencies 400 and 510 KHz was the same (-31.4 db) regardless of inductor Q.

The major reason for considering the *-1 db bandwidth* of an IFT is that total response in decibels is additive. In a typical receiver with three IFTs, all identical, a -1 db amplitude is equal to -3 db for the entire IF chain. Stopband response is also additive. What had been -31 db response for a single IFT would increase to -93 db for both IF amplifiers together.

The added insertion loss due to finite Qs can be offset by increasing the load resistance in the Figure 12-16 circuit. However, the peak amplitude over frequency increases with a greater relative depth of mid-band dip. A circuit analysis program is best to see the differences.

Converting Back to Magnetic Coupling

After the tee-section values are finalized, it can be converted back to the magnetic coupling form by:

$$L_P = L_A + L_M \qquad L_S = L_B + L_M \qquad (12-7)$$

$$L_M = \frac{(L_P \text{ aiding } L_S) - (L_P \text{ bucking } L_S)}{4}$$

Once the primary and secondary inductances are calculated, they must have some magnetic coupling. That is found by the third equation of (12-7) and requires measuring the total inductance of both primary and secondary windings connected *aiding* (providing greatest inductance), then reversing one of the winding connections so that the total inductance is *bucking* (has the least inductance). The difference of the two readings is divided by 4 and the result should equal the shunt section inductance of the tee equivalent.

While that looks easy mathematically, it is a rather time-consuming task if you've never done it before. It does, eventually, find the correct magnetic coupling to fit the tee equivalent but that

[15] A personal project some three decades prior to preparing this manuscript, to achieve slightly better fidelity with an AM BC band receiver. The original circuit had a -1 db bandwidth of about 10 KHz and that meant a 5 KHz audio output bandwidth due to double-sideband AM detection. Changing the -1 db bandwidth to 20 KHz would increase the audio bandwidth (at -1 db) to 10 KHz. A bit *crisper* in tone quality of higher registers..

isn't all. Using a shield can, even one with a cylindrical shape coil form diameter space between the winding layer and shield can inside surface, can distort the magnetic coupling as well as altering the primary and secondary winding inductances.[16]

The primary and secondary windings' individual inductances for the example IFT of Figure 12-15 and 12-16 would be 247.25 μHy.

An alternative is to use toroidal inductors which have little external magnetic field and thus are least affected by shield cans. However, in so doing, it would be just as well to make the IFT equivalent out of a 2-resonator BPF and dispense with magnetic coupling. A capacitively-coupled type of BPF would provide the DC isolation between input and output, yet have a DC path at the input and the output.[17]

In actual practice by IFT manufacturers, they have already worked out the dimensions and placement of primary and secondary windings for their selected coefficient of coupling and thus their IFT bandwidth. Some have used *pot cores* of powdered iron (totally enclosing each winding) with a link coupling to provide the IF coefficient of coupling. Others, like the J. W. Miller *K-Tran* models, have a powdered-iron outer sleeve around each winding, mainly for adjustment of center frequency of the installed IFT. The powdered-iron sleeve is no guarantor of stable coupling since the *K-Tran* enclosed data sheet recommends a certain orientation of input and output connections; reversing one of the windings' connections is said to *decrease the stage gain*. That indicates that magnetic coupling is affected in-circuit and not perfectly stable.

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[16] It has been long recommended by old-time designers to use a shield can inside dimension of at least twice the diameter of the coil form diameter. That was primarily for keeping at least 90% or so of the unshielded inductive Q. In actual practice the shield can also modifies the magnetic coupling.

[17] Suitable IF bypassing would have to be provided for the primary-equivalent and secondary-equivalent ends at the common point of the BPF.

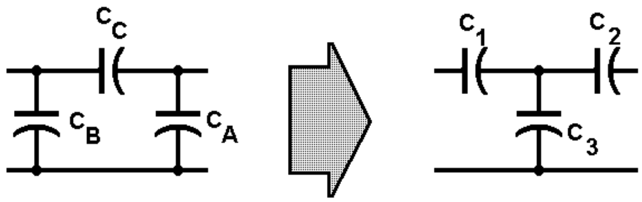
Appendix 12-1

Pi To Tee Transformation of Capacitors

Using equation set (5-5), Chapter 5 pages 6 and 7, for both resistors and inductors will work well because the reactance of an inductor, $X = 2\pi\ f\ L$, will be the equivalent of the magnitude of impedance. A resistor is equivalent to the magnitude of an impedance. For capacitors it is:

$$|Z_C| = X_C = \frac{1}{2\pi f C}$$ and requires a reciprocal to be related to resistors and inductors.

To do the Pi-to-Tee transformation can start out with equation (5-5) and use only capacitive quantities, but it *must* use reciprocals of those capacitive values: To find C of the tee, first begin with the resistor transform:



$$R_1 = \frac{R_B \cdot R_C}{R_A + R_B + R_C}$$

Convert all resistance terms to capacitive impedance by using reciprocals:

$$\left(\frac{1}{C_1}\right) = \frac{1}{\left[\dfrac{\left(\frac{1}{C_B}\right)\cdot\left(\frac{1}{C_C}\right)}{\left(\frac{1}{C_A}\right)+\left(\frac{1}{C_B}\right)+\left(\frac{1}{C_C}\right)}\right]} = \frac{1}{\left[\dfrac{\left(\frac{1}{C_B\,C_C}\right)}{\left(\frac{C_B\,C_C + C_A\,C_C + C_A\,C_B}{C_A\,C_B\,C_C}\right)}\right]} =$$

$$\frac{1}{\left[\left(\frac{1}{C_B\,C_C}\right)\cdot\left(\frac{C_A\,C_B\,C_C}{C_B\,C_C + C_A\,C_C + C_A\,C_B}\right)\right]} = \frac{1}{\left(\frac{C_A}{C_B\,C_C + C_A\,C_C + C_A\,C_B}\right)}$$

getting rid of the reciprocals:

$$C_1 = \frac{C_B\,C_C + C_A\,C_C + C_A\,C_B}{C_A}$$

Interestingly enough, the last equation above has the *same form* and subscripts as the Tee-to-Pi transform equations given in (5-6). The numerator of all numbered-subscript Tee capacitors will be identical for all three capacitors; only the denominator changes.

Avoiding Certain Conditions in Using Pi-to-Tee Transforms

In a balanced bandpass filter it was decided to transform the Pi arrangement into a Tee to yield a result where the trimmer capacitor had one side grounded. See Figure 12-19.

Unfortunately, that *did not work* for the simple reason that the Pi was already balanced and grounded at the junction of C_A and C_B. It would not matter what the value of C_3 was since the junction of C_1 and C_2 put the top of C_3 at ground potential, the same as its bottom. C_3 would have no effect on the circuit unless the values of C_A and C_B were unequal.

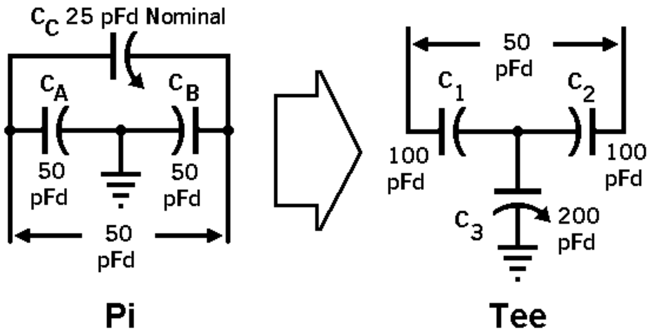


Figure 12-19 An example where shunt capacitor C3 has no relative value with a balanced configuration filter if the values of C1 and C2 are equal.

Figure 12-19 does show that the balanced configuration can effectively double the value of the fixed capacitors, C_A to C_1 and C_B to C_2. See the next chapter on practical component values for L-C filters.

Chapter 13

Practicality of L-C Filters, Stagger-Tuning

Practical L-C filter designs cannot use component values made from unobtainium. There are some simple ways to achieve obtainable component values which are outlined in here. Also covered is an old method for wide-bandwidth filter equivalents using stagger-tuning of single resonators per stage.

General

Practical L-C filter design involves mainly being able to buy, scrounge, or make realistic-value components. Those will have *tolerances* as can be observed on any L/C meter, bridge, or Q Meter Tolerance variations must be taken into account in design since those affect passband shape and insertion loss. Components, especially inductors, will have finite Q values and those can affect passband shape as well as insertion loss. In general, bandpass filters are difficult to design (if not practically impossible) at about 3% or less bandwidth with conventional inductors. Bandstop filters probably need at least 6% stopband width. Lowpass and highpass filter attenuation slopes are generally helped by adding more sections.[1]

Selection of Components for Q

In general, silver-mica fixed capacitors have Q values of 2000 and above up to (roughly) 100 MHz. Ceramic disk capacitors under 1 nFd are at least 1000 in Q up to the same frequency. For analysis purposes the capacitive Q need not be modeled for bandpass filters greater than 10% bandwidth. Inductor Q ***does*** need to be modeled for analysis since that is much lower than capacitors.

If no Q Meter is available, consult the available inductor Q specifications from manufacturers. Most listings show Q at only one frequency; one has to interpolate from that. Some of the older information in texts indicate cylindrical inductor Q through graph curves or nomographs. Those can range from 40 to 100 in Q depending on the coil form dimensions, form material, and powdered-iron adjustment core material (if any). Nearly self-supporting air core cylindrical inductors can have Qs reaching up to 200, usually because of the heavier-gauge wire used in them.

Shielded inductors will suffer Q degradation depending on the spacing between coil form outside diameter and inside wall of the shield. As a rough rule of thumb, a shield can inside dimension twice that of a cylindrical coil form outside diameter will result in inductance and Q reduction of about 10%. For more information on that, consult reference [14] listed in RHdb

[1] Unlike commercial designers, hobbyists aren't constrained by strict limits of economy to maximize production profit. One or two extra sections won't cost the hobbyist much nor add much to the time of construction.

Chapter 6, *Radiotron Designer's Handbook* chapters 10 and 11.

Toroidal forms, especially those using powdered-iron core material, offer the best Q and are least affected by nearby conductors. For more exact information on that, consult Micrometals' Catalog 3, familiarly called the *Q Book* by some (reference [18] also in RHdb Chapter 6).[2] While powdered-iron toroidal inductors are seldom found in older RF equipment despite their physical placement freedom and higher Q, they were more costly to make than cylindrical form types. The hobbyist has little such constraint on time and effort.

For applications involving high RF power, the choice must involve as heavy a wire gauge as possible and the highest Q as possible. High Q for minimum losses within the inductor. Heavy wire gauge for both current-carrying capability and decreased skin-effect from greater surface area of heavier wire. That also applies to capacitors for Q and will usually dictate air dielectric capacitors rather than small silver-mica types. Consult manufacturers' data on high RF current-carrying capability.

Modeling Q for Analysis of Filter Frequency Response

Few SPICE programs have inductor or capacitor Q model functions. Those can be added to an analysis model as a series resistance for inductors (R_{SQ}), a parallel resistance for capacitors (R_{PQ}), always at the cutoff (lowpass, highpass, or edges of bandstop types) or center frequency for bandpass filters. Q in the far stopband frequencies will have little effect on frequency response there.

For frequency - dependent Q modeling of inductors:

$$R_{SQ} = \frac{\varpi \cdot L}{Q} \qquad \text{[essentially inductive reactance / Q]}$$

For frequency - dependent Q modeling of capacitors:

$$R_{PQ} = \frac{Q}{\varpi \cdot C} \qquad \text{[capacitive reactance times Q]}$$

Where: $\varpi =$ Radian frequency $= 2\pi \cdot f$, f in Hz

L = Inductance in Hy

C = Capacitance in Fd

Capacitors can also be modeled by a series resistance, calculating the capacitive reactance divided by its Q. The difference between the two is very small and can be observed best on analysis programs having *double-precision*[3] variable handling and at frequencies close to resonance of an L-C pair.

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[2] Earlier small-quantity packages from reseller Amidon Associates had enclosed reproductions of the Micrometals Q Curves as well as inductance information as to wire sizes and number of turns needed.

[3] *Single-precision* variables are equal to 7 decimal place numerical accuracy; *double-precision* is equal to about 14 decimal places.

Effect of Distributed Capacitance of Inductors

Larger inductors, roughly 100 µHy and greater, will have some *distributed capacity* internally. That comes from cumulative capacitance between wire turns mainly, between wire layers in multi-layer coils secondarily. It can be thought of as a parallel capacitor across the inductor. As such it can be measured on a Q Meter or Vector Network Analyzer that has a calibrated variable capacitor and frequency-tuning capability. The following is a recommended method by General Radio and Boonton Electronics for measuring distributed capacity (C_D) of inductors on their Q Meters:[4]

$$C_D = \frac{C_2 - 4 \cdot C_1}{3} \quad \text{Where:}$$

$C_1 =$ Q Meter capacity reading for resonance at f_1

$C_2 =$ Q Meter capacity reading for resonance at f_2, $f_2 = 2 \cdot f_1$

All capacitances in same units, both frequencies in same units

The effect there is to put the equivalent of a parallel capacitor (due to distributed capacitance) across large inductors and create a resonant circuit, usually above the passband. This can sometimes distort the stopband response and, rarely, the passband of very narrow bandwidth bandpass filters.

End Terminations Part of Driving Stage Output and Following Stage Input

Plate resistance r_p (vacuum tubes) or collector conductance h_{OE} (transistor output conductance, common emitter configuration) will be part of the source-end terminating resistance. This is usually dismissed for source-end termination values of 5 KOhms or less since either usually have higher Ohm values than that. Any parallel reactances of the driving stage output will become part of the filter input and must be accounted for by modifying filter design values of input shunt reactances.

Load-end resistances are generally unchanged for vacuum tube or FET following stage inputs since those resistance values are quite high. Usually that is the grid return resistance (tubes) or FET gate bias resistance network total shunt resistance. For transistor base inputs it is another story since common-emitter base input impedance is quite low. Some form of impedance change, wideband transformer or reactive network, may be required at filter outputs to couple filters to common-emitter base inputs.[5] Reactances of the following stage input will also require modification of filter output shunt reactance component design values.

More on input and output impedances of active devices in later chapters. For direct-connection filtering into or out of antennas, the characteristic impedance of the transmission line can be assumed to be resistive at the filter's mid-frequency region. Antenna impedance changes over frequency will be reflected down the transmission line but that is a site-dependent condition.

Adjusting for Very Low Series Capacitances or Very High Series Inductances

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[4] Reference [7] (*ITT Green Cover*), Chapter 10, page 269.

[5] Chapters 8 and 9 preceding for some typical narrowband reactive impedance-changing networks.

Small capacitor design values, say under 10 pFd, may be difficult to obtain from dealers and distributors. A series string of larger stock values can be used to alleviate that problem. Total capacitance of a series string is the reciprocal of the sum of reciprocals of individual capacitor values.[6]

Large-value inductors can be made up of a series of smaller-value inductors. That also raises self-resonance frequencies; inductances are lower and their distributed capacity will also be lower.

Another way to achieve some relief from design-value high reactances is to make *balanced-above-ground* filter configurations instead of the common ground style described so far. In that case the physical capacitor values of series branches would double in count and value while series inductors would double in count but halve in value. As example is shown below in Figure 13-1:

The BPF of Figure 13-1 has a geometric center frequency of 9.45 MHz with a bandwidth of

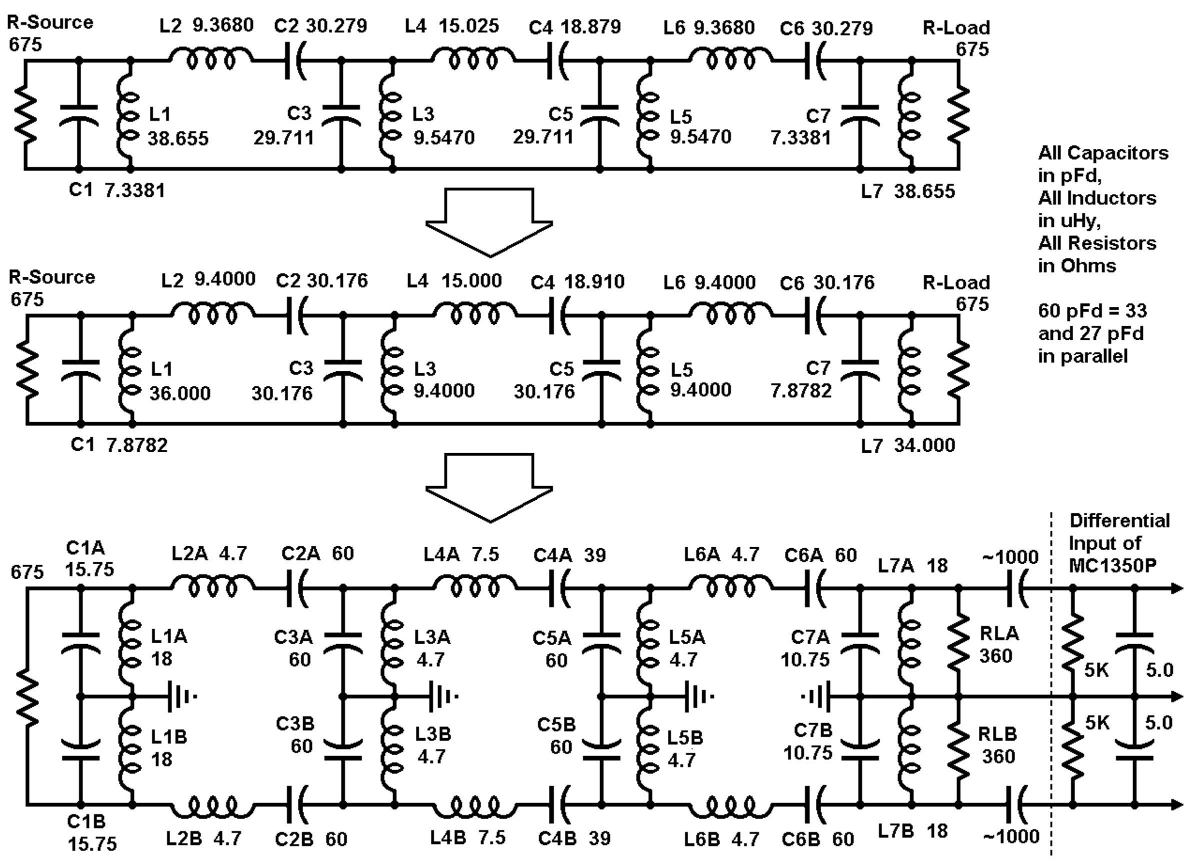

Figure 13-1 Evolution of a BPF exact design values (top), to practical values (middle), and finally the balanced above ground compromise values at bottom. The MC1350P is a differential-input, differential-output wideband video amplifier IC with quite constant parallel 5 KOhms and 5 pFd at each input. BPF source end is from a wideband RF transformer with 1:3 turns-ratio (1:9 Z ratio) from 75 Ohm coaxial line.

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[6] Using a calculator with a reciprocal key function makes this a much easier task than the conventional two-capacitor formula of total = (product of two capacitors)/(sum of two capacitors).

14.3 MHz, end resistances of 675 Ohms. That particular end resistance was based on a 1:3 turns-ratio *unbal* with unbalanced input from a 75 Ohm coaxial cable. With a 1:3 turns-ratio, the impedance ratio is 1:9, hence 675 Ohms.

The filter at the top represents the exact design values for a Butterworth response shape. Intended bandwidth at -3 db points was 4.7 to 19 MHz. Note the four inductors at 9.368 to 9.547µHy. A 9.4 µHy (two 4.7 µHy in series) would work for L2, L3, L5, and L6 and, accordingly, C2, C3, C5, and C6 were picked for resonance at 9.45 MHz (center frequency). L1 and L7 were changed to 36 µHy, equal to two 18 µHy in series, C1 and C7 duly modified in value. L4 was changed to 15 µHy (equal to two 7.5 µHy in series). All those changed values are in the middle filter of Figure 13-1. The reason for the seeming-odd choices of value changes was to permit the fully balanced-above-ground configuration shown at the bottom.

In the bottom configuration, C2, C3, C5, and C6 were changed from 30.176 pFd to 30 so that twice that value would be 60 pFd; 60 pFd can be made from a 27 pFd and 33 pFd in parallel. C4 was made 19.5 pFd so that twice that value would be 39 pFd. Analyzed again with these new values, there was very little passband response modification from the exact value version.

At the load end of the bottom configuration, an MC1390P wideband video amplifier was expected to be used in differential input and output. Since each input of that IC has an impedance of 5 KOhms in parallel with 5 pFd over the 60 MHz range of it, C7 and the load-end resistance are modified to include that. The resistive load resistance will be -0.5% from the exact design value and is not noticeable. DC-blocking coupling capacitors are required for the MC1350P since those inputs are internally biased, elevated above ground. It would be expected that the physical C7 of the BPF would be 10 pFd with 0.75 pFd accounted for in PCB trace stray capacitance (short traces there).

Passband response at edges as analyzed, resulted in the following (frequencies in MHz):

Freq.	Exact,db	Final,db	Freq.	Exact.db	Final,db
4.0000	-15.21	-15.08	14.674	0	-0.01
4.1713	-11.66	-11.41	15.302	-0.01	-0.03
4.3499	-8.22	-7.85	15.957	-0.05	-0.09
4.5361	-5.11	-4.69	16.640	-0.15	-0.22
4.7304	-2.70	-2.32	17.353	-0.45	-0.55
4.9329	-1.19	-0.93	18.096	-1.18	-1.30
5.1441	-0.45	-0.31	18.871	-2.68	-2.79
5.3644	-0.15	-0.08	19.679	-5.09	-5.17
5.5941	-0.05	-0.01	20.521	-8.19	-8.25
5.8336	-0.01	0	21.400	-11.63	-11.71

All frequencies from 6.0834 through 14.071 MHz were 0 db.

Final columns refer to the bottom configuration of Figure 13-1. *Exact* is the version at top.

The value changes contribute no noticeable change to passband edge response. This is partly due to the very wide bandwidth relative to center frequency, partly due to a Butterworth response as very forgiving of changes. Going into Chebyshev response of greater and greater passband ripple would result in more noticeable response variation from the same percentage component change.

Since this BPF was intended to be non-adjustable once fabricated, the components would be measured before construction. In light of that, a *monte carlo*-like analysis test was done to see what changes could happen, if any. This was a selection of the highest, mean, and lowest response amplitude over 10,000 full frequency sweeps with each sweep randomly varying all L and C values within ± 3 % and ± 5 % limits.[7] The major change was in bandwidth limits, where the *roll off* of the passband changes into stopband. As in nearly every Butterworth response filter, the passband mid-range remained quite flat. The -3 db bandwidths were as follows (frequencies in MHz):

	Low	Mean	High
Exact values ±3%	13.86	14.30	14.74
Exact values ±5%	13.55	14.30	15.07
Intermediate ±3%	13.78	14.23	14.68
Intermediate ±5%	13.51	14.23	14.97
Final values ±3%	13.79	14.29	14.68
Final values ±5%	13.47	14.29	14.97

Two things should be apparent: Bandwidth changes even with variation in tolerance of exact design values; there is little change in *tweaking* values of intermediate and final values since those changed within a ± 2 % boundary. The final filter was satisfactory in its application.

Every L-C filter, of any type, needs to obey the basic design values as to proportionality of each value to all other values.

A More Exotic Type of Value Change via *Bartlett's Bisection Theorem*

As applied to L-C filters this mathematical technique allows some scaling of some components with ***odd-section*** Butterworth or Chebyshev filters. It doesn't apply to the Elliptic response or Resonator type bandpass filters as described so far. It allows a change in end resistances but is **not** a full impedance transformation and should be limited to cases where some of the component values must change to fit available stock.

Using the 2 to 4 MHz BPF of Figure 13-2 as an example, the load end must be changed from 50 Ohms to 300 Ohms, a 1:6 ratio. The first step with Bartlett's Bisection Theorem applied to L-C filters is to separate L2 into two inductors, L2A and L2B, each one having half the value of L2. C2 is split into two series capacitors, C2A and C2B, each having twice the value of C2.

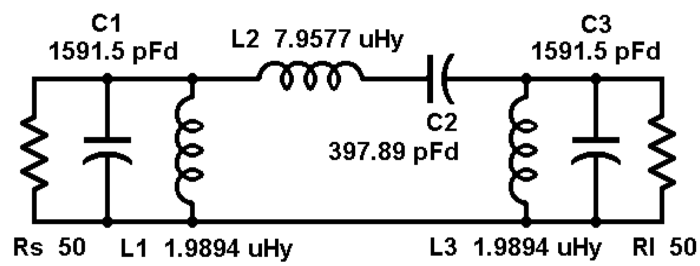

Figure 13-2 Example BPF for 2 to 4 MHz at 50 Ohms with Butterworth response.

Then L2B, which was 3.9789

[7] ± 5 % tolerance values could be purchased off-the-shelf. ± 3 % values are a composite of using simpler measuring instruments on testing self-fabricated inductors, allowing for errors of the instrumentation and the fact that measurements were done individually, out-of-circuit. Measurements might be different than installed values due to proximity of mounting structure, other wiring. Variations **will** happen in purchased parts as well as self-fabricated components. The analysis was done with the author's LCie4 program which has this component sensitivity test built-into the software. On a 1.2 GHz clock common desk PC, each sensitivity test required no more than 5 seconds to complete 10,000 sweeps of 41 frequencies each for this 7-branch BPF.

μHy, is multiplied by 6 (ratio of end resistances) to become 23.873 μHy. C2B, which was 795.78 pFd, is divided by 6 to become 132.63 pFd. R_L becomes 300 Ohms, L3 is now 11.937 μHy (from 1.9894) and C5 becomes 265.26 pFd (from 1591.5). L2A and L2B are made one inductor by adding their inductances: 3.9789 + 23.873 = 27.852 μHy. C2A and C2B are combined in a single capacitor of 113.68 pFd.[8] Figure 13-3 illustrates the sequences.

When finished and analyzed, the altered filter will have the exact passband shape as the unaltered one...but on a relative basis.

One thing that is different is that input and output impedances *swap ends.* If the bisected filter is measured for input impedance with source termination open, it will read 300 Ohms resistive at geometric center frequency. If the bisected filter is measured for output impedance with load resistance open, it will read 50 Ohms at center frequency.

Anyone using the bisection technique will have difficulty in achieving any efficient power transfer through a bisected filter. Note: Driven by a constant current RF source (any tube, transistor or FET in linear bias mode), a bisected filter will ***not achieve any voltage change input to output.*** True impedance-change networks will alter the input-output voltage or current by the square-root of the impedance ratio. The bisected filter does not do that.

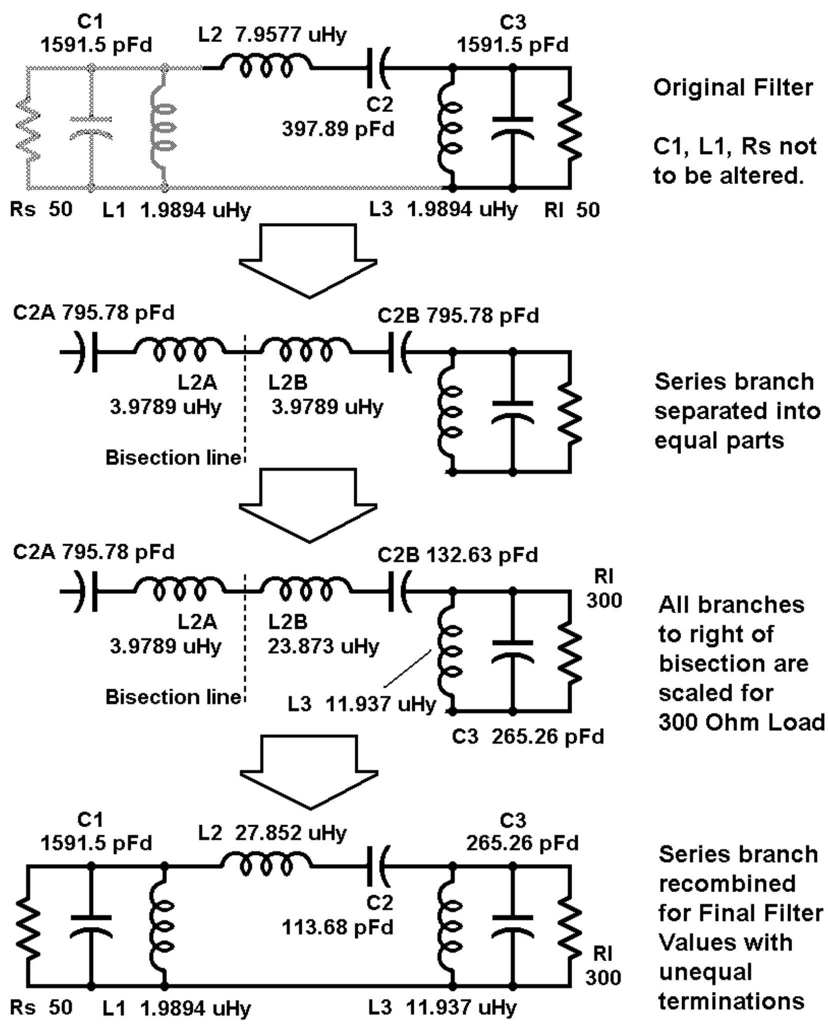

Figure 13-3 Sequence of steps in altering the BPF of Figure 13-2 for unequal terminations via Bartlett'd Bisection Theorem.

There are several sources of tables for L-C filters that include unequal source and load

[8] Combined value is the reciprocal of the sum of individual capacitors. Note: All calculations were done to more decimal places than 5.

resistances. Those should be used instead of doing the bisection technique.[9] One can use it on the middle section of a 7-section Butterworth or Chebyshev filter; a raised equivalent end resistance could step up a middle series branch, then step it down to the original equal end impedance. The bisection technique should be used only as a last resort by hobbyists, then only with the aid of an computer analysis program to double-check the result.

Stagger Tuning to Achieve Bandpass Equivalents

General

At the beginning of World War II the militaries were adopting use of then-new radar. Radar receivers required rather wide bandwidth IF amplifiers to process the short pulses from target returns. Bandwidths of 10 MHz at center frequencies of 60 MHz were typical at the time. Modern network theory in regards to L-C filter design was not yet fully developed and 60 MHz was considered a very high frequency by many vacuum tube circuit designers. The MIT Radiation Laboratory was a U.S. government *think tank* in Cambridge, Massachusetts, formed to develop radar technology in the 1940 to 1945 period. Part of that development work included use of single resonant circuits at different center frequencies (the *stagger* in the name) in multi-stage amplifier chains so that the whole chain's response was the same as a wideband bandpass filter. Stagger tuning requires control over both resonant frequency and the Q of each resonant circuit according to some simple formulas developed at the MIT *Rad Lab* and published in the 28-Volume *Radiation Laboratory Series* released to the public in 1948.[10]

How Stagger Tuning Works

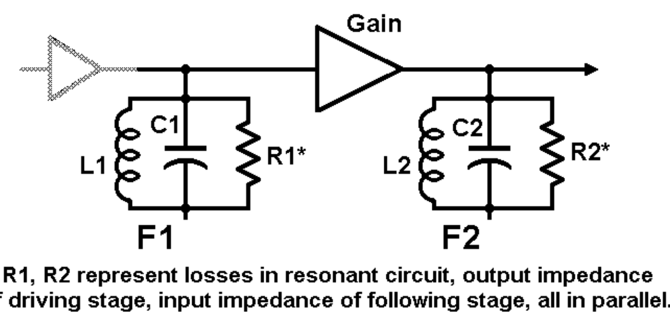

Figure 13-4 General circuit block for a stagger-tuned double amplifier. Shaded input can be an active stage or impedance step-up network.

Figure 13-4 shows the general arrangement for a stagger-tuned double with each resonant circuit *isolated from all others* by the gain stages. Each resonant circuit has its own amplitude versus frequency response as determined by its center frequency and total Q at resonance. The final output is an additive summation of each stage gain in decibels. Each stages' gain uses its output resonant circuit as the load impedance of that amplifier. Each resonant frequency will be different than all others. The total Q of each resonant circuit determines the flatness of the overall response in the center frequency region. Alignment is easy. Each resonant circuit is peaked at its predetermined resonant frequency. If the total Q of each resonant circuit has been preset,

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9 See References at the end of this chapter.

10 See References of the applicable Volume for Stagger-Tuning techniques.

usually by an added parallel resistance, the overall output response will be very similar to a Butterworth response shape.

While the stopband response does not have the attenuation of a multi-section bandpass filter, the stagger-tuning technique worked well with vacuum tube circuits. It can be used to modify an older receiver to eliminate separate *preselector* manual tuning for relatively narrow bandwidths.[11]

Determining Staggered *n-uples'* Resonant Frequency and Q

For a maximum of 20 percent bandwidth, double, triple, and quadruple stagger-tuning stages are determined by:

Where: f_O = Center frequency of desired band (arithmetic center acceptible) (13 - 1)

f_B = Bandwidth, same units as f_O

F_N = Resonant frequency of each circuit, same units as f_O

Q = Loaded quality factor of entire parallel resonant circuit, includes output impedance of driving stage, input impedance of following stage.

DOUBLE:

$F1 = f_O + 0.35 \cdot f_B$ Gain = -3 db $Q1 = (1.41 \cdot f_O) / f_B$

$F2 = f_O - 0.35 \cdot f_B$ Gain = -3 db $Q2 = (1.41 \cdot f_O) / f_B$

TRIPLE:

$F1 = f_O + 0.43 \cdot f_B$ Gain = -6 db $Q1 = (2 \cdot f_O) / f_B$

$F2 = f_O$ Gain = 0 db $Q2 = f_O / f_B$

$F3 = f_O - 0.43 \cdot f_B$ Gain = -6 db $Q3 = (2 \cdot f_O) / f_B$

QUADRUPLE:

$F1 = f_O + 0.92 \cdot f_B$ Gain = -19.4 db $Q1 = (5.25 \cdot f_O) / f_B$

$F2 = f_O + 0.46 \cdot f_B$ Gain = -6.3 db $Q2 = (2 \cdot f_O) / f_B$

$F3 = f_O - 0.46 \cdot f_B$ Gain = -6.3 db $Q3 = (2 \cdot f_O) / f_B$

$F4 = f_O - 0.92 \cdot f_B$ Gain = -19.4 db $Q4 = (5.25 \cdot f_O) / f_B$

Note: *Gain* values above are voltage gain relative to individual stage gain if that stage had been resonant at f_O.

There is data available in the References for stagger-tuned Quintuples, Sextuples, and Septuples but that seems largely academic. Doubles, Triples, Quadruples may be chained for more gain and sharper passband edges. How the individual stage gains of a staggered double form a specific passband is shown following in Figure 13-5:

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[11] The author successfully did that in the Heathkit *300 Series* amateur and shortwave receivers which had a separate manual preselector tuning control (antenna and RF Amplifier circuits using a dual-gang variable capacitor). Each band was 500 KHz wide in that series. Manual preselector was not fully ganged with the bandswitch and tuned over several bands.

The example plotted in Figure 13-5 used a staggered-double to cover the amateur 80 and 75 meter bands for a passband of about -1 db maximum fall-off at edges for 3.5 to 4.0 MHz. To do that,

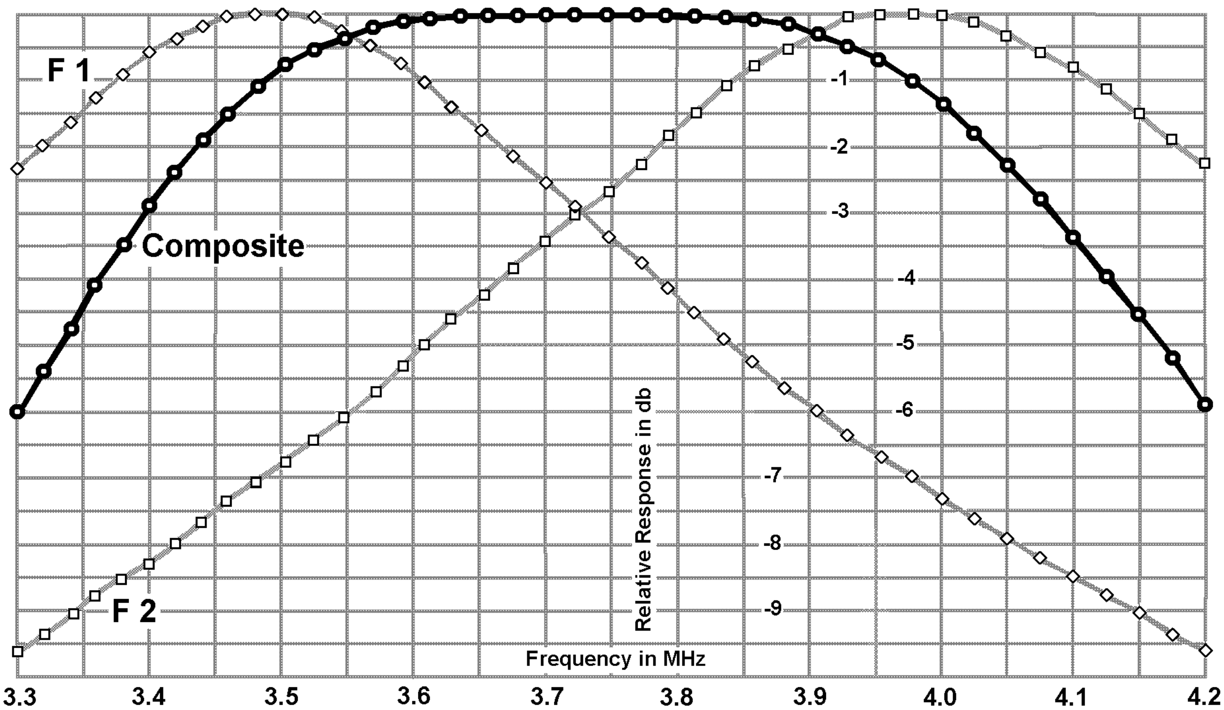

Figure 13-5 Relative responses of the stagger-tuned double of Figure 13-4. *F1* curve (grey, diamond plot points) is the lower-frequency tuned circuit, *F2* curve (grey, box plot points) the higher-frequency tuned circuit. *Composite* (dark, circle plot points) is both responses as combined into the output of the last amplifier stage. Frequency scale is linear. Note that both *F1* and *F2* curves intersect at -3 db relative response.

the total bandwidth used for calculation was 700 KHz, from 3.4 to 4.1 MHz. Center frequency was taken as the arithmetic center, partly to see how far off that would change the passband edges. As can be seen, the actual center frequency is 3.73 MHz, not 3.75 MHz, and corresponds to the geometric center frequency. For a staggered double the center frequency is where the lower and higher tuned circuits responses intersect at -3 db relative.

The example used fixed 300 pFd capacitors and a 1 MOhm resistance across each resonant circuit. The inductors were modeled with a Q = 7.576 each with L1 at 6.873 µHy and L2 at 5.290 µHy. Transconductance of each amplifier stage, modeled as a dependent current source, was taken as unity for a relative response.[12]

Total gain of any stagger-tuned amplifier chain is determined by the gain of each stage at the

[12] This results in the appearance of unobtainable high gain but for analysis purposes the relative response to band center will be the same regardless of the driving stage transconductance. A typical vacuum tube amplifier stage would have between 3 to about 5 mmhos *gm* (transconductance). Rather than set that to a specific tube type would give the appearance that this would only work with such tubes. Relative response to band center will be the same regardless of vacuum-tubes or semiconductor active devices used as active stages.

center frequency. If the Q of each stage is set to values of equation (13-1), the gain of each stage at *its resonant frequency* will be lower at center frequency by the *Gain* values given in (13-1).

Stopband relative response will be the sum total of each stage's resonant circuit (at appropriate Q) at about 30 db or more down from center frequency. It will have slightly less attenuation than a single bandpass filter but is quite suited for simple, multi-stage IF amplifier chains as used in the microwave region receivers of earlier times. Originally those were for vacuum tubes but they can be done the same way with FETs. Bipolar transistor common-emitter or common-base amplifier inputs need appropriate impedance transformation due to lower input impedance; taps or link-couplings can be used but the loaded Q from that low impedance input must be accounted for in determining the parallel resonant Q of each stage. In general, at HF and above, output resistance/impedance of tube and FET stages are so much higher than individual Q-loaded resonant circuits that they can be discounted.

Loading Resistor Calculation to Achieve Design Q

If the *unloaded Q* or Q as measured by itself, not in-circuit, is known, design Q can be achieved by shunting the tuned circuit with a parallel resistance. Carbon or carbon-film resistors at ½ Watt or less are fine for that from low-VHF (~ less than 100 MHz) on down in frequency.

Where: X = Reactance of inductor or capacitor at resonance, Ohms (13-2)

Q_S = Design Q for a stagger - tuned stage

Q_U = Unloaded Q of inductor not in - circuit

R_Q = Parallel resistor, Ohms, for setting Design Q

$$Q_U = \frac{Q_D \cdot R_Q}{R_Q - Q_D \cdot X} \qquad R_Q = \frac{Q_D \cdot Q_U \cdot X}{Q_U - Q_D} \qquad Q_D = \frac{Q_U \cdot R_Q}{Q_U \cdot X + R_Q}$$

Note: The unloaded Q must be greater than design Q; this is usually the case where the passband is no greater than 10% bandwidth.

Selection of N-Uples for Uniform Signal-to-Noise Ratio in IF Amplifier Chains

As a general rule of thumb, the lowest Q circuit would be at the beginning of an IF amplifier chain. A staggered triple would do it, placing F2 at the input. This tends to *spread* random noise over more of the total passband rather than have just one passband region have greater or lesser random noise. For a staggered quadruple, either F2 or F3 would be at the input.[13]

[13] A more refined theoretical presentation could be done to prove that, but such would only be of academic interest. Interested hobbyists could find it out for themselves by devoting many hours of analysis of a stagger-tuned amplifier chain at several frequencies. Doing that may be fun to some hobbyists but isn't strictly needed. The intuitive *rule-of-thumb* can be taken as true.

Simple Ways To Measure Inductor Q

General

Not many hobbyists can afford Q Meters at their home workshops. But knowledge of unloaded Q of inductors is needed in some filter design and, certainly, in stagger-tuned amplifiers. A very few manufacturers provide both inductance and Q values for toroidal forms.[14] Most inductor manufacturers supply only *typical* Q information at one frequency, dictated by their final product Quality Assurance testing.[15] While those are fairly constant and generally reliable, a much wider frequency range is needed. Air dielectric and dipped silver-mica capacitors generally have unloaded Qs of 1000 or more and can generally be considered lossless for approximation purposes.

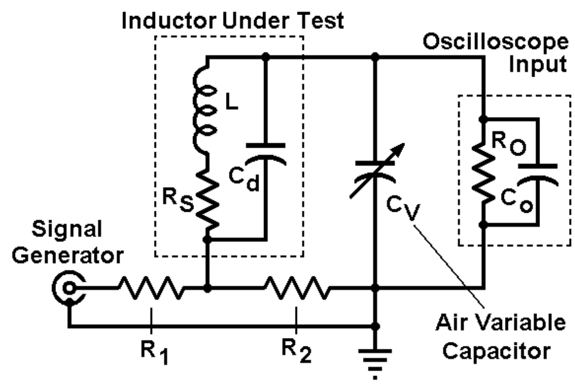

Figure 13-6 Simple Q measure set-up

The general definition of Q of a resonant circuit is the bandwidth where the amplitude is 3 db down from peak amplitude at resonance. This corresponds to a voltage that is 70.7% of maximum voltage. Either peak, peak-to-peak, or RMS voltage may be used as long as all measurements are the same units. *Proportionality* must be observed.

Using a Wideband Oscilloscope With a Known-Frequency RF Generator

Oscilloscopes with 10 to 100 MHz input bandwidth and minimum 50 mV/division sensitivity can be used to measure relative amplitudes up to 50% more than their bandwidth specifications. Their linearity is good enough that the major error lies in observation, not the instrument itself. The RF source frequency needs to be accurate, either by itself or using a bridging-connection frequency counter (recommended).[16]

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14 Micrometals and reseller Amidon Associates, Reference [ ].

15 See Appendix 2 of Chapter M1 for the *why* of those particular frequencies, plus a way to build one's own Q Meter.

16 *Bridging connections* refer to an in-line parallel connection to other equipment having high input impedance. Most frequency counter instruments of this new millennium have such, typically 1 MOhm in parallel with about 20 pFd, same as many oscilloscopes. The extra capacitance along the cable connection will only slightly mismatch the generator's output impedance but that only results in slight loss of amplitude.

A simple Q measurement set-up is shown in Figure 13-6. This requires only a ***very low*** R_2 value such as 1 Ohm from ten 10 Ohm resistors in parallel. R_1 is used to terminate the signal generator output and would be approximately equal to that impedance minus R_2.[17] C_V is any *air dielectric* variable capacitor such as a salvaged unit from an old table model AM BC receiver. The total capacitance across the inductor is the sum of C_V, oscilloscope input capacitance C_O, and the distributed capacity C_D within the inductor. C_V does not need to be calibrated just for Q measurements but it could be for measuring inductance L.[18] Oscilloscope input resistance R would affect some high-Q, large-inductance inductors since it is in parallel (shunt-loading) with the resonant circuit.. If the sensitivity of the oscilloscope is great enough, a 10:1 voltage-divider probe will typically have 10 MOhms in parallel with about 15 pFd. A direct input to the oscilloscope would be 1 MOhm in parallel with about 30 pFd, ***including the connecting cable capacity*** which could be up to 30 pFd per foot depending on the coaxial cable type. Frequency meter resolution should be better than one-tenth of the expected bandwidth. A Q of 200 at 10 MHz would have a bandwidth at -3 db points of 50 KHz and the minimum frequency meter resolution would have to be 5 KHz; 1 KHz would be better.

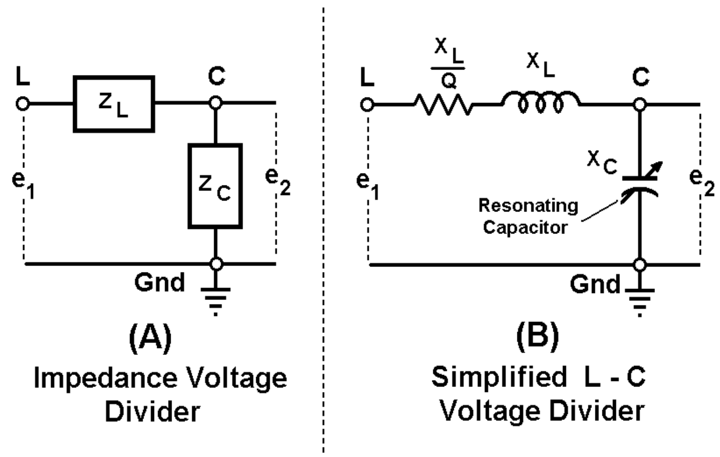

Figure 13-7 Simplified resonant circuit network yielding increase in output volts.

Determining Necessary Oscilloscope Sensitivity and Calculating Q

Using 0 dbm RF output level from a signal generator will put 223 mV RMS into R1. The level at the junction of R1 and R2 will have 4.47 mV RMS due to voltage divider action. To calculate what the oscilloscope input level will be, the simplified circuit of Figure 13-7 can be consulted. That too is a voltage divider but it will *increase the output voltage* (e) for any Q greater than unity *at resonance.*

In the simplified version of Figure 13-7 the resonant circuit action is as follows:

[17] Do not, repeat, do NOT use any wirewound resistors there, only carbon, carbon-film, or metal-film types to reduce series inductance as much as possible.

[18] Calibration of C_V requires that C_O ***always*** be the same value. That could be done by any capacitance meter capable of resolving 0.1 pFd (recommended) to 1.0 pFd. That would also be useful for measuring distributed capacity of rather large inductors.

$$\text{At Resonance: } e_2 = e_1 \left|\frac{Z_C}{Z_L + Z_C}\right| \qquad \text{Where } e_1 = \text{ input RF Voltage, } e_2 = \text{ output RF Voltage}$$

$$Z_L = \left(\frac{X_L}{Q}\right) + j X_L \qquad Z_C = 0 - j X_C \quad \text{At resonance } |X_C| = |X_L| \text{ so } Z_L + Z_C = \left(\frac{X_L}{Q}\right) + j\,0$$

$$\frac{Z_C}{Z_L + Z_C} = \frac{0 - j X_C}{\left(\frac{X_L}{Q}\right) + j\,0} = 0 - j\left[\left(\frac{X_L X_C}{Q}\right)\left(\frac{Q^2}{X_L^2}\right)\right] = 0 - j\,\frac{Q X_C}{X_L} = 0 - j Q$$

$$e_2 = e_1 Q \qquad [\text{ magnitude of } (0 - j Q) = Q\ ]$$

If the input is 4.47 mV and inductor under test has a Q of 100, the output voltage into the oscilloscope will be about 447 mV. The ratio of output voltage to input voltage is directly proportional to Q.

In actual practice there are several components left out of the simplified version. Oscilloscope resistance input will have a noticeable effect on frequencies *below* about 2.5 MHz and low-Q inductors will reduce e_1 level. See Appendix 2 of Chapter M1 for more details on this.

Determining Q by the bandwidth between -3 db points isn't affected by errors already described. That is an amplitude proportionality although more time-consuming than with a direct-reading instrument.

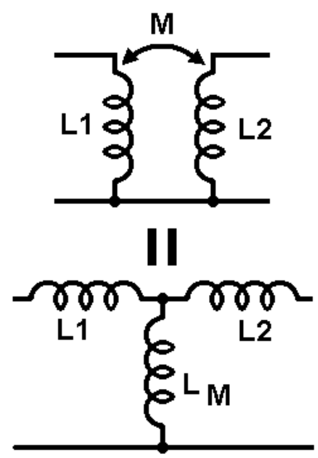

Figure 13-8 Creation of a coupling analytical model.

Input and Output Impedance Matching

In previous chapters, all filters have assumed that equal terminating resistances exist. This does not have to be the case and the limitations of space forbad the inclusion of all possible combinations. Other, specialized L-C filter textbooks have those.[19] Such dissimilar terminating resistance normalized constants will be different but the results on passband and stopband response will be the same.

In very general terms, a tolerance of about ±10% is acceptable for most hobbyist applications without unduly disturbing response characteristics or insertion loss. A check of simulations via a SPICE analysis program will confirm that. Such is certainly a check on your initial component value calculations.

If trying to use the Bartlett's Bisection Theorem technique to change parts values, be aware that altering only one end of a filter will definitely swap input and output impedances. In the Figure 13-3 example the output termination was supposedly raised to 300 Ohms from its original 50 Ohms. A check of mid-band impedance will show that it *also changed* from 50 up to 300 Ohms. Trying to use that with a 50 Ohm transmission line would result in a large VSWR, thus a power loss. The voltage

[19] References [29] *Williams* and [30] *White*.

step-up from the finished, transformed BPF of Figure 13-3 is zero. It is not the 2.45:1 step-up as a result of impedance change.

There are two relatively-easy ways to achieve a different impedance match from the equal-resistance filter constants: Broadband transformers or a tap of the end inductor for a lower impedance end. Broadband transformers can achieve decade bandwidths covering much more than the typical passband covering, at most, an octave. Mismatches well outside of the passband would only add to the stopband attenuation.

Tapping or link-coupling the input inductor works well enough in the passband for a lower input impedance. With very high coupling coefficients found in toroidal inductors, the tap position is at the point nearest common connection equal to the total turns divided by the square-root of the desired impedance ratio. With relatively-loose coupling of an extra winding of a few turns (relative to calculated inductor) on a cylindrical form, determination of the turns is a bit more complicated.

An Analytical Model of Magnetic Coupling of Two Coupled, Isolated Inductors

Figure 13-8 shows the relationship between two magnetic-coupled but DC-isolated inductors to its *equivalent analytical model* comprised of three inductors. Not all SPICE programs have an internal two-coupled-inductor model and it could be analyzed with the aid of *dependent current sources* or *DCS.* What is required now is the *mutual inductance*, **M**, and the *coefficient of coupling,* **k.** The relationships are:[20]

$$k = \frac{L_M}{\sqrt{(L_1 + L_M)(L_2 + L_M)}} \qquad (13\text{-}3)$$

$$k = \frac{M}{\sqrt{L_1 L_2}}$$

$$M = \frac{L_M \sqrt{L_1 L_2}}{\sqrt{(L_1 + L_M)(L_2 + L_M)}}$$

The problem here is to find L_M. L_1 and L_2 can each be measured separately with the other open-circuited. The solution is to measure them together with the connections shown in Figure 13-9:

All that is required is to connect the individual inductors *in-place* so as to aid total the inductance (aiding) or the total with one bucking the other. L_M can then be determined by:

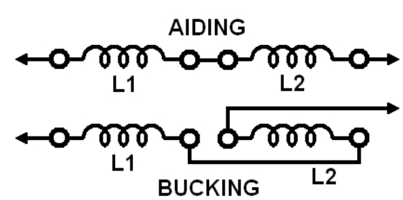

Figure 13-9 Measurements to determine L_M. Arrows go to inductance measuring device.

[20] Reference [12] *Terman*, pages 64, 65, 906, 907 as well as [4] *Landee* page 13-2.

$$L_M = \frac{\overset{\text{aiding}}{(L_1 + L_2)} - \overset{\text{bucking}}{(L_1 - L_2)}}{4} \qquad (13\text{-}4)$$

Once L_M is known, it is only a matter of substituting it into (13-3) to determine **k** and **M**. Or L_2 would be known and one would need to try various L_1 values to obtain the proper **k** and **M** and thus $\mathbf{L_M}$. This is not a quick-and-easy task but requires several trial values. It would be best to make up a tabulation of various trial values to *zero-in* on the best value of L_1.

Note: Magnetic coupling is highly influenced by physical sizes of the inductors and their positioning. One method for one size group is not always good for a different size group of components.

At least one text, reference [30], *White*, has several prototype arrangements of filters using magnetically-coupled inductors. Their only advantage for hobbyist designs is the ability to remove any possibility of DC coupling. Large values of series capacitors could do the same thing.[21]

Determination of Inductor and Capacitor Q

At least one manufacturer of powdered-iron toroidal core forms, Micrometals, has made extensive *Q Curves* graphics. Others have written and supplied free PC program for designing inductors in the VLF to UHF frequency range. This is very useful in selecting toroidal core sizes for different L-C filters.

Cylindrical air core coil forms are next-higher Q choices. A drawback to using those is the minimum sizing of nearby conductor planes such as the inside of metallic shields or close metal, even some dense plastic chassis or box components. As a general rule of thumb, the clear space between the inductor outer diameter and closest conductive plane should be greater than a coil form diameter. That assures shielded cylindrical coils have an open-air Q reduction no less than about 90%. Inductance will be reduced by shield enclosures but that can be determined by an L/C Meter.

Iron-powder slug-tuned cylindrical inductors have some variation of Q depending on their tuning slug position. Those are maximum when the powdered-iron slug is at the physical middle of the inductor winding, minimum when at maximum *out* position. A few early UHF radio designs used plated brass cores for adjustment. Their tuning slug positioning is just the opposite of powdered-iron core slug adjustments.

To be frank, measurement of capacitive Q greater than 1000, typical for air-dielectric or encapsulated silver-mica, is difficult and requires very stable frequency sources. The ***Q-Meter*** method shown in Figure 13-6 is probably best for the Hobbyist but ***Caution:*** Q values displayed must be compensated mathematically by inductor Q in resonating. In practical applications one can take the manufacturer's Q specifications at face value and use those.

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[21] Manufacturers of filters can practice economy of parts cost by omitting series capacitors, also some space requirements. They generally work with only a few physical sizes of inductor forms and have generated their own physical arrangements of coupling for their product lines. Hobbyists are not constrained by such economics and a few series capacitors for DC isolation are not going to impact their total project costs very much.

Chapter 14

Quartz Crystal Units and Narrow Bandpass Filters

A quartz crystal resonator unit is examined and detailed, along with ways to measure its parameters. The basics of very narrowband bandpass filters is described along with how to design with crystal units in those bandpass filters.

General

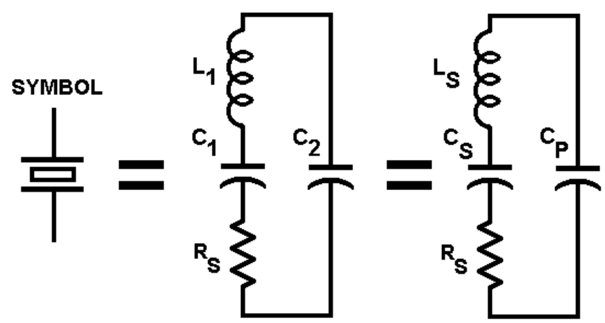

Figure 14-1 Equivalent Crystal Circuits

The quartz crystal resonator has become the prime frequency control element in radio since 1922.[1] Shown in symbolic and equivalent form in Figure 14-1, it can be seen as a two-capacitor, one-inductor, series-parallel dual-frequency circuit described in Figure 7-3 on page 7-6. It is series-resonant at a lower frequency but parallel-resonant at a higher frequency. The separation of resonance frequencies can be as short as 0.02% of either resonance frequency. The Q can be as high as 250,000. That is impossible to achieve with any known individual lumped inductor and capacitor components. At right in Figure 14-1 are the equivalent crystal components used in here.

The most familiar use of quartz crystal units may be the *quartz crystal oscillator* where the positive-feedback for the active device is controlled by the crystal unit. Less familiar is the very narrow percentage bandwidth *quartz crystal filter*. The exceptionally high Q of the crystal unit permits percentage bandwidths as low as 0.05% of a bandpass filter's center frequency. The excellent temperature stability of a quartz crystal unit assures that the passband remains stable over a large environmental range.[2]

Most common form of quartz in a quartz crystal unit is a thin plate-like slice having pressure-contact electrodes or plated-on contact electrodes that are made to wires or plug pins. Since the

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[1] Paper by Walter G. Cady, *The Piezo-Electric* Resonator, Proceedings of the IRE, Volume 10, April 1922. This paper also described the first single-crystal narrow bandpass filter which became common in narrowband selectivity enhancement of HF receivers in the 1930s.

[2] That temperature stability also applies to crystal oscillators. Temperature characteristics of a quartz crystal unit can be varied by its *cut angle* along a large quartz crystal. The most common cut angle for oscillator crystals is the *AT-cut* referring to the larger crystal's crystallographic orientation. Hobbyists seldom have facilities for cutting, grinding, finishing quartz crystals but they can be familiar with manufacturer's descriptions of them.

quartz crystal unit operates on piezo-electric properties it must be enclosed to operate properly. A variation in form is the millions of *tuning-fork* crystals made for very small 32.768 KHz quartz watch oscillators. Every color television receiver made worldwide since the 1950s has employed a quartz crystal unit in the *color subcarrier* synchronization circuit for color stability during the (analog system) horizontal sweep duration.

During the World War II years, the United States was the major supplier of quartz crystal units for the Allies' side and Brazil (a neutral) was the major source of raw quartz crystals. Production of quartz crystal units achieved an average of one million units per month in the last three years of that war.[3] Research into man-made crystal growth after WWII resulted in larger crystals of more uniform purity than natural quartz. Such man-made quartz enabled more economical crystal units from the better yield and purity until quartz crystal units became as common as conventional passive components.

Measurement of Equivalent Circuit Component Values

Quartz crystal units are specified by the type of resonance, series or parallel. If parallel resonant the external capacitance is given as a recommended value, typically 20 to 32 pFd. It is rare for manufacturers to supply any other quartz crystal unit equivalent circuit values.[4] The external parallel-resonant capacitance is not the entirety of C_P but becomes a part of it when in the circuit. To apply quartz crystal units to bandpass filter designs requires a temporary test set-up to measure equivalent circuit values. The *temporary* set-up ***requires*** an RF source that has ***very fine tuning*** and the frequency ***must*** be monitored by a frequency counter. A wideband oscilloscope (at least 30 MHz) with a 10:1 low-capacity probe is almost mandatory. The probe input capacity should be known fairly accurately; manufacturers specifications are adequate.

Figure 14-2 shows the simple schematic of the test fixture. R_{IN} can be any value from 220 Ohms to 2.2 KOhms for series resonance of MF to HF quartz crystals; it

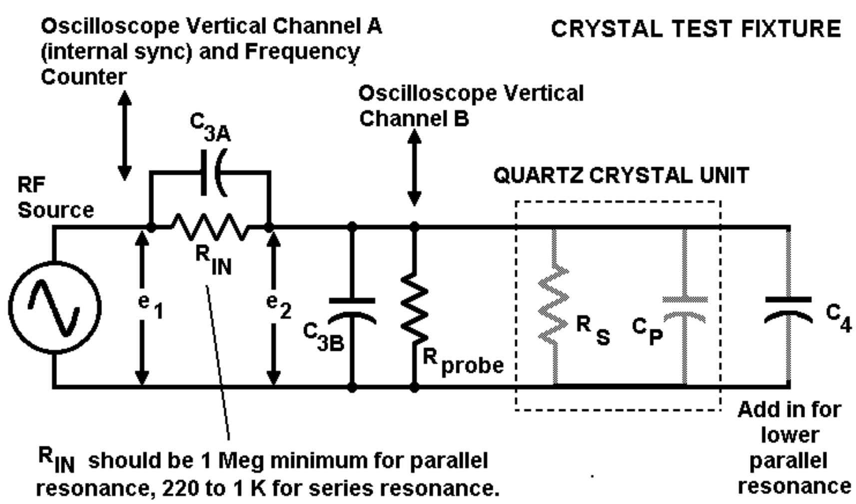

Figure 14-2 Test fixture for determining equivalent circuit values of a quartz crystal unit. Calibrate by measuring C3 total with crystal unit removed, Channel B probe connected.

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[3] See Reference [43] for a fascinating history of WWII crystal production described by a participant..

[4] If and when that is published it is usually given as a *range of values* for stock parts. Manufacturers will supply units to more specific values provided the buyer requests a large quantity, usually greater than 1000 units.

must be at least 1.0 MOhm for parallel resonance measurements. If the oscilloscope allows, connect the frequency counter to a vertical channel auxiliary output of its channel A. An oscilloscope 10:1 voltage divider probe typically has about 10 pFd to common at the probe end, in parallel with 10 Meg. Probe resistance will not be a factor since parallel resonance does not require an accurate output voltage measurement. The value of R_{IN} is not critical but should be at least 10 times higher than the expected R_S. A calculator of at least 10 digits in display should be used for calculation; a scientific type calculator capable of large exponent range is required for HF crystals. The frequency counter monitoring the RF signal source should have a resolution of ± 1 Hz or better.

The text fixture, with probe for oscilloscope channel B connected, quartz crystal unit disconnected, should be measured (as accurately as possible) for capacitance to common at the crystal unit connections. $C_3 = C_{3A} + C_{3B}$ with C_{3A} being the capacitance across R_{IN} at the high-resistance needed for parallel resonance and C_{3B} the scope probe. C_4 is perhaps best as a small plug-in unit with its total capacitance measured (including the holder).

The phase across e_2 will ***lag*** e_1 when below series resonance, above parallel resonance; it ***leads*** e_1 when the source frequency is above series resonance but below parallel resonance. This phase change is abrupt in each resonance and that makes it easier to determine than a dip or peak in e_2 amplitude. Series resonance produces a dip in e_2 amplitude, a peak with parallel resonance. Ability to see this phase shift is enabled by using internal synchronization of the oscilloscope to the source at e_1. Once synchronized to the source RF the phase can be seen to shift at each resonance frequency.

C_{3B} can be about 12 pFd total with typical oscilloscope probes. C_4 can be any value from 10 to 20 pFd but ***must*** be measured accurately to use as the second external parallel resonance capacitor.

The procedure:

1. Oscilloscope probe to junction of R_{IN} and crystal unit, R_{IN} at minimum value.
2. Adjust the RF source frequency for *minimum* e_2 level, final frequency setting at zero phase shift relative to e_1. That frequency is f_1. Record the frequency.
3. Change R_{IN} to the higher resistance.
4. Adjust the RF source frequency for *maximum* e_2 level, final setting to zero phase shift. That frequency is f_3. Record that frequency. It will be slightly higher than f_1.
5. Plug in C_4. Re-adjust frequency for zero phase shift at newer, slightly lower parallel resonance and record that frequency as f_4. That frequency will be between f_1 and f_3.

Calculation of Equivalent Circuit Values

The value of R_S will be found at series resonance by: $$R_S = R_{IN}\left(\frac{e_2}{e_1 - e_2}\right) \qquad (14\text{-}1)$$

Peak-to-peak voltages are fine there and afford the highest signal level at series resonance.

$$k_3 = \frac{f_3^{\,2}}{f_1^{\,2}} \quad k_4 = \frac{f_4^{\,2}}{f_1^{\,2}} \quad \varpi_1 = 2\,\pi\,f_1 \quad \varpi_3 = 2\,\pi\,f_3 \quad \varpi_4 = 2\,\pi\,f_4 \qquad (14\text{-}2)$$

f_1 = series resonance frequency due to L_S and C_S values.

Equation set (7-6) establishes the distinct relationship between all three C-L components of the crystal unit but, to find two of them, the third must be selected. The two-frequency parallel resonance measurement will find C_2 by:[5]

$$C_P = \frac{C_4 (k_4 - 1) - C_3 (k_3 - k_4)}{(k_3 - k_4)} \qquad (14-3)$$

As an example, an arbitrary-value (but nearly typical[6]) 8 MHz crystal unit was used with a frequency-domain analysis program. R_S was 20.0 Ohms, f_1 was 8.000 000 MHz, L_S was 79.157 mHy, C_S was 5 fFd (yes, femtoFarads). C_P was 12 pFd and f_2, parallel resonance frequency, was 8.001 675 MHz. Oscilloscope probe capacity was 10 pFd and that taken as total C_3. C_4 was picked as 10 pFd. The analysis program used can resolve 1 Hz steps in frequency.

Series resonance frequency was 8.000 015 MHz as checked by the zero phase shift point. With 1.0 V source voltage and 220 Ohms for R_{IN}, the voltage across e_2 was 83 1/3 mV. Using (14-1) that calculated out to exactly 20.0 Ohms, equal to the R_S value of the circuit model. Parallel resonances were read and used with (14-2) for the following:

$$f_3 = 8.000904 \cdot 10^6 \qquad k_3 = \frac{(8.000904)^2}{(8.000015)^2} = 1.00022226$$

$$(k_3 - 1) = 222.26193 \cdot 10^{-6} \quad \text{[using HP - 32S calculator values]}$$

$$f_4 = 8.000628 \cdot 10^6 \qquad k_4 = \frac{(8.000628)^2}{(8.000015)^2} = 1.00015326$$

$$(k_4 - 1) = 153.25558 \cdot 10^{-6} \qquad (k_3 - k_4) = 69.006350 \cdot 10^{-6}$$

$$C_3 = 10 \cdot 10^{-12} \qquad C_4 = 10 \cdot 10^{-12}$$

Using the values immediately above, C_P was solved using (14-3):

$$C_P = \frac{10^{-11} \cdot \left(153.25558 \cdot 10^{-6}\right) - 10^{-11}\left(69.006350 \cdot 10^{-6}\right)}{69.006350 \cdot 10^{-6}} = 12.208910 \cdot 10^{-12}$$

That value is only +1.74% in error from the circuit model of 12 pFd

Finding the remaining two component values uses identities from (7-6):

$$C_S = C_1 = (C_P + C_3)(k_3 - 1) \qquad L_S = L_2 = \frac{1}{\varpi_1^2 \, C_S} = \frac{k_3}{\varpi_3^2 \, (C_S + C_3)(k_3 - 1)} \qquad (14-4)$$

With several term groups already calculated, it is a matter of plugging in those into (14-4):

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5 For derivation of this formula see Appendix 14-1 at the end of this chapter.

6 Skewed to values that would cause the most error in measurement and calculation. In reality the equivalent circuit component values would vary greatly among different frequencies, type of cut, and (to a lesser degree) with the type of holder and its electrode connections.

$$C_S = (22.208910 \cdot 10^{-12})(222.26193 \cdot 10^{-6}) = 4.9361952 \cdot 10^{-15}$$

The value of C_1 is only -1.28% error from circuit model of 5 pFd

$$L_S = \frac{1.00022226}{(50.2655767 \cdot 10^{6})^2 \cdot (4.9361952 \cdot 10^{-15})} = 80.1800526 \cdot 10^{-3}$$

The value of L_S is only +1.29% error from circuit model of 79.157 mHy

Calculations carried out to many decimal places may seem overdone to some, but it is a requirement in this case. The frequency difference between f_S and f_3 is only 889 Hz at a geometric center frequency of 8.000459 MHz. That is equal to 111.119 PPM or 0.011 111 9%. Due to the skewed values of C_P , C_S, and L_S, the actual parallel resonant frequency of this hypothetical crystal unit (without C_3 and C_4) is about 8.001692 MHz and not quite twice that of f_3. The resulting small value of any (k - 1) term can itself be a source of potential error.

If the frequency counter's time base is slightly off by a few PPM, that is not a cause for concern; it does not contribute to any significant errors of these calculations. What is more important is to set the RF signal source frequency for the relative zero-phase condition. That requires a very fine tuning ability of the source. For a group of crystal units to be used in a narrow passband filter, it may be necessary to build a special, very-fine-tuning oscillator circuit to use as the source. This is for the tester's convenience in adjusting parallel resonance frequency.

Error Conditions and Fixture Tolerances

If all three frequencies of the example are off by +5 Hz due to frequency counter miscalibration but C_3 and C_4 are exactly 10.0 pFd, the calculated C_2 value will be 12.2089128 pFd, an error of only $+230.11 \cdot 10^{-9}$ compared to previously-calculated value of 12.208910 pFd. Slight miscalibration of the frequency counter will not cause much error.

But, if C_3 and C_4 are ± 5% in calibration (equal to ± 0.5 pFd in this example) the values of C_2 will be 12.819358 and 11.598467 pFd respectively, indicating that C_2 will also be off by ± 5% compared to the exact calculation value.

Series resonance frequency f_1 will remain the same regardless of parallel loading of capacitance across the series L-C circuit. The only caution for parallel resonance testing is that the parallel capacity added by the fixture cannot be so much as to move the parallel resonance frequency too close to series resonance frequency.[7]

Measure Parameters via ± 3 db Amplitudes

Measure R_S first, then series resonance bandwidth (f_X) finding crystal center frequency (f_1) to find crystal Q (Q_X) by:

$$Q_X = \frac{f_1}{f_X} \quad \text{then} \quad L_S = \frac{Q_X \cdot R_S}{2\pi f_1} \qquad \text{[Hz, Hy, Ohms]} \qquad (14-5)$$

[7] See chapter 7 on the two-capacitor, one inductor series-parallel combination.

Series resonance capacitance C_S is then found by:

$$C_S = \frac{1}{(2\,\pi\, f_1)^2 \cdot L_S} \qquad [\text{Fd, Hy, Hz}] \qquad (14\text{ - }6)$$

Parallel resonance capacitance C_P can be derived using equation set (7-9) on page 7-6 since both C_S and L_S are now known along with f_1 and f_2.

Reference [37] includes an active-device test set that includes a *gain-setting switch* for 0 db or +3 db relative gain. When measuring the amplitude minimum at 0 db gain, the switch is set for +3 db gain to find the *same display amplitude* when adjusting frequency for the +3 db up-from-peak.

The Simplest Quartz Crystal Filter, Only One Crystal Unit

Figure 14-3 is adapted from Reference [42] and basically uses the series resonance of the crystal unit to pass a tiny bandwidth at that frequency. The crystal unit symbol has been replaced by the equivalent circuit to show the series tuned circuit enabling the passing of mainly the fundamental frequency of a fixed-frequency input waveform.

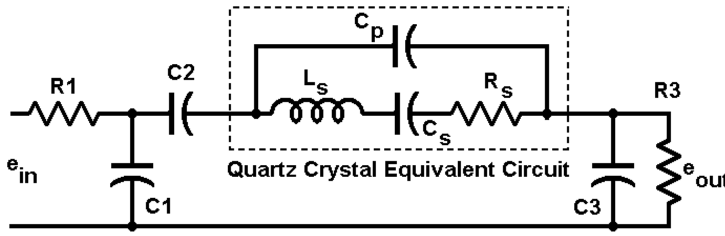

Figure 14-3 Simplest crystal filter circuit.

According to Intersil AN9815 description, $R_!$ and C_1 form an R-C lowpass filter such that the reactance of C_1 is equal to the resistance of R_1. In reality this is not a critical calculation and the actual values can vary ± 50% without much degradation of operation. Another voltage divider is R_S of the crystal unit and R_3, the load or input impedance of the following stage. C_3 is a relatively high value capacitance supposedly to reduce the effects of variations of crystal unit equivalent circuit values; it acts mainly to reduce the amplitude of e_{OUT} over all frequencies above series resonance. The total resistance of C_1, C_2, and C_3 was described as equal to the crystal unit's parallel resonance loading capacity when all are in series:

$$\frac{1}{C_{LOAD}} = \left(\frac{1}{C_1}\right) + \left(\frac{1}{C_2}\right) + \left(\frac{1}{C_3}\right) \quad \text{All capacitances in same units}$$

That is not critical either since the series resonance of the crystal unit does the filtering, not the parallel resonance. On a circuit analysis examination using the 8 MHz example crystal unit described before, the following values were used in the circuit:

R1 = 1.5 KOhms R3 = 5 KOhms
C1 = C2 = 100 pFd C3 = 220 pFd
10 pFd in parallel with crystal unit for an equivalent 22 pFd C_P
L_S = 79.157 mHy C_S = 5.0 fFd R_S = 20 Ohms

The -3 db bandwidth was quite sharp, about 62 Hz, centered at 8.000 327 MHz. Relative response at 8.000 262 and 8.000 380 MHz was -22 db. Off-center response was -32 db at exactly 8.0 MHz and -34 db at 8.000 500 MHz with a -70 db dip at 8.000 915 MHz. The latter from the new parallel resonance with added C_P. Insertion loss at series resonance was 16 db.

An Older-Design Single Crystal Filter

The circuit of Figure 14-4 was popular in communications receivers of about pre-1950 times for its ability to provide a very narrow filter for on-off keyed CW mode reception. This arrangement uses an IFT with a center-tapped secondary. Depending on the trimmer or manually-variable capacitor *balance* setting, the sum of output IF voltages from the crystal unit (inductive) will match those from the adjustable capacitor (capacitive) to yield a sharp peak for the input of the following stage. Note that with the center tap grounded the secondary phases are in opposition; an inductive phase shift will match an opposite-polarity capacitive phase shift. Peak frequency will be somewhere between series and parallel resonances of the quartz crystal unit. The bandwidth of the output peak is very narrow, typically around 200 Hz with old crystal units of the 455 KHz variety.

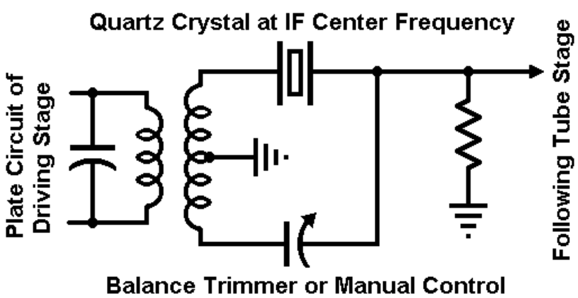

Figure 14-4 Simple very-narrow bandwidth crystal filter for CW reception.

While this filter would definitely pull out on-off keyed carrier signals from input noise, its far-from-resonance selectivity was no better than the single crystal filter of Figure 14-3. This circuit was relatively inexpensive to produce other than the single crystal unit's cost and designer-manufacturers liked it for that reason back in vacuum tube days. It is of no value in filtering voice signals needing 2.4 KHz minimum bandwidth for a single AM signal sideband.

Lattice Arrangements of Multiple Crystal Units for True Sharp Selectivity

A typical four-crystal-unit arrangement for specific narrow bandwidths is shown in Figure 14-5. T1 and T2 are used to convert from single-ended input and output to the balanced crystal unit interconnection. That interconnection is called a *lattice* and most texts will show X2 and X3 on a diagonal rather than all-horizontal as in Figure 14-5.

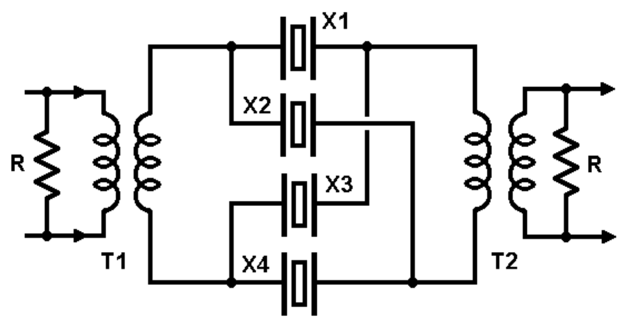

Figure 14-5 A typical 4-crystal lattice filter

T1 and T2 are somewhat broadband (in relation to the desired bandwidth) and their primary-secondary turns ratios are arranged so that the balanced center arrangement *sees* the correct resistive end terminations. The four-section arrangement can be chained for stricter requirements on bandwidth and selectivity. This sort of crystal filter was used often in the 12 KHz bandwidth commercial-government-military SSB radios from the early-1930s until after WW II was over in 1945.

The mathematics needed for lattice filter design is quite involved, much more so than any other bandpass filter so far presented. At lower center frequencies, say around 100 KHz, they might be duplicated with L-C series-resonant circuit for percentage bandwidths on the order of 5 to 10 percent. Older texts from before 1960 have the necessary design equations already worked out if any hobbyist wishes to spend a few months on paper designs alone. The *Dishal / Cohn / Minimum-Loss* crystal bandpass filter is considerably easier to design and also allows selection of the same-frequency quartz crystal units.

Dishal Ladder Filters

Dishal's paper of 1963[8] showed the way to making simpler quartz crystal bandpass filters, popular at that time for single-channel single-sideband HF transceivers that had come into vogue a decade before.[9] Based on *modern network theory and design* they are relatively simple structures in an expandable ladder arrangement as shown following.

The two types in Figure 14-6 are named after their unsymmetric output versus frequency response. In the upper filter, the low-impedance series resonance passes more lower frequency components. The high-impedance parallel resonance attenuates the higher frequency components. For the lower filter the parallel resonance is in shunt and passes more higher frequencies while series resonance shunts lower frequencies to common. The carrier frequency is depicted by the dash line for application in a radio.

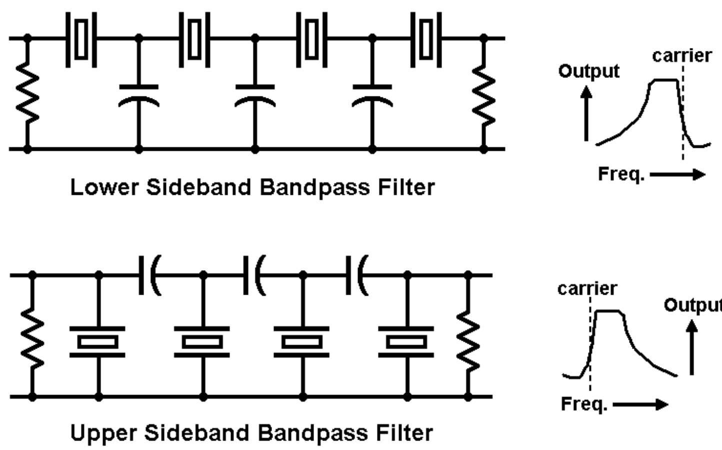

Figure 14-6 Generalized 4-crystal bandpass filters.

These ladder configurations may be expanded by adding more L-sections in either filter. Resistor terminations are always required at each end in practical bandpass filters within the HF region. Capacitive coupling or shunting is desirable in either case since fixed capacitors have a much higher Q than inductors. The very small percentage bandwidth requires components with very high Q. Quartz crystal units have that.

---

8 Reference [34]

9 The new USAF Strategic Air Command desired HF SSB radios for its *loiter alert* announcements and single-channel SSB radio contracts were let to Collins Radio and RCA Corporations around 1950. Other than that, SSB radio technology was centered around the multi-channel 12 KHz wide *commercial* format for HF long-distance communications begun in the early 1930s. See Reference [ ] for more history on that.

Synthesizing the Dishal Configuration From L-C Filter Data

Figure 14-7 shows the relationship between the crystal filter symbols and the equivalent circuit of just the series resonance components within the crystal unit. When transformed to the bottom circuit, the resemblance to conventional lumped-constant L-C filters becomes more apparent.

Intuitively, the very low series resonance impedance will make something like an R-C voltage divider having very little attenuation around the series resonance frequency. Adjacent shunt-connected capacitors will act to decrease the equivalent circuit series capacitance just above the series resonance frequency.

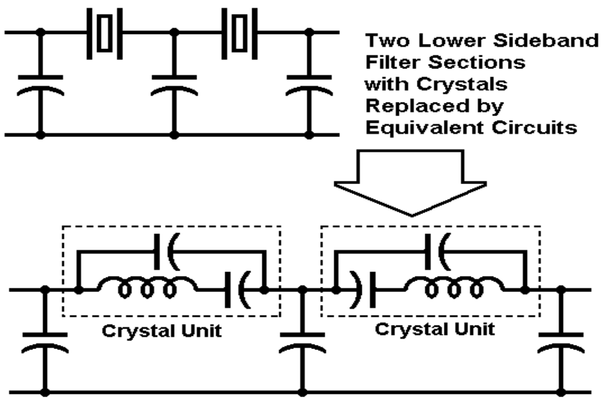

Figure 14-7 Transformation of two sections of a lower sideband filter to the series resonance equivalents of the crystal units.

At the crystal parallel resonance frequency (modified slightly by the adjacent shunt capacitors) the equivalent circuit impedance is very high and there is a decided notch in amplitude versus frequency response at parallel resonance.[10] The overall filter response will have a lower frequency limit at the crystal unit series resonance and extend upward to a point determined by the crystal unit equivalent circuit and the shunt capacitor values to be selected. *Skirt selectivity* on the high side of this configuration is extremely steep and that gives rise to the configuration name of *lower sideband*; i.e., the notch frequency is equivalent to an AM (or SSB) carrier frequency yet the passband is still good for 200 to 300 Hz voice frequency components.

The *upper sideband* quartz crystal configuration of Figure 14-8 passes frequencies from between series and parallel resonance on up slightly past crystal unit parallel resonance. It has a sharp notch or null at the series resonance frequency. Intuitively it can be thought of as another R-C voltage divider chain with high parallel resonance impedance versus lower-reactance series capacitors. Attenuation is minimum at parallel resonance. Amplitude versus frequency response is the mirror image of the lower sideband configuration.

The upper sideband configuration has lesser ability to compensate for inevitable stray or parasitic capacitance of either driving circuit or the load. Also, the practical high input and output impedances require a step-up and step-down transformer to operate with either tube or transistor circuitry. In actual practice the lower sideband configuration is quite capable of filtering up to 5 KHz bandwidths at HF and with sharp skirt selectivity at either passband end.

Adjustment of Crystal Unit Resonance Frequencies

Crystal ladder bandpass filters will require some crystal sections to be higher or lower than their inherent series and parallel resonant frequencies. That can be done by test-selection of units'

[10] Actual attenuation can be only theoretical at the notch since it is far beyond normal laboratory metrology instruments ability to measure.

frequency or by capacitive trimming using added series or parallel capacitors as shown in Figure 14-9.

This frequency trimming is limited and the amount of capacitance to be used requires a rather cumbersome mathematical calculation. Worse yet, adding a series frequency-shifting capacitor will result in changing *both* series and parallel resonance frequencies. Adding a parallel capacitor shifts mainly the parallel resonance frequency lower, very little effect on series resonance.

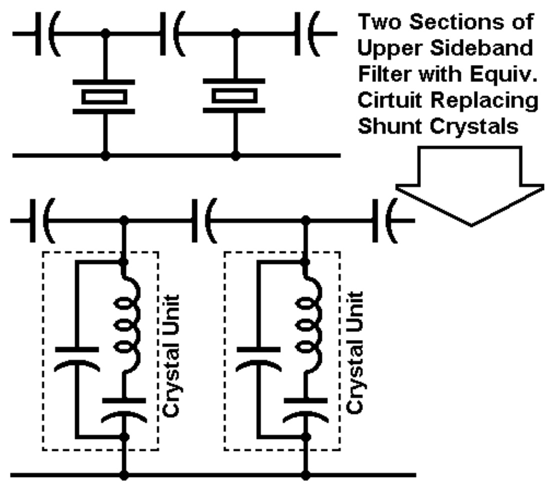

Figure 14-8 Equivalent configuration of upper sideband crystal filter.

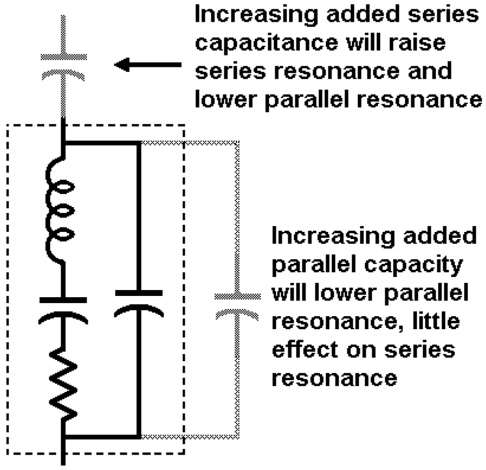

Figure 14-9 Trimming of crystal unit (within dash lines) resonant frequencies by series or parallel capacitors (shown in grey).

Inductors could have been used except that their Q values, even for exceptional toroidal form coils with Q of 500 is *not good enough.* High-Q capacitors of 5000 or greater (silver-mica) or 10,000 or greater (air dielectric with heavy plates) is the only possible solution. Too low an added component Q will make the overall filter frequency response round its frequency response corners and increase insertion loss.

Cohn or *Minimum-Loss* Ladder Configuration

The Cohn configuration is categorized by a literal minimum of components. All crystals should be as identical in resonant frequencies as possible and crystal units in series with *same value* shunt capacitors between crystal units. A Cohn bandpass filter is generally suitable for the 200 to 500 Hz bandwidths favored for radiotelegraphy signals. Trying to use it at wider bandwidths results in a passband of many peaks and valleys, not suited for voice grade single sideband filtering. While various amateur radio articles have stated that Cohn filter configurations are suitable for voice-grade SSB or AM reception, those statements seem subjective or are done with undescribed test equipment. With adequate selection of crystal unit series-parallel equivalent circuit frequencies it is theoretically possible. However, that requires an extensive test-and-selection of a large quantity crystal units. For radio hobby purposes the time and cost considerations might be better spent in adding fixed capacitors for trimming passband response, thus resulting in a Dishal configuration filter.

Equalizing the passband response requires the various sections of a bandpass filter to have different resonances. Those frequencies are either from selected-frequency crystal units or those that were trimmed by added fixed capacitors.

Designing Dishal or Cohn Crystal Ladder Filters

Personal Computer Design

The mathematics operations will be intense for this singular area, more so than with L-C filters. There will be numerous trade-offs between termination resistance, quartz crystal unit measured parameters, percentages of possible variation due to manufacturing tolerances of capacitors, and the ever-present stray capacity of driver and load circuits. All of those will affect the passband response and the stopband attenuation. The small variation between series and parallel resonance frequencies of all crystal units will require not only careful measurement of crystal unit parameters but also the best precision possible in numeric calculation aids.

There is one excellent design program available for free download on the Internet as of the year 2007, Reference [41]. This very comprehensive design program that does all the calculation and includes a variety of output response plotting, plus a *Monte Carlo* sensitivity plot (random value variation within specified percentage tolerances), along with the ability to change section values to see their effect on passband response. It will do in seconds what normally takes hours with a scientific calculator and manual entry of various values, then recording those answers on paper. *Filter Design* by Neil Heckt includes the ability to print out all schematic diagram values (including entered crystal parameters) for easy reference later.[11] Its calculations and frequency responses compare very favorably with the same schematic analyzed in any analysis program as well as with hardware itself.[12]

Some Things To Be Aware Of Prior To Actual Design

The lower-frequency corner of the crystal ladder filter passband is fixed by the crystal unit series resonance frequency. Passband width is dependent on the coupling capacitor values and the crystal unit parallel resonant frequency. That parallel resonance frequency difference should be at least twice to three times the desired bandwidth.

Series resonance resistance R_S will determine how *flat* the passband response is; i.e., the variation in amplitude versus frequency in the passband. Too high a resistance will result in rounding of the passband edges, very similar to an L-C bandpass filter. Very narrow passband widths on the

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[11] This is not a paid endorsement of Mr. Heckt's program. It is based on actual use of the only free program of its kind available on the Internet and a comparison of calculated schematic values in other analysis programs as well as performance of hardware in actual circuits. The PC program requires some effort by users to get used to due to its comprehensive capability, but once that is accomplished it works very well.

[12] To properly evaluate tangible hardware filters requires a very stable, very-fine-tuning signal source and a calibrated logarithmic detector sensor such as a logarithmic detector IC capable of greater than 80 db dynamic range. To properly check a frequency response over a 5 KHz sweep needs an accurate setting capability to ± 1 Hz at filter center frequencies of 3.5 to 9.5 MHz. This is not a trivial task and cannot be satisfied with old radio service shop equipment. Capacitance measuring equipment requires measuring small capacitors to resolutions of a tenth of a picoFarad with reasonable accuracy.

order or 250 to 500 Hz will require crystal units with very high Q.

Expect some very small added fixed capacitor values paralleling crystal units. Those values can sometimes equal stray wiring capacitance. Physical mounting of crystal units will have to take that into account.

Lower sideband Dishal configurations will have lesser stopband attenuation on the low side than on the high side. There is a very high attenuation at the crystal units' parallel resonance which increases slightly towards higher frequencies. That *ultimate attenuation* at far-from-center frequency stopband will remain lower than low-side attenuation. For Upper sideband filters the stopband attenuation reverses.

How Many Crystal Units will be Needed?

That depends mostly on the desired stopband attenuation at what frequencies. An approximate *rule-of-thumb* might be derived from trying various 3.58 MHz *color-burst AFC* crystals in several 2.4 KHz bandpass filters.[13] The crystals for these examples have an R_S of 12.3 Ohms, L_S of 64.47 mHy, C_S of 30.7 fFd, and C_P of about 4 pFd. The series resonance would be 3.577 438 MHz and parallel resonance would be at 3.591 140 MHz for a series to parallel resonance frequency separation of 13.7 KHz. Crystal Q at series resonance would be 117,816. Examples were Dishal Lower Sideband (series-crystal configuration) Filters with *Filter Design* program capacitor values approximated by the nearest 5% tolerance fixed type; all added crystal parallel capacitors were kept at program exact values. Attenuation in db is relative to 0 db at filter center frequency at the given bandwidth increments away from that center frequency.

Bandwidth	Approximate Relative Levels in db at each Increment					
Increments	4-Crystal	5-Crystal	6-Crystal	7-Crystal	8-Crystal	9-Crystal
-4	-51	-69	-77	-94	-108	>120
-3	-46	-58	-71	-87	-101	-116
-2	-39	-45	-60	-74	-85	-100
-1	-22	-30	-38	-47	-55	-65
0*	-0.4	-0.3	-0.7	-0.6	-0.9	-0.8
+1	-42	-60	-73	-89	-101	-119
+2	-93	>120	>120	>120	>120	>120

* Insertion loss at center frequency
>120 denotes relative level lower than -120 db

Note: The attenuation slope above the higher passband corner is extremely rapid and reaches a minimum level that is physically impossible to attain in any practical structure. A 120 db difference in levels corresponds to a receiver's antenna input of 1.0 µV versus 1.0 V at the detector.

The tabulation above is only approximate but it can be a general guide in choosing the number of crystal units for a given radio design application. It should also be noted that *even number crystal units* will have the output resistance termination higher than the input termination; *odd number*

[13] The old USA NTSC color television subcarrier reference frequency is 3.579 545 454 545 ... MHz and every color TV receiver built for the NTSC standard had one. After the changeover from NTSC to DTV broadcast in the USA after February 2009 such color-burst crystals might be available as *NOS* or *New, Old Stock* surplus at some distributors.

crystal units will have equal resistance terminations. All bandpass filters must be terminated with resistors at both ends to preserve bandpass shape.

Influence of Crystal Unit Series Resonance Q

As the crystal unit Q is lowered, the -3 db bandwidth will shrink and the insertion loss will increase. The passband corner frequency versus amplitude response becomes more rounded with decreasing Q although there is little change in mid-passband shape; that is the major cause of shrinkage of the -3 db bandwidth. To illustrate that, a 7-crystal, 0.2 db ripple Chebyshev response filter using the mentioned 3.58 MHz color-burst crystals, 2.4 KHz design bandwidth, was examined in *Filter Designer* using 5% tolerance nearest-value capacitors substituting for exact program values. Crystal Q was varied from 30K to 180K, R_S varying inversely from 48 Ohms to 8 Ohms.

Crystal Q	-3 db BW, Hz	Insertion Loss, db
180K	2382	0.4
120K	2308	0.6
90K	2294	0.8
60K	2258	1.2
30K	2128	2.4

At the two higher Qs the Chebyshev passband ripple was easily seen. At Q of 90K the ripple was barely visible and upper-frequency region of the passband was beginning to round off more than the lower-frequency passband corner. For the two lower Qs the passband response was rounding off with a definite higher-frequency-corner rounding compared to the lower-frequency-corner.

There was no appreciable affect of crystal unit Q on stopband attenuation beyond one bandwidth from filter center frequency.

Choice of Chebyshev versus Butterworth Response

For SSB voice-frequency filtering (2.4 to 2.8 KHz bandwidths), the Chebyshev response with 0.1 to 0.5 db passband ripple will yield the greatest stopband attenuation. Attenuation will increase with higher design ripple values but the variation in passband shape will require a more careful choice of physical fixed capacitor values with higher design ripple. A choice of 0.2 db ripple was considered a good compromise in all the example filter designs.

Butterworth response is mentioned as better for very narrowband (250 to 500 Hz bandwidth) filters in Reference [37] due to the definite rounding of the passband corners. The *group delay* variation is less with Butterworth response over the passband since that has less phase distortion than Chebyshev response. It should also be noted that Group Delay is not necessarily related to amplitude variation over frequency, the shape of the passband response. *Group Delay* is the incremental phase change divided by the incremental frequency difference. This results in an actual time delay from input to output of any filter. Most L-C and Crystal bandpass filters have more Group Delay at the passband edges than at the center. This can result in some distortion at the output, most noticeable with on-off, step-impulse signals such as radiotelegraphy or certain data (teleprinter) sideband content.

In practical terms for the hobbyist, Chebyshev response filters have more stopband attenuation than Butterworth response and stopband attenuation is the primary goal of narrow crystal bandpass filters.

Practical Design Using *Filter Designer*

Beginning inputs to the program can be any choice of all crystal parameters or part of the crystal parameters and a desired termination resistance or desired bandwidth. That choice is up to the user. The next step establishes the number of crystals (*filter order*) and, if a Chebyshev response was desired, the db ripple in the passband. Finally, a factor that is known as *r0v3* is requested.[14] That is a *trial value* to begin the process; it may be changed later at will but requires beginning over in the calculations. The *r0v3* value can be anything between 2 and 10 and can include decimal fractions. If all the crystal parameters have been entered first, the most noticeable change will be in termination resistance with different *r0v3* values.

With *r0v3* entered, the program displays a schematic of the crystal filter along with several options to the basic fixed components. Those options can be chosen ahead of time and can become default settings without the program requesting them again. Certain conditions may result in *negative* fixed component values (indicated in red). No plotting can be done if negative component values result. That is usually caused by an unworkable entry of *r0v3* and all entries must begin over.

A right-click on the mouse will bring up a sub-menu of response shape plots over a requested frequency change (horizontal scale) and vertical scale (db in case of amplitude v. frequency). There is a small click box that allows keeping the scale factors as default values, good for comparing small changes in the branch values of the same filter design. Any number of plots may co-exist on the screen. Schematics and all plots may be printed out as desired with each plot selected by a sub-menu that appears. The ladder branches are called *dipoles* in the program. Each dipole/branch can be changed in value by putting the cursor on the schematic dipole and clicking the right button on the mouse.

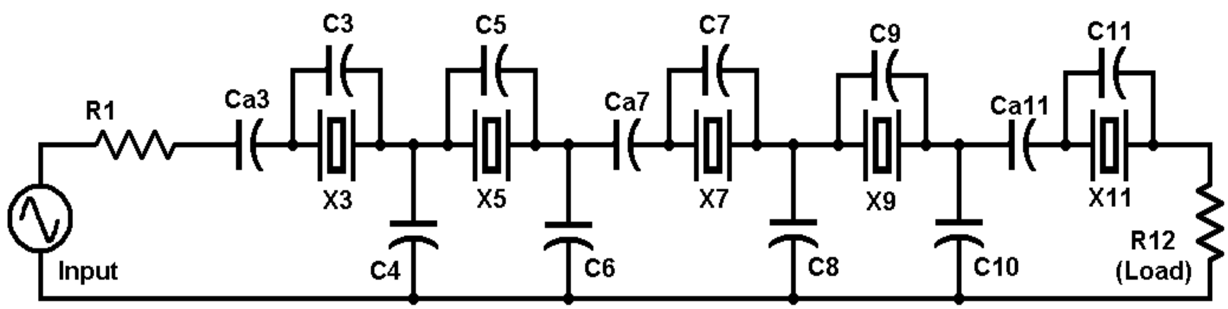

Figure 14-10 Schematic of an example 5-crystal Dishal Lower Sideband bandpass filter as it would appear in *Filter Designer.* The five parallel-to-crystal capacitor values (C3, C5, C7, C9, C11) will adjust the upper end of the passband in frequency. All other capacitors affect the shape of the passband, ripple and peaks/valleys.

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[14] This is not unknown in the calculation of very inter-related values such as found in ladder filter design. With L-C filters certain inductance or capacitance values can be fixed and all others dependent on the cutoff or center frequencies and bandwidth if bandpass or bandstop filters are desired. The equivalent circuit values of all quartz crystal units are very fixed and of rather rigid values and cannot be varied without a complicated remaking of the crystal unit.

As an example, The five-crystal Dishal Lower Sideband filter of Figure 14-10 was derived from *Filter Designer.* The crystal units again were the *color burst* variety used before, all equal and having a Q of 120K. Filter bandwidth was specified as 2.4 KHz and Chebyshev response with 0.2 db ripple in passband was selected. The end terminations would be equal with odd-number crystal units. Since the program will calculate 5-decimal-digit capacitor and resistor values, the nearest 5% tolerance capacitors were used, 10% tolerance resistors selected as follows:

Reference Designation	Program Value	Selected Value
R1, R12	1.2675 K	1.5 K
C3, C5, C7, C9, C11	3.1720 pFd	3 pFd
Ca3, Ca11	50.364 pFd	51 pFd
Ca7	161.27 pFd	165 pFd
C4, C10	38.379 pFd	39 pFd
C6, C8	50.364 pFd	51 pFd

Note: While 1.2 KOhms for end terminations would be closer, 1.5 KOhms allows a little more gain from the driving stage; there is more flexibility in end termination resistor values than with the capacitor values.

Worst-case passband ripple was 0.3 db and insertion loss was 0.4 db. Changing the values of C3, C5, C7, C9, and C11 could adjust the *upper passband* frequency rather independently from the lower passband frequency. The lower corner is rather fixed by crystal unit series resonance. As a comparison of what happens with those capacitor value changes, the following table indicates bandwidth to -3 db points and the frequency in MHz of the upper -3 db point. All 5 capacitor values were changed in unison.

C3, C5, C7, C9, C11 Fixed Value	Bandwidth, Hz	Upper -3 db Freq.
3.3 pFd	2280	3.580213
2.7 pFd	2365	3.580298
2.2 pFd	2439	3.580378
1.8 pFd	2497	3.580442
1.2 pFd	2594	3.580546

Variation of -3 db Frequency was no more than 19 Hz

The above suggests that small 1 to 5 pFd trimmer capacitors could be used for C3 to C11 to set the upper -3 db frequency. In actual practice the measured values of the crystal unit characteristics would vary and such trimmers could be used to *tweak* the passband width to the desired frequency value.

Insertion loss varied only from 0.38 to 0.42 db with the above C3-C11 changes. Passband ripple remained constant at about 0.3 db. The low side *skirt response* (slopes of attenuation outside of passband) was not great with only 5 crystals, reaching -63 db in 7.9 KHz below the lower -3 db point of the passband. High side skirt response is steep, -63 db in about 1.6 KHz from upper -3 db point.

Varying C4, C10 and C6, C8 from 39 and 51 pFd to 43 and 56 pFd, then to 33 to 47 pFd resulted in a slight passband shape difference and a slight bandwidth change. Varying Ca3, Ca11 and Ca7 from 51 and 165 pFd to 56 and 180 pFd, then to 47 and 150 pFd also had only a slight passband shape variation and slight bandwidth change. Going beyond those variations would result in more pronounced passband changes as well as bandwidth changes. It is important to keep the *proportions* of the fixed shunt and series capacitor values and keep their values within similar percentage limits.

All tabulations on this page applied only to the example filter; other filters or other bandpass specifications would have different results.

Different crystal units, even those marked for a specific frequency, can yield different characteristics. Those of a slightly higher resonance frequency could be placed at all branches that have a series capacitor, i.e., those with the ***Ca****N* reference designations on *Filter Designer* schematics. The program assumes all crystal units have the same characteristics so analysis trials would require another program (such as *LTSpice*) to see amplitude versus frequency performance. There would have to be different adjacent capacitor values to set the desired passband shape.

Testing a Prototype Filter

The least testing time is accomplished with a relatively slow rate sweep generator capable of accurate frequency control at very small sweep frequency widths. A DDS or Direct Digital Synthesis RF generator IC would be at the heart of such a source, driven by a programmed microcontroller. Such an RF source is a whole new project in itself. However, once tested, a DDS RF source can be stand-alone, not needing a frequency counter for monitoring frequency.

As an interim substitute for the RF source sweep, an L-C oscillator using a voltage-variable capacitor driven by an op-amp from a repetitive sawtooth waveform oscillator could be used. Those are reasonably stable at room temperature and could be checked with a frequency counter with DC voltages at the variable capacitor's voltage control input. A caution is that the amount of capacitance change for a very small sweep width is correspondingly very small. For a 3.575 to 3.585 MHz sweep (10 KHz) for *color burst* crystals, the frequency change ratio is only 1.0027972:1. The needed capacitance change ratio is the square of that or 1.00560223:1. That is equivalent to only 0.56 pFd change of an oscillator tank circuit having 100 pFd capacitance.[15]

Taking a tip From old economy-model TV alignment sweep generators, the variable capacitor could be a metallic disc glued onto the cone of a loudspeaker. The loudspeaker-with-disc would then be placed in the vicinity of two fixed plates of the L-C oscillator tank. The loudspeaker could then be driven by a sawtooth waveform audio source through any low-frequency audio speaker amplifier. While that was good enough for TV alignment in the 1950s covering 6 to 10 MHz sweep widths at VHF, the sweep linearity is hard to control and a DC control input can result in overheating a loudspeaker.[16] The sweep rate should be under 20 Hz for these narrow sweep widths and high-Q crystals for any of the swept RF sources.

Amplitude versus frequency of a filter output can be observed using an oscilloscope having a vertical deflection bandpass equal to the crystal center frequency. Most modern oscilloscopes have at least a 20 MHz -3 db upper limit with 50 mV per division sensitivity. Output amplitude would be linear, not logarithmic and a temporary scale overlay might be needed for convenience in reading decibel changes.

As usual with measurement of crystal unit characteristics, the RF source should be under 10

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[15] For more on tuning ratios and how to calculate them, see the next Chapter on Tuning of L-C circuits.

[16] Another alternative is a small motor driving a small, continuously-rotary-variable capacitor at 1200 RPM. The sweep rate would be fixed at 20 Hz. Very few rotary-control capacitors have continuous rotation so that would require some manual construction with good bearings for the capacitor rotor shaft.

mW. One milliWatt output into a 50 Ohm load (0 dbm) is equal to 223 mV RMS.

A Notation on Analysis Program Modeling

Most textbooks and *Filter Designer* show the RF source as being a constant-voltage source in series with a termination/source resistance as at left in Figure 14-11. Since nearly all active device drivers of crystal bandpass filters are really *constant-current* sources, as at right in Figure 14-11, a more realistic total circuit model should use the constant-current source. The source termination resistance would be a parallel of all active-device driver resistances and a fixed resistor. Theoretically both are equal and produce the same RF voltage at the input of a filter.

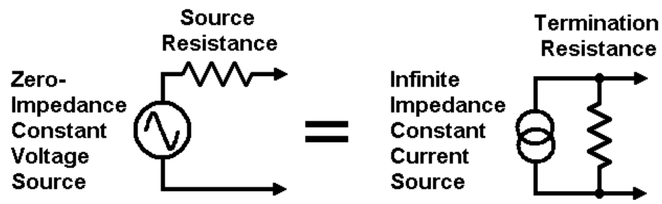

Figure 14-11 Comparison of constant-voltage versus constant-current source inputs.

Stray Circuit Capacitance at Termination Ends

Connecting any crystal bandpass filter into a circuit will place some stray circuit and driver or load capacitance in parallel with each end. This causes only a very slight change in overall filter response. On an analysis of a 7-crystal Dishal Lower Sideband filter having 1.5 KOhm terminations at 3.58 MHz center frequency, adding 4 pFd to each end degraded the performance only -0.1 db at the passband and -0.9 db at the -15 db roll-off frequencies beyond the passband. Passband response shape was essentially unchanged.

The amount of response amplitude change depends on the filter frequency and the reactance of the stray circuit capacity. At 4 MHz, the reactance of 4 pFd is only about 10 KOhms. Such causes a minor change in the terminating resistance if that is low. Terminations of 1.5 KOhm resistive would have their magnitude reduced to 1.483 KOhms. Higher resistance terminations at higher frequencies would be reduced more but those can be modeled separately (such as in *LTSpice*) if there is some concern.

Appendix 14-1

Derivation of the Equivalent Circuit Component Values

General

The general form of Chapter 7 three-component doubly-resonant L-C configurations is used here with identities at resonance involving only the imaginary parts of impedance or admittance. Since the quartz crystal unit is naturally of very high Q, the real part of impedance or admittance can be omitted without any great loss of accuracy.

Basic Resonances

Series resonance in the L_1 - C_1 path is not disturbed by any value of C_2 in parallel with it. Parallel resonance occurs above series resonance. There are only two resonance frequencies. All three reactive components are related to one another at those two resonance frequencies; once one component value is derived correctly, the other two values can be found.

Series resonance frequency is f_1. At any other frequency $k_N = \frac{f_N^2}{f_1^2}$ and:

$$X_{L1C1} = \frac{\varpi_N^2 L_1 C_1 - 1}{\varpi_N C_1} \qquad \text{At } f_1: \ \varpi_1^2 L_1 C_1 = 1 \text{ since } X_{L1C1} \text{ must go to zero.}$$

$$\text{At } f_2: \ k_2 = \frac{f_2^2}{f_1^2} \qquad f_2^2 = k_2 \cdot f_1^2 \qquad X_{L1C1} = \frac{\varpi_2^2 C_1 L_1 - 1}{\varpi_2 C_1} = \frac{k_2 \varpi_1^2 C_1 L_1 - 1}{\varpi_2 C}$$

$$\text{But, } \varpi_1^2 C_1 L_1 = 1 \text{ so } X_{L1C1} = \frac{k_2 - 1}{\varpi_2 C_1} \text{ and } B_{L1C1} = \frac{-1}{X_{L1C1}} = \frac{-\varpi_2 C_1}{k_2 - 1}$$

$$B_{C2} = \varpi_2 C_2 \qquad \text{Parallel resonance } Y_2 = B_{C2} + B_{L1C1} = \varpi_2 C_2 - \left(\frac{\varpi_2 C_1}{k_2 - 1}\right) = 0$$

$$\text{Since } Y_2 = 0 \text{ at } f_2: \ \varpi_2 C_2 = \frac{\varpi_2 C_1}{k - 1} \text{ and } \varpi_2 C_2 (k - 1) = \varpi_2 C_1 \text{ and } C_1 = C_2 (k - 1)$$

With C_1 and C_2 related by k, it is an easy progression to other identities:

$$L_1 = \frac{1}{\varpi_1^2 C_1} = \frac{k_2}{\varpi_2^2 C_1} = \frac{k_2}{\varpi_2^2 C_2 (k_2 - 1)} \qquad \text{and} \qquad k_2 = \frac{C_1 + C_2}{C_2}$$

The derivations above are, by themselves, good only for establishing equation set (7-6). What is possible now is to *add parallel capacitance* to C_p and measure the new, slightly-lower parallel resonance frequency. That still doesn't establish any solution for C_p value, but doing it at a second new frequency with even more parallel capacitance added to the test fixture will do that. Each new parallel resonance frequency will result in a new **k** value, each relating the new resonance frequency to the series resonance.

About the only difficulty there is the necessary ***low*** added capacity and that only in terms of accurate measurement. Fortunately, that is alleviated by several direct-reading capacity meters capable of resolving ± 0.1 pFd or testing it by substitution on a resonance instrument such as a Q Meter with calibrated capacitance scale. The low value of added capacitance is forced by the low C_p values of typical quartz crystal units.

At any parallel resonance frequency f_N that results in $k_N = \frac{\varpi_N^{\;2}}{\varpi_1^{\;2}} = \frac{f_N^{\;2}}{f_1^{\;2}}$, $C_1 = C_P\ (k_N - 1)$ so:

When $C_P{'} = (C_2 + C_3)$ results in new parallel resonance frequency f_3 and ϖ_3, $C_1 = C_P{'}(k_3 + 1)$

When $C_P{''} = (C_2 + C_3 + C_4)$ has another parallel resonance f_4 and ϖ_4, $C_1 = C_P{''}(k_4 + 1)$

But, C_1 hasn't changed value. As C_P increased, parallel resonance frequency had to decrease so that both C_1s have the same value. C_1 is part of the series resonant arm and the series resonance frequency does not change.

Equating the two C_1 terms: $C_P{'}(k_3 + 1) = C_P{''}(k_4 + 1)$ then, substituting for $C_P{'}$ and $C_P{''}$:

$(C_2 + C_3)(k_3 - 1) = (C_2 + C_3 + C_4)(k_4 - 1)$ Removing equals on both sides:

$C_2\,k_3 + C_3\,k_3 = C_2\,k_4 + C_3\,k_4 + C_4\,k_4 - C_4$ Solving for C_2 :

$C_2\,(k_3 - k_4) = C_3(k_4 - k_3) + C_4(k_4 - 1)$ Knowing that $k_3 \rangle k_4$:

$$C_2 = \frac{C_4\,(k_4 - 1) - C_3\,(k_3 - k_4)}{(k_3 - k_4)} = C_P$$

What was important above is divorcing the need to directly access C_2 by itself (which is physically impossible in an oscillator circuit) and relating it to the two new parallel resonance frequencies obtained through adding to the quartz crystal unit equivalent parallel capacity. Removing C_3 and C_4 will result in the *natural* equivalent parallel capacitance (C_2 by itself) and ω_2 and k_2 could be calculated from now-known C_2:

$$k_2 = \frac{C_S + C_P}{C_P} \qquad \text{and} \qquad f_2 = f_1\,\sqrt{k_2}$$

Some Comparisons to Reference Texts

In Reference [35] Worthie Doyle relates the following relationship of equivalent crystal unit values as (variables renamed to equivalent circuit in this chapter):

$$f_1 = \frac{1}{2\,\pi\,\sqrt{L_1\,C_1}} \qquad \text{and} \qquad f_2 = \frac{\sqrt{(1/C_2) + (1/C_1)}}{2\,\pi\,\sqrt{L_1}} \qquad \text{Removing the square roots:}$$

$$4\,\pi^2\,f_1^{\,2}\,L_1 = \frac{1}{C_1} = \varpi_1^{\;2}\,L_1 \quad \text{and} \quad \varpi_2^{\;2}\,L_1 = \frac{C_2 + C_1}{C_2 \cdot C_1} \quad \text{But } \varpi_2^{\;2} = k\,\varpi_1^{\;2} \text{ so:}$$

$$\frac{k}{C_1} = \frac{C_2 + C_1}{C_2 \cdot C_1} \quad \text{and} \quad k\,C_2 = C_2 + C_1 \quad \text{or} \quad C_1 = C_2\,(k - 1)$$

That equates this chapter's identity to that in [35]. This chapter concentrates on the fixed relationship of C_2 , C_1 , and L_1 so that a sequence of calculations can determine the equivalent circuit of a quartz crystal unit knowing only frequencies of resonance in a separately-calibrated test fixture capacitance and calibrated external parallel capacitor.

The test fixture described by Hayward[17] uses a switchable-gain amplifier for 0 db or +3.0 db relative gain. While that is as accurate, it does require an active-device amplifier plus RF detector; the phase shift method uses a passive test fixture and most any modern dual-trace oscilloscope.

Measuring equivalent parallel capacity of the crystal unit by itself is only going to result in an inaccuracy. A *substitution method* measurement to find the difference in parallel resonance frequencies by *two* known test fixture capacitances as described herein will be the most accurate.

The difference frequencies between series and parallel resonance is so small compared to series resonance frequency that the frequency counter must be able to resolve 1 Hz frequency. The excitation source must have an extremely fine tuning capability to avoid user frustration during test.

Measuring Equivalent Circuit Parallel Capacity By Other Means

While this can be done, it has potential inaccuracy due to two things: The capacity meter operating frequency (generally not indicating such) or, if in a bridge circuit, operating at a very low frequency (1 KHz or 10 KHz); the characteristics of the crystal unit well away from either resonance frequency. A common Q Meter instrument such as a Boonton 260A can be set to a particular known frequency (separate dial and control) and has manual adjustment for its RF measurement level (the *XQ* or *Times Q* control).

At instrument test frequencies below series resonance the reactance of the series arm becomes more and more capacitive as the frequency becomes lower. Measuring capacitance across a crystal unit becomes the sum of C_P and C_S . At frequencies above parallel resonance, C_P will predominate since the series arm becomes more and more inductive as the test frequency increases.

A Q Meter of pre-1970s design could be used directly to measure C by use of their calibrated variable capacitor dial and adjustment of the test frequency. In general, their frequency control is too coarse to permit easy tuning over a very small frequency increment; a more elaborate test fixture is needed to couple a 1 Hz resolution frequency counter for accurate frequency settings.

Test Fixture RF Power Input Maxima

Never use any RF source power level greater than manufacturer's maximums for any testing. A maximum of 1.0 V RMS (+12 dbm from a 50 Ohm generator) is high enough to permit oscilloscope viewing and would be quite acceptable for most crystal units with 1 KOhm series resistance and up to 300 or so Ohms R_S.

[17] Reference [37] page 23.

Appendix 14-2

Comparison of Amplitude versus Phase Change at Resonance

Traditional radio technology has relied on the ± 3 db amplitude change away from peak or dip in order to quantify the bandwidth of a resonant circuit. While that was good in the 1920s, modern oscilloscopes (since the 1960s) with bandpasses up to 25 MHz and dual-trace sweeps allow tracking the *phase change* that occurs when an excitation source frequency passes through resonance.[18] By using a dual-trace oscilloscope with internal triggering on Channel A, the horizontal sweep will be referenced to the excitation source regardless of the input to Channel B. Channel B can observe *both* amplitude and phase changes when the source is tuned through resonance of a circuit. At resonance of a simple circuit the phase shift goes to *zero degrees*. For a series resonant circuit (as in the crystal unit in the described test fixture in main text) the phase shift *increases with increasing frequency.* Parallel resonance will show the phase shift *decreasing* with increasing frequency. Oscilloscope observation allows setting the excitation frequency very close to zero phase shift.

As a quantifying test, the Hayward example[19] *color subcarrier crystals* with his measured equivalent circuit values was analyzed with a model of the test fixture described herein. Both amplitude and phase shift results of the analysis program were used for comparison.[20]

At series resonance maximum dip the Channel B voltage was 23.2 mV. At the +3 db points the frequencies were 3.577409 MHz and 3.577466 MHz for a delta of 57 Hz; phase shift was -44.68° to -42.94° for a total of 87.62°. That is a rate of change of about 1.5°/Hz.

At parallel resonance the phase shift was +44.48° at 3.581057 MHz and -45.78° at 3.581138 MHz, also the approximate -3 db from peak frequencies. Rate of change was about 1.11°/Hz for 90.26° shift over 81 Hz. Peak output voltage was 0.27 V.

The ability to resolve phase shifts depends on the available oscilloscope sweep times. Most have a variable sweep rate adjustment. The period of a 3.58 MHz sine is about 280 nSec so a setting of 0.1 μSec per division would allow a 360° full sinewave in 2.8 divisions; at 50 nSec/division that would show one sinewave in about 5.6 divisions or about 64° per division. Superimposing the B Channel trace on the A Channel trace would allow an optical resolution of roughly ± 2° of relative phase shift.

---

[18] AC phase shifts and their amounts were known prior to year 1900 but measurements, even semi-accurate observation was difficult for RF for the average hobbyist before about 1940. More modern oscilloscopes were available for less than $500 by about 1970, with dual-trace sweeps and internal or external sweep triggering.

[19] Reference [37]; the crystal equivalent circuit series arm had 64.47 mHy, 30.7 fFd, and 23 Ohms with 5 pFd parallel capacity. The Channel B oscilloscope probe was 10 MOhms in parallel with 10 pFd. Fixture series resistor was 1 KOhm for series resonance and 1 MOhm for parallel resonance. Excitation source impedance was terminated in 50 Ohms with 1.0 V RF RMS across the termination resistor.

[20] The analysis program used 12-decimal-digit accuracy variables in an admittance matrix solution core and would read out frequency to 6 digits, amplitude to 5 digits, and phase to 0.01°resolution. Any SPICE derivative program with the proper *wrapper* (added input/output control/display) in frequency analysis mode can do the same.

References for Chapter 14

(In general order of publication)

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[41] *Filter Design [Computer Program],* Neil Heckt, Almost All Digital Electronics (AADE) 2006. A marvelous PC-compatible computer program covering nearly all filter designs with a section on quartz crystal ladder filters with most of the references listed here attached in its Tutorial section. Tutorial has how-to-use-program instructions. Free download from http://www.aade.com in 2007.

[42] *Crystal Filter Yields Pure Sine Waves,* Intersil application note AN9815, June 1998.

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[44] A tutorial of 297 slides on all significant aspects of quartz crystal units by John R. Vig, January 2007 at http://www.ieee-uffc.org/freqcontrol/tutorials/vig3/vig3_files/frame.htm

[45] *Crystal Clear: The Struggle for Reliable Communications Technology in World War II,* Richard J. Thompson Jr., Wiley-IEEE Press, 2007, 230 pp. Good historical account of the huge effort to produce military radios and millions of quartz crystals for them back then.

Chapter 15

Variable L-C Tuning Methods

Direct or manual frequency control of L-C resonance requires a variable capacitor or inductor as part of a parallel or series tuned circuit. Indirect control of frequency from a PLL synthesizer is usually done by a voltage-controlled variable capacitance diode. In all such tuning circuits, the frequency ratio is related to the square-root of the capacitance or inductance change ratio.

General Tuning Relationship

The change of capacitance or inductance variation (one fixed, the other variable) must correspond to the inverse square of the frequency change. Consider the basic resonance equation:

$$f = \frac{1}{2\pi\sqrt{LC}} \quad \text{and} \quad f^2 = \frac{1}{4\pi^2 LC} \quad \text{From that we can say:}$$

If: f_1 = Lowest frequency of tuning range

f_2 = Highest frequency of tuning range

C_1 = Capacitve resonance value at f_1 with L fixed

C_2 = Capacitive resonance value at f_2 with L fixed

Then: $$\left(\frac{f_2}{f_1}\right)^2 = \left(\frac{C_1}{C_2}\right)$$

If C is fixed and L variable, substitute L in the above ratio. The ***ratio*** must be observed. If one tunes a frequency range of 2 to 3 MHz, the frequency change ratio os 1.5:1 and the corresponding capacitive change ratio must be 2.25:1, the square of the frequency change ratio.

Single Parallel Fixed Capacitor

Figure 15-1 shows a single fixed capacitor in parallel with a variable capacitor. That forces the total capacitance change ratio to be less than that of the variable capacitor by itself. The parallel capacitance must be found to fit a desired frequency change ratio. It may include stray capacitance in wiring or circuit board or connected component capacitance. To find the fixed value:

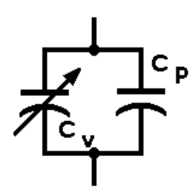

Figure 15-1

$$C_P = \frac{C_V\,(V - D)}{D - 1} \quad \text{or} \quad C_V = \frac{C_P\,(D - 1)}{V - D} \quad \text{Where:} \qquad (15\text{-}1)$$

V = Maximum to minimum variable capacitor ratio

D = Square of desired maximum to minimum frequency ratio

C_V = Minimum capacity value of variable capacitor

C_P = Fixed capacitor value, same units as C

Given a desired tuning of 12 to 15 MHz, D will equal 1.5625 (1.25 squared). The variable capacitor is 10 to 50 pFd so V equals 5.000 and C_V will be 10 pFd. The fixed parallel capacitance calculates to:

$$C_p = \frac{10\,(5.0000 - 1.5625)}{1.5625 - 1.0000} = \frac{34.3750}{0.56250} = 61.111 \text{ pFd}$$

Simple addition will find the total minimum and maximum capacitances as 71.111 to 111.111 pFd. Dividing maximum by minimum gives the total change ratio of 1.5625 which is a check on the arithmetic. Note that several decimal digits are used in calculation for accuracy; depending on the numeric value of D the denominators of (15-1) can become very small before division is done. Also, the value of C_P is in pFd because the value of C_V is also in pFd. Only ratios are involved and capacitance values can be anything as long as all are in the same units.

In actual practice C_P would probably be a trimmer variable or a trimmer plus a fixed capacitor in parallel so that alignment of the trimmer will result in the correct frequency change ratio.

Single Series Capacitor

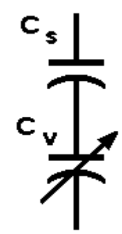

Figure 15-2

The arrangement in Figure 15-2 will result in less total capacitance since both capacitors are in series. Variables **V** and **D** and $\mathbf{C_V}$ would be the same as in (15-1) plus $\mathbf{C_S}$ and $\mathbf{C_D}$ with $\mathbf{C_D}$ referring to the minimum *total* capacitance.

$$C_S = \frac{V \cdot C_V\,(D - 1)}{V - D} \quad \text{and} \quad C_D = \frac{V \cdot C_V\,(D - 1)}{D\,(V - 1)} \qquad (15\text{-}2)$$

Using the same variable[1] and frequency range as in the previous example:

$$C_S = \frac{5 \cdot 10\,(0.5625)}{3.4375} = 8.181818 \text{ pFd} \qquad C_D = \frac{5 \cdot 10\,(0.5625)}{1.5625 \cdot 4} = 4.5 \text{ pFd}$$

---

[1] *Variable* is a short-form, generally spoken version of *variable capacitor*. That is quite common.

Total circuit capacity range would be 7.03125 to 4.500000 pFd, again a ratio of 1.5625:1. The single series capacitor has no compensation for total circuit *stray or other* capacitances so this must be used in circuit design with some caution. It helps to have a combination arrangement such as in Figure 15-3.

Series-Parallel Arrangement

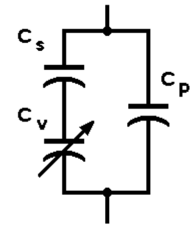

Figure 15-3

Figure 15-3 is better in a practical tuned circuit since the series capacitor could be a fixed component very close to and with short leads to the variable. That would allow the parallel capacitor to be trimmable for alignment. There are two ways to calculate values: The obvious way is to use some fixed value for C as in (15-2) and Figure 15-2, yielding a lower V' value (but higher than D) than the variable by itself. Another way, more calculation-intensive would be to use the formulas of (15-3):

Let: V = Maximum to minimum ratio of C_V
D = Square of maximum - to - minimum change ratio of desired frequency
K = An intermediate value numerically between V and D
C_V = Minimum value of variable
C_S = Fixed series capacitor
C_P = Fixed / trimmable parallel capacitor
C_T = Total circuit capacity with C_V at minimum

Then:

$$K = \frac{V(C_V + C_S)}{V \cdot C_V + C_S}$$

$$C_P = \frac{C_V \cdot C_S (K - D)}{(C_V + C_S)(D - 1)} \qquad (15\text{-}3)$$

$$C_T = \frac{[V \cdot C_V (K - 1)] + [K \cdot C_P (V - 1)]}{K(V - 1)}$$

A problem here is that there are an infinite number of values for **K.** It might be better to pick some standard value of $\mathbf{C_S}$ and calculate the **K** of the result. It could be useful to include the tolerance limits of $\mathbf{C_S}$ to determine the variations in **K** and thus find the limits of $\mathbf{C_P}$ needed for alignment.

As an example, keep the previous values of the variable and desired frequency range. Using the equation set of (15-2), a D' value of 2.7 could be chosen (it is within 5 and 1.5625). Then:

$$C_S = \frac{5 \cdot 10 \cdot (2.7 - 1)}{5 - 2.7} = \frac{50 \cdot 1.7}{2.3} = 36.957 \text{ pFd}$$

That might be suitable, in between the standard values of 33 and 39 pFd. Choosing 33 pFd, the value of **K** from equation set (15-3) is then:

$$K = \frac{5\,(10 + 33)}{5 \cdot 10 + 33} = \frac{215}{83} = 2.590361 \qquad \text{and}$$

$$C_P = \frac{10 \cdot 33 \cdot (2.590361 - 1.562500)}{(10 + 33) \cdot (1.5625 - 1)} = \frac{330 \cdot 1.027861}{24.1875} = 14.02354 \text{ pFd}$$

At least 6 fractional digits are necessary above in the terms having **K** and **D** close in value or **D** approaching unity.

Using the geometric center of the square root of **V** times **D** as **K** may be an easier way to determine $\mathbf{C_S}$. For **V** = 5 and **D** = 1.5625, this **K** would be equal to 2.795085 and the resulting $\mathbf{C_S}$ would be 40.706 pFd. That is very close to 39 pFd, a standard value. Using $\mathbf{C_S}$ as 39 pFd would require $\mathbf{C_P}$ to be 16.842 pFd.

Taking ± 5 % values of 39 pFd as tolerance limits, the calculated $\mathbf{C_P}$ values would be 17.696 pFd for the +5% limit and 15.958 pFd for the -5% limit. Those $\mathbf{C_P}$ variations turn out to be +5.1% and -5.2%, respectively.[2]

A Parallel-Series Arrangement

This remaining arrangement shown in Figure 15-4 might be useful but it has the usual series capacitor circuit inability to include compensation for stray external circuit capacity, the same as the single series capacitor case. A more practical equation set can be when $\mathbf{C_S}$ is picked as a known, standard value, and $\mathbf{C_P}$ calculated to fit:

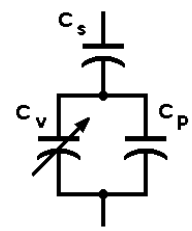

Figure 15-4

Let: D, V, C_V remain as in other equation sets, C_S a known value.
Intermediate terms of:

$$S = D - 1$$
$$M = S\,[C_S + C_V\,(V - 1)]$$
$$P = C_V\,[C_S \cdot (V - D) - (V \cdot C_V \cdot S)]$$

Then:

$$C_P = \frac{-M + \sqrt{M^2 + 4 \cdot S \cdot P}}{2 \cdot S} \qquad (15-4)$$

The parallel-series arrangement is not a practical one where a tuned frequency must be aligned precisely in any circuit having an unknown variation of stray capacity. It could be used with a large capacity change ratio variable with a small frequency change ratio, treating all three as a variable and then adding a final parallel capacitor to set the final frequency ratio.

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[2] Mathematics can get one very precise numbers using enough digits but sometimes it is easier to just subdivide the circuit into smaller parts and calculate those values. Equation sets (15.-1) and (15-2) could be used here with an intermediate V' and get the same results with less calculating effort.

Dual Conditions For Variable Inductors

General

There are few examples of variable inductors used in frequency tuning other than some AM broadcast receivers for automobiles in the 1950s to 1960s era and in Collins Radio communications equipment of the 1950s through the 1980s. The latter was principally in the fabled Collins *PTO* or Permeability Tuned Oscillator that featured linear frequency tuning, also the military HF receivers R-390 through R-392 which used the PTO ganged with octave-range RF amplifier tuning.

One variable inductor in series with a fixed inductor has a tuning range and calculations the same as a variable capacitor in parallel with a fixed capacitor. One variable inductor with a parallel fixed inductor is the same as a variable capacitor in series with a fixed capacitor. The following are the simpler duals of variable inductors included for completeness.

Single Series Fixed Inductor

This arrangement is that of Figure 15-5 and would have the following variables:

V = Ratio of maximum to minimum inductance of the variable

D = Square of ratio of maximum to minimum frequencies desired.

L_V = Minimum inductance of variable inductor

L_S = Inductance of series fixed inductor, same units as L_V

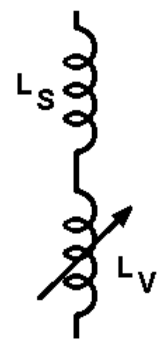

Figure 15-5

$$L_S = \frac{L_V\,(V - D)}{D - 1} \quad \text{and} \quad L_V = \frac{L_S\,(D - 1)}{V - D} \qquad (15\text{-}5)$$

Note that the above is similar to equation set (15-1).

Single Parallel Fixed Inductor

With the arrangement of Figure 15-6 use the above variable conditions but substitute L_S for L_P as the inductance of the parallel fixed inductor. Then:

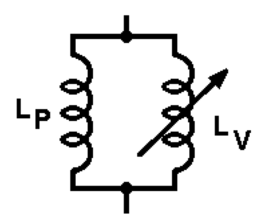

Figure 15-6

$$L_P = \frac{V\,L_V\,(D-1)}{V-D} \qquad (15\text{-}6)$$

Note that the above is similar to equation set (15-2).

SUPERHETERODYNE RECEIVER TUNING

The *Superhet*[3] was invented by Edwin Howard Armstrong in 1918 while in Paris, France, on Army duty during World War I. It would revolutionize receiver design from then on, becoming the standard architecture for receivers that would remain for at least the next 9 decades!

While this subject is in advance of the topic flow in this work because of the shift of frequency of the Local Oscillator (LO) relative to the RF Signal input tuning frequency. by the amount of the Intermediate Frequency or *IF*. The *Mixer* stage accepts the Signal input frequency (low level) and the LO injection (constant higher power) frequency to produce the sum and difference frequencies of the two. Amplitude information (modulation) is maintained in both mixing products.

Armstrong's invention allowed selectivity of the received signal to be constant, something that wasn't there before. Selectivity of an antenna input stage varies in proportion to resonance frequency; at lower frequencies a tuned frequency bandwidth is narrower than at higher frequencies. In a common AM broadcast receiver tuning over a 1:3 ratio frequency band, it would have a 3 times wider bandwidth at the higher end relative to the lower end of that band. The high end would be more susceptible to interference from other signals near the desired tuning frequency.

Armstrong's superheterodyne allow the IF stages, fixed tuned, to handle all of the selectivity in his design. Selectivity of the antenna-input tuned circuits could be poor since the IF amplifiers could be made narrow and ***constant.*** But all was not good once the more-convenient multi-gang variable capacitor using only a single manual control for tuning was invented by others. The problem was that there was no easy way to keep the LO shifted in frequency relative to the antenna input resonant tuning range; both had the same tuning range but there was a need to shift that range for the LO to maintain ***tuning tracking***.[4]

As an example of tracking, use the simple single-band AM BC receiver. Its RF Signal input would be 550 to 1650 KHz for a 3:1 frequency ratio. If its IF is 455 KHz and the LO is above the RF Signal input, then the Local Oscillator tuning range must be 1005 to 2105 KHz. The LO tuning ratio is 2.094 527:1. To use the D ratios from (15-1), the RF Signal input ratio D would require a value of 9:1 and the LO ratio D of 4.387 055 875:1. If a two-gang variable capacitor of 35 to 420 pFd each gang were used, V = 12:1 for both Signal and LO.

The circuit of Figure 15-7 shows the RF Signal input and LO resonators using only parallel capacitors to change tuning ratios. The RF Signal input (to the left) tunes 550 to 1650 KHz and variable capacitor Cv will be 95.156 844 pFd at mid-band frequency of 1100 KHz. The LO circuit (to the right) must tune 1005 to 2105 KHz and it does that. But the mid-band frequency of 1100

---

[3] Colloquial expression. The ***super*** part may have been a result of Reginald Fessenden's earlier experiments of using a small RF generator near the desired frequency to boost sensitivity. Fessenden may have accidentally created a form of ***heterodyning*** as in the later common BFO or Beat Frequency Oscillator in receivers. The heterodyning principle had not been researched much and a Spark transmitter was definitely not a Continuous Wave RF generator. That all changed with wider use of vacuum tubes later. Some surmise that Armstrong wanted all to know his receiver was superior to others, hence the addition of the ***super*** prefix.

[4] The common AM BC band receiver uses a variable dual-gang tuning capacitor but the section used for its local oscillator have different capacitances relative to the antenna-input tuning capacitance at each variable rotational position. For a single band receiver that is both good and economical but multi-band receivers need simpler solutions to maintain tracking for each band..

KHz shifts upward to 1555 KHz to match the RF Signal input mid-band frequency. That requires tuning variable Cv to be at 129.632 pFd and that is rather far from the mid-band frequency resonating capacity of 95.157 pFd. At that capacity position the LO is at 1702.23 KHz and results in a 147.23 KHz tracking error!

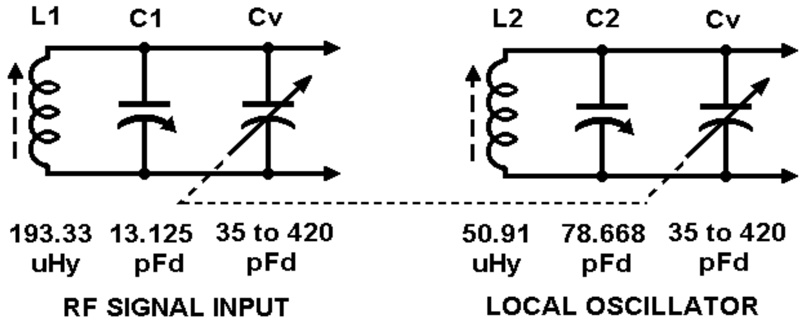

Figure 15-7 A simple ganged variable capacity tuning for 550 to 1650 KHz with IF of 455 KHz and LO above RF Signal at 1005 to 2105 KHz.

Note that each L-C circuit is at the correct end frequencies but the middle frequency is skewed off by a considerable amount. That can be corrected by using the arrangement of Figure 15-8, adding only one fixed capacitor called a ***padder***.[5]

L1, C1, and Cv are the same values in both Figures 15-7 and 15-8. The difference between the two Figures is addition of C3 to change the series capacitance of Cv and C3 and the value change of C2 so that the total capacitive change ratio of C2, C3, and Cv corresponds to the required 4.387 055 875:1 for the LO as stated before. Further, the total capacitance of the LO circuit plus the value of L2 results in a ***third zero-error frequency*** (besides the end frequencies) at mid-band. With the correct values of C2, C3, and Cv of Figure 15-8 the maximum tracking error is reduced to -5.88 to +8.70 KHz at extremes of (roughly) 1/4 and 3/4 positions of variable capacitor Cv. The comparisons are shown in Table 15-1.

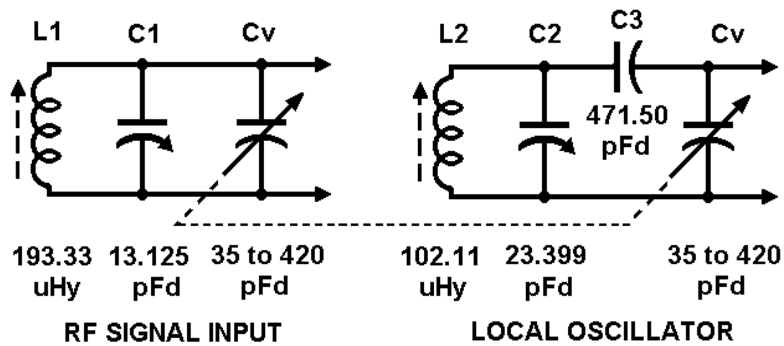

Figure 15-8 Modification of Figure 15-5 with addition of C3 to greatly reduce the *tracking error* by changing the total series capacitance of C3 and Cv.

Note that both L1 and L2 are trimmable inductors, C1 and C2 are trimmable capacitors, and C3 is a fixed value capacitor. This is for alignment. L2 of the LO circuit is adjusted for exact tuning frequency while C2 adjusted for exact tuning at band high end. That is repeated at each band end, a closing sequence. L1 is adjusted for maximum signal at its low end while C1 is adjusted for maximum signal at the high end, again a closing sequence due to interdependence of L and C for each circuit.

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[5] The author has no idea where the name ***padder*** came from; the extra capacitor for the LO has always been there and called that since 1947. The terminology may have come from differentiating the padders from the common ***trimmers*** used for compensating for vacuum tube electrode capacity tolerances.

Table 15-1
Capacities and Frequencies for Figure 15-6

Tune MHz	Cv+C1 pFd	RF Sig. KHz	Cv + C3 pFd	Cv+C3+C2 pFd	LO KHz	Track Error KHz
550	433.13	550.001	222.198	245.597	1005	0
770	202.94	778.715	141.902	165.301	1225	8.7
1100	95.16	1100.00	79.1874	102.586	1555	0
1430	51.48	1424.13	46.4122	69.8112	1885	-5.9
1650	48.13	1650.00	32.5828	55.9818	2105	0

Note: ***Cv+C1*** **column has a parallel connection total while the**
Cv + C3 **column has Cv and C3 in series connection.**
Cv+C3+C2 **has C2 in parallel with series of Cv and C3**

Track error swings positive at lower quarter of the band, swings negative at the upper quarter of the band, relative to the Local Oscillator tuning. Any tuning frequency display will take its frequency from the LO, whether mechanical dial or a frequency counter.

Table 15-1 is taken partly from the author's own program, *SuperTrak 3.* It is presented to show the general variation of capacitance value change of the L1-C1-Cv circuit versus the L2-C2-C3-Cv circuit used for the LO.

Choices of Networks

Figure 15-8 uses the same configuration of the *series-parallel* network of Figure 15-3. The *parallel-series* network of Figure 15-4 could also be used but has some disadvantages in terms of adjusting that circuit. Since multi-section variable capacitors can be manufactured quite uniform section versus section, there is less of a need for a trimmer capacitor across a variable. With the Figure 15-6 circuit, C2 can compensate for both tube and semiconductor stray and interelectrode capacity changes with replacements of tubes or semiconductors.

Accounting for *Image Frequencies*

Normal heterodyning action of frequency conversion of the sum and difference mixing products will also result in ***images*** when the IF amplifiers are fixed-tuned.

$$F_{LO} - F_{SIGNAL} = F_{IF} \quad \text{and} \quad F_{SIGNAL} - F_{LO} = F_{IF}$$

For a signal input frequency of 1200 KHz, the LO could be either 1655 KHz or 745 KHz. If the LO is 1655 KHz, the opposite mix product could be a signal of 2110 KHz coming in through the antenna. That unwanted signal frequency is called the ***image.***

Images always exist. Designers can attenuate images as they wish, sometimes with variable tuned circuits or with fixed filters as their choice. The amount of image attenuation is generally called ***image rejection*** (relative to desired signal) and generally described in decibels.

As a general rule images are found at ***twice the Intermediate Frequency*** away from the desired signal frequency. 2110 KHz is 910 KHz above 1200 KHz and 910 KHz is twice the IF.

Have the LO Above or Below the Signal Frequency?

Design practice for decades had considered the LO resonance circuit on the *high side of the Signal*, that is, the LO frequency tuning range is above the RF Signal input. This was to advantage in multi-band receivers using the same multi-gang variable capacitor for tuning. Bandswitching circuitry for that was simpler, more profitable. For monoband designs it is sometimes an advantage to have the LO resonance on the *low side of the Signal*, covering the same tuning span at a lower frequency. To do this requires an exchange of the RF Signal and LO circuits with the LO tuning the lower frequency span with a larger capacitance change ratio. The RF Signal circuit would use the C3 padder capacitor for a lesser capacitance change ratio.

For one-of-a-kind projects that only means that RF Amplifier stages would each require the padder capacitor for each RF stage tuning. There is a slight advantage in having the LO at a lower ratio since there is error from frequency drift with a lower LO tuning span.[6]

There is more ***image attenuation*** (or rejection) with the LO on the low side. That comes from the normal frequency response of resonant circuits.

With the LO on the low-side of the Signal, its images are twice-the-IF *below* the Signal. The AM BC receiver example cannot be used here. Its LO frequency range would have to tune 95 to 1195 KHz to be on the low side. That squared frequency ratio of 158:1 is impossible at achieve in either a variable capacitor or variable inductor.

Ratios of RF and LO Tuning Compared

In old general purpose *communications receivers*[7] it was common practice to use a 3-gang variable capacitor for tuning, each section or *gang* having the same ratio of maximum to minimum capacity. That can present some design difficulty due to differing frequency tuning ratios as shown in Table 15-2:

Table 15-2 Tuning Ratios of a Typical Old Multi-Band Receiver

RF Tuning	LO Tuning	Squared RF Ratio	Squared LO Ratio
0.55 - 1.65	1.005 - 2.105	9:1	4.387 045:1
1.5 - 4.5	1.955 - 4.955	9:1	6.423 826:1
4 - 12	4.455 - 12.455	9:1	7.816 135:1
10 - 30	10.455 - 30.455	9:1	8.485 338:1

Note that the RF Capacity Change Ratio is always the same. The LO Capacity Change Ratio

[6] SuperTrak 3 allows for the LO both on high-side and low-side of RF Signal frequency span.

[7] A *communications receiver* was expected to tune over all of the HF radio spectrum, was called that since, before communications satellite relays, only the HF spectrum was used for long-distance radio paths. It was also common practice to include the AM Broadcast band as the lowest band. A few receivers had a 5th band covering approximately 140 to 420 KHz

begins less than half the RF Ratio at the lowest band but, with each successive band the LO Capacity Change Ratio approaches the same value as the RF Ratio. That was as a consequence of the lessening offset between RF and LO frequencies with higher bands.

With the LO on the high side of the signal, the LO tuning ratio is always less than the RF tuning ratio. If the LO is on the low side, its tuning ratio is always larger than RF tuning ratio. Usually only capacitors, fixed or trimmable, were used to change tuning capacitor ratios.[8]

Another Condition to be Accounted For In Terms of *Gain*

The relative gain of a Mixer input from an antenna input is almost entirely dependent on resonance magnitude. Adding an RF Amplifier ahead of the Mixer would make the amplifier gain approximately *transconductance*[9] *times resonance magnitude.* Since transconductance is a known value, overall gain at any frequency is still dependent on resonant circuit impedance magnitude.

Given the same Q values of inductors and capacitors of the RF Signal input path, there is a surprising difference in overall gain for the typical *Communications Receiver* example of Table 15-2. Each band would have a 0 to 9.54 db difference between low end and high end sensitivity.

In comparison of the four bands of the example, the 2nd band would have -8.71 db gain relative to the first band, the 3rd band would have -8.52 db relative to 2nd, highest band would have -7.96 db relative to the 3rd band. Overall there could be a 25 db difference in gain over all bands, the worst gain occurring on the highest band. All gain change is dependent on resonant impedance magnitude.

One way to *spoil* such differences in resonance magnitudes is using low-Q inductors at higher bands or differences in coupling of tuned circuits at higher bands in order to (roughly) equalize the overall gain changes.[10]

Why is *Tracking Frequency Error* Considered Important?

A low RF versus. LO tracking error is necessary for the synchronously-tuned resonant circuits to not fall down much on the resonance skirt response at the Mixer signal input. In the initial example the 2-Point Tracking example had a mid-band tracking error of 147 KHz. A single resonant circuit with a Q = 55 would have about a -23 db response from that error! With the same Q but a maximum of 8 KHz tracking error with a 3-point tracking, worst-case signal magnitude would be only -2.1 db. That little amount is hardly noticeable.

Important consideration: Tracking error conditions are dependent on a ganged variable capacitor's maximum and minimum capacitance values. Those ***must be known*** to at least 3 decimals, preferably measured by a quality capacitance bridge. That achieves a baseline V ratio

---

[8] That was for parts-cost economy since trimmable inductors cost more than fixed components. The same was true for equal-gang-value variable capacitors for tuning. The 3:1 ratio on all switched bands was to fit the 9:1 capacitance change ratio of common variable capacitors.

[9] *Transconductance* in vacuum tubes is rated in ***gm*** of ***mhos*** and is the ratio of plate current change over the control grid voltage change. Since that is the inverse of Ohms, it is, naturally, rated in mhos. A typical vacuum tube gain model is transconductance times load impedance with *plate resistance* in parallel with load impedance.

[10] In truth designers of lower-cost receivers before about the 1960s were not careful in trying to equalize gain in a typical communications receiver.

upon which to base all tracking calculations.

THREE-POINT TRACKING CALCULATIONS

General

Only a few personal computer programs have been written to solve that problem. The author presents his own version here called ***SuperTrak 3.*** Written in the simpler BASIC it can achieve good accuracy and does not need the more esoteric higher-level language understanding. The basic *guts* of the 3-point tracking calculations are given following, extracted from *SuperTrak 3.* Input and output routines have been omitted for simplicity; those can be included with relatively simple BASIC commands added to the source. All reference designations of capacitors and inductors follow those given in Figure 15-8. All calculations are in 14 to 15 decimal double-precision arithmetic using Ohms, Farads, and Henries.

Format of the source code presentation is a sort of generic BASIC.[11]

Basic Calculation Source Code

```
* Superheterodyne receiver 3-point tracking program calculating all
* component values for an antenna input stage and Local Oscillator
* stage tuned by a multi-gang variable capacitor.
*                      Copyright 2010 by Leonard H. Anderson K6LHA
*
*   Assume that tuning frequency range limits are entered as FRMIN
*   and FRMAX, Intermediate frequency entered as FIF, positive if LO
*   is above Signal frequency, negative if LO is below Signal.
*   Ganged tuning capacitor limits are entered as CVMIN, CVMAX.
*
*       Program constants
    TWOPI = 6.28318530717958  * <- 2 Pi to 15 decimals
    LOGCON = 8.685885638      * <- log. conversion for voltage db
    QL = 80                   * <- Default value of inductive Q
    QC = 1500                 * <- Default value of capacitive Q
*
    FRMID = (FRMAX+FRMIN)/2   * <- set for middle of tuning band
*
*   FLMIN, FLMID, FLMAX are LO frequencies offset by IF.
*
    FLMIN = FRMIN+FIF
    FLMAX = FRMAX+FIF
    FLMID = FRMID+FIF
*
    IF FIF < 0 THEN
```

[11] It has been written in three forms of BASIC plus older MS Fortran 5.1. It has not been translated to any form of C, C+, or C++ although the simple algebraic statements can be converted to those more familiar with C languages. Line numbers have not been included and the asterisk substitutes for the single quote character used on one form of BASIC.

```
        ABOVE = -1
        FIF = ABS(FIF)
        X = FRMIN       *     Exchange FR and FL for LO below Signal
        Y = FLMIN       * With LO above RF Signal, LO capacitive change
        FLMIN = X       * ratio will be smaller than RF Signal tuning.
        FRMIN = Y       * With LO below RF Signal, LO change ratio is
        X = FRMAX       * larger than RF Signal tuning.  Since the
        Y = FLMAX       * resulting L1, C1, Cv resonance is the baseline
        FLMAX = X       * for all calculations, L1, C1, Cv is the LO
        FRMAX = Y       * tuning circuit for LO range BELOW RF Signal.
        X = FRMID       * Reversal of FR and FL entries is necessary for
        Y = FLMID       * calculation with LO below RF Signal.
        FLMID = X
        FRMID = Y
    ELSE
        ABOVE = 1
    END IF
    V = CVMAX/CVMIN       * V is the change ratio of variable capacitor
    D = (FRMAX/FRMIN)^2   * D is change ratio of lower frequency range.
    DL = (FLMAX/FLMIN)^2  * DL is change ratio of higher frequency range.
*
NEW
*   **************   Calculate ALL the RF circuit values   ***************
*
    BMIN = UNITY/CVMIN      * BMIN, BMAX are non-frequency sensitive
    BMAX = UNITY/CVMAX      * temporaries as used here
*
*     Calculate lower-frequency, higher-ratio L-C circuit (L1-C1-Cv)
*
    C1 = CVMIN*(V-D)/(D-UNITY)    ' calculate fixed parallel capacitor C1
    CRTMIN = CVMIN+C1           * <- total resonating cap. @ max. frequency
    T2 = (TWOPI*FRMAX)^2
    L1 = UNITY/(T2*CRTMIN)   * <- resonating inductance @ max. frequency
    T1 = (TWOPI*FRMID)^2
    CRTMID = UNITY/(T1*L1)    * total cap. @ mid-frequency
    CVMID = CRTMID-C1         * Variable capacity at tuning mid-point,
*                     needed as middle (third zero-track error reference)
*               and must be kept for lower-ratio (L2,C2,C3,Cv) circuit.
*
*     Calculate higher-frequency, lower-ratio L-C circuit L2, C2, C3, Cv
*
    T4 = (TWOPI*FLMAX)^2     * T4 and T5 are temporaries
    T5 = (TWOPI*FLMID)^2
*
*       C3 has a lower limit with C2 = 0.  C3 has no upper bound.
*
    C3LL = CVMAX*(DL-UNITY)/(V-DL)          * <- C3 limit if C2 = 0
    C3 = C3LL                               * <- start value of C3 iteration
*
*       Do a "search" the brute-force way by doing every value of C3 possible,
*       beginning with C3LL and incrementing it by DELTC3...calculating C2
*       along the way and checking for FLMID zero track error, stopping when
*       interim track error at FRMID changes error polarity.
*
*       Brute-Force calculation of C2 & C3 calculation by incrementing C3 0.1
```

```
*       pFd at each loop and temporary calculation of C2 with a check of zero
*       tracking error at mid-frequency.  Not elegant but effective and fast
*       enough since each loop takes about 60 microseconds execution on an
*       average personal computer with a 1.2 GHz clock rate.  Maximum value
*       of C3 value is 50 nFd.
*
    FOR T = 1 TO 500000 STEP 1
        C3 = C3 + (10^(-13))      * <- increase C3 by 0.1 pFd each iteration
        B3 = UNITY/C3             * <- temporary value
*
*             Find the series arm (C3 and Cvar) capacitance extremes
*
        CLSMIN = UNITY/(B3+BMIN)      * These are an interim set of min/maxima
        CLSMAX = UNITY/(B3+BMAX)      * to aid calculation in each iteration
        VS = CLSMAX/CLSMIN         * VS temporary ratio of C3 and Cv in series
        C2 = CLSMIN*(VS-DL)/(DL-UNITY)           * temporary value of C2
*
*             L2 can be calculated when both C2 and C3 are known
*
        L2 = UNITY/(T4*(C2+CLSMIN))
*
*             Find mid-point LO frequency to compare with RF circuit resonance
*             to compare temporary Tracking Error value.
*
        CSMID = C3*CVMID/(C3+CVMID)   * series of C3 & Cv at mid-frequency
        CLTMID = CSMID + C2           * total capacity for mid-band resonance
        FLOMID = UNITY/(TWOPI*(SQR(CLTMID*L2)))      * resonance frequency
*
        FERR = FLOMID-FIF-FRMID     * Tracking ERROR Frequency
*
*             Break out of the FOR loop if the track error polarity changes.
*             That will be very close to a value of 0 Hz.
*
        IF T>1 THEN
             IF FERR>0 AND OLDFERR<0 GOTO [BREAKOUT]
             IF FERR<0 AND OLDFERR>0 GOTO [BREAKOUT]
             OLDFERR = FERR
        END IF
    NEXT T
*
BREAKOUT
*
GOTO TABULATE
*                   [ Continue to Tabulation Display ]   \/
*
*       Routine below is to allow changing C3 value and also recal-
*       culating L2, C2; Arrives here via Change menu "P" [for Pad]
*       entry.  Since this is used after a full calculation there is
*       no search routine.  It is intended to check tolerance
*       variations of C3 as a fixed capacitor.
*
PADNEW
    BMIN = UNITY/CVMIN
    BMAX = UNITY/CVMAX
    T4 = (TWOPI*FLMAX)^2
```

```
    T5 = (TWOPI*FLMID)^2
    B3 = UNITY / C3
    CLSMIN = UNITY / (B3+BMIN)
    CLSMAX = UNITY / (B3+BMAX)
    VS = CLSMAX/CLSMIN
    C2 = CLSMIN*(VS-DL)/(DL-UNITY)
    L2 = UNITY / (T4*(C2+CLSMIN))
*
TABULATE
*         \/     Tabulation Display of all the values
```

Some Particular Parts of the Source

Default condition is to have the LO frequency range above the RF Signal input range. This makes the Signal have the lower-frequency tuning with the largest capacity change ratio. The LO then has the higher frequency tuning range but the lower capacity change ratio. To have the LO on the low side of the signal, it was a simple matter of exchanging the FR and FL prefixed variables.

As entered, a positive IF frequency denoted an LO above the signal. A negative IF denoted the LO below the signal. A flag variable, ABOVE, was kept at +1 if LO above, -1 if LO was below, then the Intermediate Frequency variable FIF was made ABSolute (always positive).[12]

The higher-capacitance-change-ratio circuit of L1-C1-Cv was the baseline to establish the mid-band variable capacitor value. This would be kept for the lower-capacitance-change-ratio circuit.

The large loop to get C3 and C2 values close to correct was necessary since there are an infinite combination of those two capacitors. Only one combination is correct for the mid-band frequency zero tracking error. A choice of 0.1 pFd increment value of C3 seemed practical enough to cover many different combinations of frequencies at differing ratios of capacitances. A ±0.1 pFd tolerance was found to be good enough for most HF tuning applications.

Using personal computers of the 2008-era, one iteration through the FOR loop would take about 41 uSec to execute 18,743 total iterations in 0.765 Seconds.[13] A solution for C3 = 50 nFd would take about 20.5 Seconds total to solve.[14] The key to getting a solution is the mid-band zero-tracking-error condition originally found by the L1-C1-Cv circuit as variable CVMID.

An Example Application Using *SuperTrak 3*

For an actual project the author wanted a single-conversion front-end to tune 3.4 to 4.1 MHz using a 3-gang 32 to 92 pFd per section ganged variable capacitor.[15] The IF was chosen to be 455

---

[12] That was primarily for the display routine not shown here.

[13] eMachines PC with 1.2 GHz clock rate, 100 MHz RAM access rate.

[14] On a practical application on HF with frequency-change-ratios within 9:1 and 500 pFd variables with ratios less than 12:1, it is unlikely that C3 padders will ever get as large as 50 nFd. The 0.1 pFd iteration increase was picked more for accuracy at rather low ratios of tuning capacitance.

[15] Receiver covered in more detail in Chapter 73.

KHz center with the LO on the low side of the signal. The program display (on one page) was:

```
            SuperTrak 3 Superheterodyne Tuning Tracking Program
               Copyright 2009 by Leonard H. Anderson, K6LHA

           Calculations for 3-Point Tracking in Superheterodynes
                  Done on Mar 06, 2010 at 17:19:58 local

 RF INPUT   GAIN  Track Error  Image Reject, db @  Local Osc. Var.Capacity
---------- ------ -----------  ------------------ ----------- -----------
3.4000 MHz  -1.6      0.0  Hz  -33.6 @ 2.4900 MHz  2.9450 MHz  92.000 pFd
3.4350 MHz  -1.5   41.373  Hz  -33.5 @ 2.5250 MHz  2.9800 MHz  87.959 pFd
3.4700 MHz  -1.4   69.069  Hz  -33.4 @ 2.5600 MHz  3.0150 MHz  84.061 pFd
3.5050 MHz  -1.3   84.870  Hz  -33.3 @ 2.5950 MHz  3.0500 MHz  80.297 pFd
3.5400 MHz  -1.2   90.493  Hz  -33.2 @ 2.6300 MHz  3.0850 MHz  76.662 pFd
3.5750 MHz  -1.1   87.597  Hz  -33.1 @ 2.6650 MHz  3.1200 MHz  73.149 pFd
3.6100 MHz  -1.1   77.782  Hz  -33.0 @ 2.7000 MHz  3.1550 MHz  69.753 pFd
3.6450 MHz  -1.0   62.597  Hz  -32.9 @ 2.7350 MHz  3.1900 MHz  66.469 pFd
3.6800 MHz  -0.9   43.539  Hz  -32.8 @ 2.7700 MHz  3.2250 MHz  63.291 pFd
3.7150 MHz  -0.8   22.057  Hz  -32.7 @ 2.8050 MHz  3.2600 MHz  60.216 pFd
3.7500 MHz  -0.7      0.0  Hz  -32.6 @ 2.8400 MHz  3.2950 MHz  57.238 pFd
3.7850 MHz  -0.6  -22.602  Hz  -32.5 @ 2.8750 MHz  3.3300 MHz  54.353 pFd
3.8200 MHz  -0.6  -43.094  Hz  -32.4 @ 2.9100 MHz  3.3650 MHz  51.557 pFd
3.8550 MHz  -0.5  -60.637  Hz  -32.3 @ 2.9450 MHz  3.4000 MHz  48.847 pFd
3.8900 MHz  -0.4  -73.980  Hz  -32.2 @ 2.9800 MHz  3.4350 MHz  46.220 pFd
3.9250 MHz  -0.3  -81.911  Hz  -32.1 @ 3.0150 MHz  3.4700 MHz  43.671 pFd
3.9600 MHz  -0.3  -83.245  Hz  -32.0 @ 3.0500 MHz  3.5050 MHz  41.197 pFd
3.9950 MHz  -0.2  -76.831  Hz  -31.9 @ 3.0850 MHz  3.5400 MHz  38.796 pFd
4.0300 MHz  -0.1  -61.545  Hz  -31.8 @ 3.1200 MHz  3.5750 MHz  36.464 pFd
4.0650 MHz   0.0  -36.292  Hz  -31.8 @ 3.1550 MHz  3.6100 MHz  34.200 pFd
4.1000 MHz   0.0      0.0  Hz  -31.7 @ 3.1900 MHz  3.6450 MHz  32.000 pFd

        RF Circuit |Z| magnitude = 24.227 KOhm at 4.1000 MHz

    ----- LO Section -----               ----- RF Section -----
        L1 = 16.900 uHy           L2 = 12.382 uHy, C3 = 1.4708 nFd
        C1 = 80.807 pFd                   C2 = 90.370 pFd

         o--o----o                         o--o--C3--o
         |  |    |       Cvar range        |  |      |
        L1  C1  Cvar     is 32.000 pFd    L2  C2    Cvar
         |  |    |       to 92.000 pFd     |  |      |
         o--o----o                         o--o------o

  Inductor Q = 80  Capacitor Q = 1500     IF = 455.00 KHz, LO Below RF
         Tracking Error extremes are -83.245  Hz to  90.493  Hz

       Total Iterations for Solution = 14408, taking 0.621 Seconds
```

Note that the tracking frequency error stays within ±100 Hz throughout the band. Given a Q of 80 at 3.4 MHz, the -3 db bandwidth will be about 43 KHz so that ±100 Hz tracking error won't be noticeable. The value of C3 is very close to 1470 pFd which can be made up of two ±5% mica capacitors, one 1000 pFd and one 470 pFd. To see the effect of making C3 nominally 1470 pFd, the tracking error resulted in -80.958 to +92.780 Hz extremes, hardly unchanged.

Five percent tolerance capacitors for C3 can vary between 1396.5 and 1543.5 pFd. Checked for tracking error on both C3 values, the error went from 0.0 Hz to +379.15 Hz at lower tolerance limits to -342.41 Hz to 0.0 Hz at higher tolerance. That turned out to be within ± 0.9 percent of being centered on the tuned carrier.

There was only a slight variation in aligned L2 and C2 values from C3 fixed at ±5% tolerance. L2 exact values would be 12.336 to 12.435 uHy and C2 from 90.800 to 89.888 pFd, hardly noticeable in normal alignment.

Worst-case image rejection was 31.7 db at a tuned frequency of 4.1 MHz (image frequency at 3.19 MHz. Using two tuned circuits, both tuned alike, antenna input to RF Amplifier input plus RF Amplifier output to Mixer signal input, overall image rejection would be 63.4 db worst-case.

The *Gain* column represents a single stage RF input gain based on the impedance magnitude change of the L2-C2-C3-Cv circuit from 4.1 MHz to 3.4 MHz. Impedance magnitude is about 24.2 KOhm between RF Amplifier output to Mixer signal input so that voltage gain would be 72.6 times (or +37.2 db) at 4.1 M Hz.[16] Any voltage gain at the antenna input connection would depend on the impedance change in that network.

An Impedance Magnitude Source Coding Addition

```
        IF ABOVE > 0 THEN
            TPIL = TWOPI*L1      * Higher-ratio lower-frequency circuit
            TPIC = TWOPI*CAPR    * Reactance multiplier
        ELSE
            TPIL = TWOPI*L2      * Lower-ratio higher-frequency circuit
            Y = ((CAPV*C3)/(CAPV+C3)) + C2
            TPIC = TWOPI * Y     * Reactance multiplier
        END IF
        OMEGLR = FTUN * TPIL
        OMEGCR = FTUN * TPIC
        GY = ((OMEGCR*OMEGLR*QL)+QC)/(OMEGLR*QL*QC)
        BY = ((OMEGCR*OMEGLR)-UNITY)/OMEGLR
        ZMAGR = UNITY / (SQR(GY^2 + BY^2))
*
        OMEGLI = FIM * TPIL
        OMEGCI = FIM * TPIC
        G2 = ((OMEGCI*OMEGLI*QL)+QC)/(OMEGLI*QL*QC)
        B2 = ((OMEGCI*OMEGLI)-UNITY)/OMEGLI
        ZMAGI = UNITY / (SQR(G2^2 + B2^2))
        Y = ZMAGI / ZMAGR     * <- ratio of image magn. to RF magn.
        X = LOGCON * LOG(Y)   *      make it in db
        Y = (INT(X*10))/10    *  only allow tenths of db
        DBIMA(AT) = Y         * relative volt. gain of image, adjusted
```

FTUN is the floating-point value in Hz for a tuning frequency tabulation display line. FIM is the image frequency in Hz. DBIMA() array holds the image rejection in db for the display. QL and QC are Q values for all inductances and all capacitances, respectively. The first conditional ***IF*** decides whether LO is above the Signal or below it depending on flag variable ABOVE.

---

[16] Voltage gain of any tube or MOS FET would be its transconductance times load impedance magnitude.

LO on High Side of RF Signal Frequency

This was easy to do with SuperTrak 3 and has the RF Signal input at the low-frequency, high-ratio side. While this is the conventional design plan, it has slightly less image rejection but no real surprises. This run was the fourth in a series of 8 calculations; using a computer saves time.

```
           SuperTrak 3 Superheterodyne Tuning Tracking Program
              Copyright 2010 by Leonard H. Anderson, K6LHA

          Calculations for 3-Point Tracking in Superheterodynes
                 Done on Mar 06, 2010 at 17:21:42 local

 RF INPUT   GAIN  Track Error  Image Reject, db @  Local Osc. Var.Capacity
---------- ------ -----------  ------------------ ----------- -----------
3.4000 MHz  -1.6       0.0  Hz  -31.2 @ 4.3100 MHz  3.8550 MHz  92.000 pFd
3.4350 MHz  -1.5    28.446  Hz  -31.1 @ 4.3450 MHz  3.8900 MHz  88.101 pFd
3.4700 MHz  -1.4    47.548  Hz  -31.0 @ 4.3800 MHz  3.9250 MHz  84.322 pFd
3.5050 MHz  -1.3    58.500  Hz  -30.9 @ 4.4150 MHz  3.9600 MHz  80.655 pFd
3.5400 MHz  -1.2    62.456  Hz  -30.8 @ 4.4500 MHz  3.9950 MHz  77.098 pFd
3.5750 MHz  -1.1    60.539  Hz  -30.8 @ 4.4850 MHz  4.0300 MHz  73.646 pFd
3.6100 MHz  -1.1    53.837  Hz  -30.7 @ 4.5200 MHz  4.0650 MHz  70.293 pFd
3.6450 MHz  -1.0    43.405  Hz  -30.6 @ 4.5550 MHz  4.1000 MHz  67.037 pFd
3.6800 MHz  -0.9    30.269  Hz  -30.5 @ 4.5900 MHz  4.1350 MHz  63.874 pFd
3.7150 MHz  -0.8    15.426  Hz  -30.5 @ 4.6250 MHz  4.1700 MHz  60.800 pFd
3.7500 MHz  -0.7       0.0  Hz  -30.4 @ 4.6600 MHz  4.2050 MHz  57.812 pFd
3.7850 MHz  -0.6   -15.522  Hz  -30.3 @ 4.6950 MHz  4.2400 MHz  54.906 pFd
3.8200 MHz  -0.6   -29.765  Hz  -30.2 @ 4.7300 MHz  4.2750 MHz  52.079 pFd
3.8550 MHz  -0.5   -41.983  Hz  -30.2 @ 4.7650 MHz  4.3100 MHz  49.329 pFd
3.8900 MHz  -0.4   -51.304  Hz  -30.1 @ 4.8000 MHz  4.3450 MHz  46.653 pFd
3.9250 MHz  -0.3   -56.875  Hz  -30.0 @ 4.8350 MHz  4.3800 MHz  44.047 pFd
3.9600 MHz  -0.3   -57.863  Hz  -30.0 @ 4.8700 MHz  4.4150 MHz  41.510 pFd
3.9950 MHz  -0.2   -53.455  Hz  -29.9 @ 4.9050 MHz  4.4500 MHz  39.039 pFd
4.0300 MHz  -0.1   -42.857  Hz  -29.8 @ 4.9400 MHz  4.4850 MHz  36.632 pFd
4.0650 MHz   0.0   -25.292  Hz  -29.8 @ 4.9750 MHz  4.5200 MHz  34.286 pFd
4.1000 MHz   0.0       0.0  Hz  -29.7 @ 5.0100 MHz  4.5550 MHz  32.000 pFd

        RF Circuit |Z| magnitude = 22.315 KOhm at 4.1000 MHz

     ----- RF Section -----                ----- LO Section -----
         L1 = 11.405 uHy           L2 = 8.5932 uHy, C3 = 1.8993 nFd
         C1 = 100.11 pFd                   C2 = 110.60 pFd

          o--o----o                          o--o--C3--o
          |  |    |        Cvar range        |  |      |
         L1  C1  Cvar     is 32.000 pFd     L2  C2    Cvar
          |  |    |       to 92.000 pFd      |  |      |
          o--o----o                          o--o------o

   Inductor Q = 80  Capacitor Q = 1500     IF = 455.00 KHz, LO Above RF
          Tracking Error extremes are -57.863  Hz to  62.456  Hz

        Total Iterations for Solution = 18743, taking 0.775 Seconds
```

Padder capacitor C3 can be changed to a parallel of an 1800 pFd and 100 pFd ±5% mica for 1900 pFd nominal. That value is so close to the calculated 1899.3 pFd that there will not be perceptible change in operation. Tracking error extremes are slightly less than for the version with LO below the Signal frequency.

Varying C3 to extremes of its 5% tolerance results in maximum track errors of +289 Hz (C3 at -5%) to -268 Hz (C3 at +5% tolerance). Those errors are also well within the expected -3 db bandwidth of the tuned circuit with a Q of 80. L2 would require value adjustments of 8.57 µHy to 8.62 µHy, C2 adjustments of 110.2 to 111.0 pFd, either of which are easily obtainable.

References for Chapter 15

[46] *Bandspreading Techniques for Resonant Circuits*, by the author, Ham Radio magazine, February 1977. This appeared to be the first instance of simple calculation by ratios to achieve a reduced total capacitance-change ratio modification.

[47] Hewlett-Packard User's Library Program 3974D, *Variable Capacitor Bandspreading,* Summer 1978 to Fall 1979, by the author, useable on an HP-67 and HP-97. This user's group library for H-P programmable calculators has been eliminated but several program collections are still available in corporate libraries. Both HP-67 and HP-97 calculators are no longer in production. The HP-35S (an anniversary model number) was in production as of the beginning of 2009.

[48] Javascript on-line calculator by Phil Rice, VK3BHR, with C source code available 2012 at http://sites.google.com/site/vk3bhr/ Nice simple Windows calculator without all the features of *SuperTrak 3.*

Chapter 16

Low-Frequency Transformers

Transformer is a generic name for magnetic devices coupling power, transforming the primary winding voltages to different voltages at secondary windings. The general nature of transformers used in power supplies and audio coupling is discussed on the basis of *rewinding* an existing *iron-core* transformer. *Low Frequencies* here refer to the range of 50 Hz through about 15 KHz.

Common Characteristics

All ***transformers*** change only ***potential*** or voltage by the ***turns ratio*** of their primary to secondary winding of a magnetically-coupled device. Because they are passive and exhibit slight loss, they also change secondary ***current*** by the inverse of their turns ratio. As a result of transformation of voltage and current they have an ***impedance ratio*** that is the ***square root*** of the turns ratio. Operating frequency ratio is largely a function of transformer ***core materials.*** Narrow-band resonant circuits can also have transformer characteristics, thus allowing control of impedance-transforming (or ***matching***) to different source or load impedances but only at a very narrow band of frequencies. General relationship of windings are:

$$\frac{e_P}{e_S} = \frac{\text{Turns}_P}{\text{Turns}_S} \qquad \text{Where subscript P = Primary, S = Secondary, } e_N \text{ = VAC RMS}$$

Turns = Number of wires each winding

and

$$\frac{i_S}{i_P} = \frac{\text{Turns}_P}{\text{Turns}_S} \qquad \text{Where i = Amperes, RMS; Turns = winding wire turns}$$

At very light current loads on Secondary winding(s)

Note that left term has inverse relationship.

Impedance ratio relationship is generally expressed mathematically as::

$$\frac{|Z_P|}{|Z_S|} = \sqrt{\left(\text{Turns}_P / \text{Turns}_S\right)} \qquad \text{and} \qquad |Z| \text{ ratio } = \sqrt{\text{Turns ratio}}$$

Impedance transformation applications have been done at audio frequencies, particularly with audio output vacuum tube stages to low-impedance speakers.

Common Core Material and Transformer Shapes

The most common core material is found in power transformers and consists of thin iron-alloy ***lamination*** pieces are assembled in a ***core stack*** as in ***E-I*** shapes as shown in Figure 16-1. ***Magnetic flux*** flows through the stack evenly in cross-section with the actual wire windings contained in a non-conductive, non-magnetic ***bobbin*** that is physically held in place by alternating E-I laminations. The bobbin is shown as white in Figure 16-1.

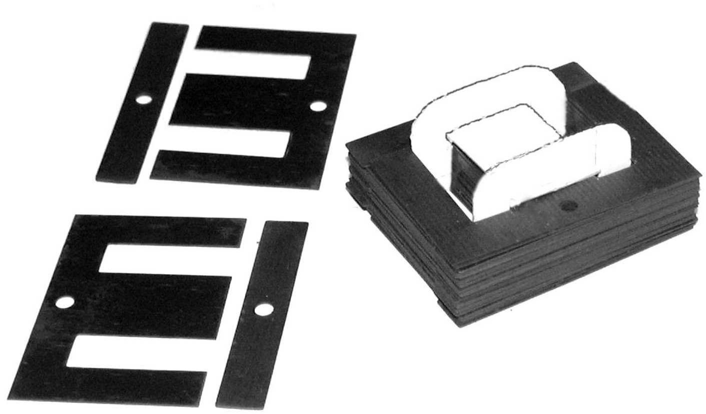

Figure 16-1 Small transformer with individual E and I laminations at left, loosely assembled with a wire winding bobbin at right. Two to four layers of laminations are stacked the same way, then alternated for the next layer group. Physically small transformers may have only two securing bolts (small holes). Larger transformers usually have four bolt holes, one at each corner.

Other shapes and cores are used but this chapter concentrates on so-called *audio range* frequencies between 50 and 15,000 Hz. Low-frequency transformers are typically used in *power supplies* to transform normally-constant AC voltage power distribution voltages into those needed for a variety of to-be-rectified-into-DC voltages for circuits or, for electronics using vacuum tubes, heater filament voltages.

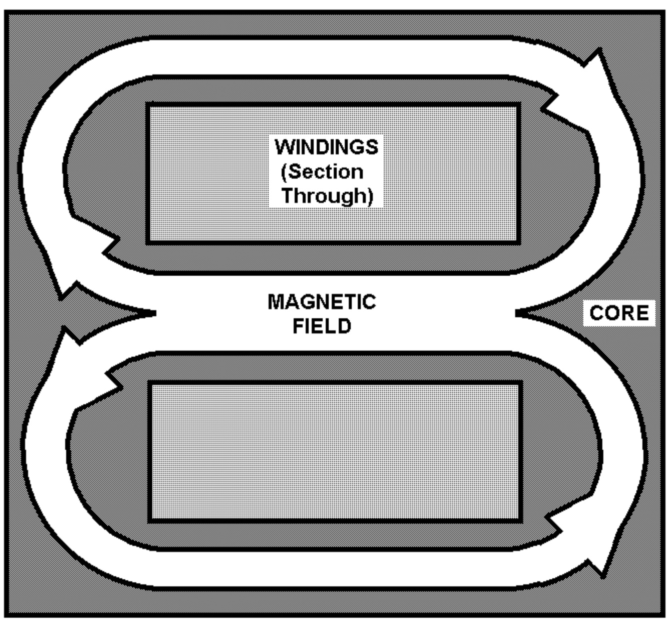

Figure 16-2 Section through E-I core to show magnetic field lines of force at one point in time.

In very old tube radios the audio amplifier stages were coupled solely by transformers. Transformers have the unique ability to ***insulate and isolate*** primaries and secondaries for ***both*** AC and DC voltages.

Approximate relationship of magnetic lines of force relative to windings is shown in Figure 16-2. Wire direction of windings in into/our-of the page. Magnetic field is in two paths, guided by the core material. It shows why the ***tongue*** E-piece is twice as wide as any other parts of the core.

Laminations are steel alloy with 1 to 4 percent silicon added to the alloy, usually around 0.018 to 0.120 inches thick, depending on general application. High-fidelity audio transformers might use thinner laminations. Power transformer applications at 50 or 60 Hz power-line frequencies can use less-costly steel.

E-I core laminations are sold in bulk. Those are not available through distributors on a pack-per-transformer basis. As such this chapter is largely about ***rewinding*** old E-I core transformers to fit a project's needs.

Table 16-1
Simplified E-I Transformer Constants

AWG	Dia. Mils	Circ.Mils	Ohm/1000Feet	Turns/Inch	Min.Insul,Inch
12	82.7	6530	1.588	10	0.010 K
14	65.9	4107	2.525	13	0.010 K
16	52.4	2583	4.016	16	0.010 K
18	40.3	1624	6.385	19	0.007 K
20	33.4	1022	10.15	24	0.005 K
22	26.6	642.4	16.14	29	0.005 K
24	21.3	404.0	25.67	35	0.0022 G
26	16.9	254.1	40.81	42	0.0022 G
28	13.5	159.8	64.90	51	0.0015 G
30	10.8	100.3	103.2	60	0.0015 G
32	7.95	63.21	164.1	70	0.0013 G
34	6.90	39.75	260.9	81	0.0013 G
36	5.50	25.00	414.8	92	0.0010 G

Notes:

MIL **= One-thousandth of an Inch**

Diameter in Mils = Diameter of single-enamel-covered coating of wire.

Circular Mils = Area of wire cross-section (determines heat rise)

Turns per Inch = Approximate for single-enamel-covered coating of wire

Minimum Insulation thickness for physical support of one layer winding, K = Kraft paper, G = Glassine.

Other Common Core Shapes

The next most common transformer shape is the ***toroidal*** (*doughnut)* shape resembling a roll of tape which it actually has, a tape of thin iron-alloy tape continuous in length. Another material is ***ferrite*** in one piece, made from a powder of iron-alloy and other metals compressed and ***sintered*** together under high heat to fuse the material together. Millions of *horizontal output transformers* from old analog TV sets were made of ferrite core material, windings wide-spaced (due to very high voltages developed) in the ***core style*** of winding on two legs of a continuous loop of ferrite material.

In toroidal formers, nearly all the magnetic field stays within the core material of the torus..

Toroidal structures are the hardest to wind individually. Every wire winding must pass through the center hole of the torus and is difficult to wind manually. Special toroid winding machines which use a ***shuttle,*** pre-loaded with wire, which goes through the torus hole and unwinds as it rotates. Such winding machines are expensive and very few hobbyists have them.

RF inductors and transformers for LF through VHF use ***powdered-iron***, but this powder is in an epoxy or similar binder the insulates most powder pieces. Ferrite is better for internal heat dissipation while powdered-iron has a better Q or Quality factor at HF and higher frequencies. Powdered iron cores and some ferrite materials are covered in the next chapter.

Class of Transformer Described Here

The simplest hobbyist design and rewinding task is to work with is the everyday 40° Celsius internal heat rise version for power and small audio transformers. Nearly all insulations will survive an internal heat up to 160° F or roughly 40° C rise over an ambient temperature typically found around vacuum tube structures.

There are several higher-class types made but those involve full enclosure or encapsulation or both, involving several different high-temperature materials. Those take much more time to construct and are seldom needed.[1] What is more needed is a source of suitable-size laminations (normally bought in bulk orders) from which to design and construct your own special transformer. Search and select an old *junker,* discard the bobbin containing all the windings, separate and clean the old laminations and your raw material is ready.

The physical size of a transformer to be re-made can be estimated by the total secondary windings' power output. Power output determines the iron core cross-section. Get one slightly larger in power output than what you want to use. This allows for some damage to laminations possible when disassembling an old transformer.

Rewinding Old Transformers - Set-Up Needs

1. A ***sturdy*** winder to hold the bobbin securely; wire tension during winding should not move anything. Not elaborate, but requires some way of counting turns even if just a stick to show rotation of one revolution. It helps to have a small clear place toward a bobbin side wall for both starting and ending of each winding.

2. A bobbin of non-conductive material such as 1/32" thick epoxy-fiberglass substrate. End pieces of bobbin must be sturdy to prevent wire build-up to bow-out sides.

3. Paper insulation for use between wire layers, cut for close fit of paper width to bobbin inside width. Kraft paper (light-medium brown color) is perhaps easiest to obtain.

4. Personal choice of thin masking tape for temporary hold-downs.

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[1] Repairing a *potted* transformer can be done but will take hours and hours of just removing the potting material, then more hours to melt that compound and pour it back in.

5. Personal choice of varnish or epoxies for final cementing of wires and insulations. Higher-volatility lacquers or similar quick-drying paints or acetone-based cements undergo greater dimensional change during curing, are not recommended from practical experience. Cyano-acrylic cements (***Crazy-Glue***) are good but need a wipe-off towel to keep it off fingers.[2]

6. ***Magnet wire***, of course. Preferably measured by micrometer for diameter over enamel insulation. Spool sizes of wire will be larger than what one is accustomed to using for point-to-point wiring.

7. A paper pad for notes. Rewinding takes a long time for beginners and a home environment is usually full of interruptions. Once a transformer bobbin is fully wound, it is quite difficult to try and rebuild it if a mistake has been made...it is much quicker to rewind another bobbin in that case. A notepad is good to jot down things when interruptions happen.

Rewinding Old Transformers - Core Lamination Removal and Cleaning

Rewinding requires full disassembly of core laminations in order to install a new bobbin. Some older transformers have been impregnated with various formulations of a varnish-like material, even including the entire core stack. Some may have just a paint over the completed transformer causing just the edges of laminations to stick together. Either condition requires a sturdy sharp knife, such a replaceable-blade utility knife to begin prying apart laminations.

Begin at one edge of a lamination stack and force the utility knife blade between the two outermost laminations. A small hammer may be needed to force the knife blade in...***Caution: Use safety goggles to protect eyes!***

Up to the first six laminations may be destroyed in this process. This is unavoidable to achieve working room for separating all the others. That lessens the core stacking dimension for the next step, ***Initial Calculations***. Damaged laminations are good for testing solvents to remove varnish.

One by one, the laminations will separate.[3] Keep the removed laminations in a fairly airtight place. A reclosable plastic kitchen bag is good for that. Laminations are not normally plated with anything to avoid oxidation and iron oxides can form, increasing the total stacking dimension. That does not change the core cross-section (iron oxides aren't magnetic) but it will make the re-stacked core physically larger.

Varnish-impregnated cores may not be useable. For those the best bet is to soak the laminations in a solvent such as acetone or toluene. Again, ***Caution: Keep adequate ventilation in soaking area.*** If a test soaking appears to work, put the kitchen to work again using a coverable glass cooking pan or tray to hold laminations with solvent. Glass is not affected by solvents. The removable cover keeps the solvent from evaporating. Glass lets one inspect the soaking process; that can take many hours.

Paint-finished-on-edges cores are salvageable although a few laminations will be damaged in

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[2] Use an absolutely ***minimal amount*** of cyano-acrylic cements since, once it sets, it is ***set*** and does not come off easily.

[3] The process only seems to take forever subjectively.

that process too. Usually, finish paint will collect along bobbin and window edges to grab edges of laminations. More work for the utility knife.

Useable, cleaned laminations may or may not have to be lightly sanded to remove any accumulated oxide. If so, use a very fine-grit ***finishing (emery) paper*** in an oscillating hand sander. Sand only enough to restore visual appearance of clean iron; check appearance on the damaged laminations for visual clue.

Re-stack the laminations without the bobbin to check the total stacking size, using the same mounting hardware. Tighten-down all holding bolts as hard as considered safe. Total stacking size should be within 5 percent of original measurements less the damaged laminations.[4] It is a good idea to keep the core together until the bobbin winding is complete. There is less chance for oxide build-up if most of the surfaces are covered.

Rewinding Design - Initial Calculations

This is done to first determine the wire total resistance, then secondly to compensate for wire resistance. The number of turns of wire per primary or secondary, each winding operating at 60 Hz, is determined from:

$$N_{TURNS} = \frac{4.939 \cdot e_{RMS}}{A_{CORE}} \qquad \text{where:} \qquad (16\text{-}1)$$

$$e_{RMS} = \text{RMS voltage} \qquad A_{CORE} = \text{Core cross-section, square inches}$$

For each winding operating at 50 Hz:[5]

$$N_{TURNS} = \frac{5.927 \cdot e_{RMS}}{A_{CORE}} \qquad (16\text{-}2)$$

$$\text{where:}$$

$$e_{RMS} = \text{RMS voltage} \qquad A_{CORE} = \text{Coss-section, square inches}$$

Note that the number of turns decreases in inverse proportion to operating frequency.[6] RMS Voltage on primary winding will include primary wire resistance loss and secondary winding(s) will include secondary wire loss in series with output.

This initial calculation finds the number of turns but not the wire size. To find wire size the ***VA*** or Volt-Ampere rating of all windings must be calculated, then the wire gauge selected for wire

[4] The 5 percent value is the author's empirical value based on practical rewindings. Primary criterion is getting the turns ratio between primary and secondary(s) correct.

[5] Both (16-1) and (16-2) derived from Reference [4} formula (14.5).

[6] A major reason for increasing aircraft AC power buss frequency to 400 Hz in the mid-1950s. Core mass and reduced wire size would make electromagnetic devices on aircraft ***lighter.***

circular mils at the RMS current at that **VA** value[7] from Table 16-1. That table has RMS current of twice the Circular Mil area for a 40° C heat rise.

Primary winding (considered as energy input winding) must include an approximation for ***Power Factor*** as well as overall efficiency.[8]

$$VA_{PRIMARY} = \frac{\text{Output Power}}{\text{Efficiency x Power Factor}} \qquad (16-3)$$

where:

$$\text{Efficiency} \sim 0.85 \qquad \text{Power Factor} \sim 0.90$$

Secondary current is the design current known prior to initial calculation. From those two currents, the largest wire gauge is selected from Table 16-1 whose ***Circular Mil*** that meets the 40° C rise at this current by:

$$\text{CircularMils} = \frac{\text{RMS Current, milliamperes}}{2} \qquad (16-4)$$

Note that this formula is expressed in **milli**Amperes, not Amperes as most others.

At this point the windings' number of turns is known, and the wire gauge is known, so the total winding resistance can be calculated from:

$$R_W = \frac{N \cdot R_{TF} \cdot \left[B + C + \left(pi \cdot D\right)\right]}{6000} \qquad (16-5)$$

where:

N = Number of turns $\qquad R_{TF}$ = Wire resistance per 1000 feet

B = Width of Tongue, inches $\qquad C$ = Depth of lamination stack, inches

D = Distance from core to center of first layer, any winding

Wire resistance per thousand feet is a standard measurement and found on all wire tables. Dimension D is from the nearest core lamination through bobbin to center of the first layer of any winding; an initial trial can begin with dimension D at the middle of the Window aperture and *fine-tune* it later when more information is known about winding patterns.

With each winding's resistance approximately known, the voltage drop from wire resistance at maximum current can be calculated. Keep that data on the notepad. For the moment, concentrate on the Window opening. This example has a Tongue that is 1.50 inches long. Let the Window

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[7] ***Volt-Ampere*** calculations are done the same as Watts. Transformer people apparently want to separate Watts of losses from **V-A** of electrical energy applied. ***Circular Mil*** is the cross-sectional area of wire given in thousandths of an inch. Table 16-1 has twice the Circular Mils per gauge to ease wire selection for typical 40° C heat rise in wires.

[8] ***Power Factor*** is the phase shift between AC RMS voltage and RMS current. The overall ***efficiency*** of small transformers of about 50 to 150 Watts is approximately 85 percent. Apparently that efficiency is derived from many transformer designs done over many years by transformer designer-manufacturers.

width fill space occupy 0.9 of the actual Window width for wire winding. The remainder os that space is used for unkempt windings plus insulation covering wires coming from the end of a winding to the beginning of the next one. Actual layer winding length would have a target of 1.35 inches for 90 percent of available space.

Multiply the Turns Per Inch (Table 16-4) for the selected wire gauge by the 90th percentile of the Turns Per Inch table value. That will be the number of turns per layer of that winding. Divide it into the initial Number of Turns for that core cross-section to find all the layers of that one winding. Do that for all other windings.

Get the magnet wire *diameter* in inches and note that on the notepad. Multiply each wire stack by the number of layers. Select a safe voltage breakdown between layers and between isolated windings and choose a safe material for that. Each insulator tape accompanies each wire layer. Add those up. Add a primary to secondary winding insulator. Add the thickness of the center spool of the bobbin material. Add some insulation over the outside surface of the bobbin, that side which will be exposed to the outside.

Total dimension of wire diameters and insulation pieces ***must be less than the maximum window opening.*** If it is, fine, continue to fine-tuning the secondary output voltage. If not, then recheck calculations or choose a core lamination with a larger Window opening.

Insulation Between Windings and Layers

Hobbyists accustomed to point-to-point wiring are sometimes amazed at the ***thinness*** of interior insulation in transformers. That is a misinterpretation of not realizing that point-to-point wiring has to flex and rub up against mechanical structures in chassis and boxes, therefore insulating jackets are made much thicker than needed as a safety factor.

Various paper products are used in transformers for inter-winding and inter-layer insulation. Table 16-2 has an abbreviated list of some paper types and their ***peak-to-peak*** voltage breakdowns.[9] All such papers have good strength along the plane of the paper. This is necessary to provide a flat surface for following layers of round wires without having wires underneath acting as guides for upper layers. Each layer should be wound on as flat a surface as possible to avoid distortion of a whole winding as it is built up.

Don't be afraid of using ***more*** insulation between windings and layers than tables suggest unless the Window gets too full. It will be near-impossible to add more insulation once a bobbin is fully wound.

Polyethylene tapes aren't recommended due to their deformability under pressure during winding. Polyamide tapes (Kapton) are excellent. Mylar tapes are generally good but some *Mylar-like* tapes may deform under heat. Check Mylar for that with a soldering iron. Teflon tapes are good to high temperatures but can ***cold-flow*** over time and deform. Widths narrower than Window length will require overlaps to keep withstanding voltage up but that results in lumpy surfaces on which to wind the next layer. Polymer packing tapes with embedded fiberglass threads will cause the same uneven surface for a winding. Packing tapes are very strong and could be used as the top-most

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[9] Taken from many different sources of ***RMS*** voltage breakdown listings. Conversion to ***peak-to-peak*** was considered better since AC power line inputs aren't all nice sinewaves and differing voltage regulating circuits may have definite non-sinusoidal waveform-changes. Divide by 2.8 to get RMS Voltage breakdown. Some more information on common transformer papers are given in Appendix 16-1.

insulation on a bobbin's winding.

AN EXAMPLE TRANSFORMER DESIGN

A single-secondary transformer of 18 VAC RMS at 2.0 A output was desired with primary working from 115 VAC RMS 60 Hz power line. An old *6.3 VAC at (about) 6.0 A* was available to sacrifice its iron core for this cause.[10] VA rating would have to be 18 x 2 = 36 VA (without core or wire losses). A drawing of the measured E lamination is shown in Figure 16-2. Based on stacking height of 1 1/16 inch and a probable loss of 4 laminations in disassembly, the core cross-section would be 1.0 square-inches, stacking height reduced to exactly 1.0 inches..

Using (16-1) the number of initial primary winding turns is 568.0 turns, the number of initial secondary turns is 88.90. Using Table 16-1, known secondary current of 2 A results in #20 AWG. For the primary winding, (16-2) is used for initial current in primary:

$$VA_{PRIMARY} = \frac{36.0}{0.85 \text{ x } 0.9} = 47.06 \qquad i_{PRIMARY} = \frac{47.06}{115.0} = 0.4092 \text{ A}$$

Since half the primary current is 204.6 Circular Mils, #26 AWG is chosen (254.1 Mils, a little oversize). That *double-enamel covered*[11] wire has a diameter of 0.0180 inches and 40.81 Ohms per 1000 feet resistance. First of all, there is the possibility of winding 52 turns per inch or 78 turns on a 1.50 inch long Tongue. Dropping that to 90 percent or 70 turns per 1.50 inch allows some slack. Dividing 70 into 568 turns for the primary yields 8.114 layers. The turns per 1.50 inch could be compressed slightly to 72 turns per layer for 8 total layers in the primary.

Resistance of the primary winding uses (16-5) and yields an initial resistance value of:

$$\text{Dimension B} = \text{C} = 1.00 \text{ inch, D} = 0.25 \text{ inch}$$

$$R_{TF} = \text{Ohms / 1000 feet of \#26 AWG} = 40.81 \text{ Ohms}$$

$$R_{W\text{-}PRI} = \frac{N \cdot R_{TF} \, [B + C + (pi \cdot D)]}{6000} = \frac{568 \cdot 40.81 \cdot [1 + 1 + (3.1416 \cdot 0.25)]}{6000} = 10.76$$

Resistance of the secondary winding will also use (16-5) and has an initial resistance value of:

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[10] Obtained in a trade, was probably purchased in the late 1950s by first owner. One of the previous owners had spray-painted the entire transformer black so markings were obscured. Approximate VA rating determined by lamination size in an old catalog. It was not tested in operation since only the laminations were desired.

[11] There is no easy way to tell *single-covered* from *double-covered* so the author merely uses the larger diameter and hopes for the best on assembly time. Enamel covering has been colored a dark red-brown for over half a century and that is expected to remain for another half century.

$$R_{TF} = 8.051 \text{ Ohms } / \text{ } 1000 \text{ feet for \#20 AWG}$$

$$R_{W\text{-}SEC} = \frac{89 \cdot 8.051 \cdot [1+1+(3.1416 \cdot 0.25)]}{6000} = \frac{89 \cdot 8.051 \cdot 2.785}{6000} = 0.3326 \text{ Ohms}$$

Secondary has less wire on it and, although a larger wire size, it can be wound at 36 to 37 turns per layer for 2 1/2 layers, 3 full layers accounting for insulation between layers.

One problem is that the primary winding has a fair amount of series resistance for this small a transformer of 36 Watts total power transfer. Copper wire resistance will waste 4.4 Watts in heat at 0.409 A RMS primary current. That can be reduced by using larger wire: #24 will result in 6.768 Ohms series resistance and 2.768 VAC RMS drop; #22 has 4.255 Ohms and 1.740 VAC drop. Number of Turns per Layer may also need rechecking. Core Window build-up will depend on the number of winding layers.

To start a build-up check, assume there are 8 layers of primary winding at 115 VAC RMS nominal across all and 3 layers of secondary with 18 VAC RMS nominal across all of those. Between-layer insulation must withstand 40.7 V peak-to-peak on the primary layers, 17.0 V peak-to-peak on the secondary layers. Either 0.005 inch kraft paper or 20-pound bond paper (0.0038 inch thick) can handle that. Kraft paper is slightly cheaper and masking tape sticks to it slightly better for temporary holding. Each layer's insulation can then be 5-thousandths kraft.

Double-enamel covered #26 wire is 0.018 inch diameter, #20 is 0.0346 inches diameter. With 8 layers of the primary, wire build-up alone for that is 0.144 inches. Secondaries 3 layers will need 0.119 inches of #20 with single thickness of insulation.

Insulation between primary and secondary adjacent ends should be at least 115+18 = 133 VAC RMS plus 10 percent more to compensate for high AC line voltage or 146 VAC RMS. To allow for peak voltages, multiply that by 2.83 to get 413 Volts. Three layers of 5-thousandths kraft paper would handle that although four layers were used as a safety factor.

The bobbin is made from 0.032" epoxy-fiberglass substrate edge-glued with epoxy, allowance made for 0.033 inches final thickness..

Primary wire layer build-up = 0.144
Primary insulation layer build-up = 0.040
Primary-Secondary insulation layer (4) = 0.020
0.204 inches for all of Primary

Secondary wire layer build-up = 0.104
Secondary insulation layer build-up = 0.015
Bobbin hub thickness = 0.033
0.152 inches for all of Secondary

Total = 0.356 [Window is 0.500 inches maximum]

There's at least 0.144 inches of unused open space at the Window top. That is quite safe but the primary winding size could be made larger to reduce the primary wire resistance.

Changing to #22 AWG would have its diameter at 0.0177 inches, resistance at 16.14 Ohms per 1000 feet. Using (16-5) primary wire resistance would be:

$$R_{W\text{-}PRI} = \frac{568 \cdot 16.14 \cdot 2.785}{6000} = 4.255 \text{ Ohms}$$

However, #22 would hold only about 45 turns per layer and 568 turns are needed. That will take 12.6 layers. An even 44 turns per layer (except last) can hold each layer smooth for 568 total turns. This estimate with #22 will take 0.360 inches for wire build-up and 0.065 for insulation layer build-up or 0.425 inches for the new or about 0.100 inches higher than Window opening.

Another trial can try using #24 AWG wire with 0.0224 inches diameter, 25.67 Ohms per 1000 feet, and about 57 turns per layer. That could handle 568 turns in 10 layers. Build-up is:

Primary wire layer build-up = 0.224
Primary insulation layer build-up = 0.050
Primary-Secondary insulation layer (4) = 0.020
0.294 inches for all of Primary

Secondary remains the same as original and is 0.152 inches. Total build-up is then 0.446 inches. There is only 0.054 inches left open so winding must be done with taught wire winding.

Total primary winding resistance has changed to:

$$R_{W\text{-}PRI} = \frac{568 \cdot 25.67 \cdot 2.785}{6000} = 6.768 \text{ Ohms}$$

Since there are no spare laminations, no further changes can be done with a core stacking area increase. Normally that could be done to increase the core cross-section and thus reduce the number of turns in both primary and secondary windings.

Secondary induced voltage at ***no load*** will be the excitation voltage (115 VAC nominal) divided by the ***turns ratio*** of primary to secondary windings, in this case 568:89 = 6.3820. 115 V divided by 6.3820 equals 18.019 VAC RMS. That is within 0.108 percent of 18.0 and is quite useable to prove both winding's turns were correct.

With ***full load*** there is that copper loss resistance to contend with on both windings. The primary has a 2.768 Volt drop with 0.409 A RMS current and nominal primary voltage will be 112.23 VAC rather than 115 VAC. Turns ratio hasn't changed but induced secondary winding voltage is 17.59 VAC. Further, secondary voltage drop of 0.6652 VAC from its 0.3326 Ohm wire resistance at 2.0 A current will take that down to 16.92 VAC at ***full load.***

That seemingly over-large voltage drop is a consequence of trying to get the transformer compact but also assuming it will have a fairly constant load resistance. An equalizing influence is to assume some target or nominal load resistance exists and ***design to that target***. The design part is to go back through things again and choose a different turns ratio between primary and secondary. For example, a slight reduction in primary winding turns is still within the acceptable flux density of the core yet will decrease the turns ratio and increase secondary voltage under some load.

The only way to make a minimal voltage change between no-load and full-load is to use a transformer body with laminations nearly twice as large in area. That allows a larger Window, thus a larger wire gauge and reduces the wire resistance. If higher permeability core material is available that can reduce turns but more comprehensive transformer texts must be consulted for effects of higher-permeability iron laminations.

What the Author Finally Did

The original project need was for a somewhat odd-value of output voltage for a solid-state regulated supply. A large variable autotransformer (to adjust AC line voltage) and a heavy-duty 1:1 isolation transformer proved out the full-wave rectifier and voltage regulator on the bench. The salvage operation of rewinding an old filament transformer was done in objection to a $24 cost of a new 18 V, 2 A transformer. After proving the design was viable and could be made, the author came across an imported transformer of only $8 cost from a well-known distributor. It fit and rewinding was cancelled.

Since electronics, especially consumer electronics, use fewer and fewer *heavy iron* transformers in this new millennium, they have become rare and quite costly for some of the old multiple-secondary-winding ***plate-filament*** supply combinations common in the vacuum tube era.[12]

Another Transformer Winding Example

This was a multiple-secondary winding intended for use in a later project for all the various power supplies. It used a well-oversized core obtained from the scrap-out pile at an employer many, many years ago. Width and depth of the core laminations was 3.75 by 4.50 inches with the window opening 0.750 by 2.250 inches. The core stack is variable and could be 1.50 inches high; although enough laminations were available to stack to 2.00 inches with quite rusty laminations.[13]. Secondary AC RMS voltages and currents would be approximately:

1: 130 VAC at 60 mA = 7.8 W (plate-screen supply)
2: 20 VAC at 1.5 A = 30 W (filament supply with generous peripheral supply capability
3: 16 VAC at 0.2 A = 3.2 W (-15 VDC regulated, -5 VDC regulated detector bias)
4: 7 VAC at 0.6 A = 4.2 W (+5 to +6 VDC supply for digital logic, other things)
5: 16 VAC at 0.2 A = 3.2 W (+12 VDC supply for VCOs or any similar device added)

Secondary winding #5 might not ever be used but that project is experimental and some other future circuits might be added.

Total maximum load wattage is 48.4. From (16-3) the maximum primary VA is 63.3. From (16-4) the first try at wire sizes for required secondary currents are:

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[12] A check of *Hammond Transformer* catalog in Canada in early 2010 yielded single-piece costs of $60 to $120 each for models that cost only one-tenth the price of those available elsewhere five decades before.

[13] An unforeseen problem was how to take rust off about 84 laminations, rusted on each side. A solution for that was two packets of steel wool and lots of ***elbow-grease*** on a scrap of stone counter-top. That scrap had a flat top and would not nick any laminations. A little bit of 10W-30 automobile engine oil dripped onto the steel wool helped the rust scrub-off effort. Laminations would have to be cleaned afterwards. Paint thinner was good for that.

	Current ÷ 2	**Wire, AWG**	**Circ.Mils of Wire**
Primary	275	#24	404
Secondary #1	**30**	**#32**	**63.7**
Secondary #2	**750**	**#20**	**1022**
Secondary #3	**100**	**#28**	**160**
Secondary #4	**300**	**#24**	**404**
Secondary #5	**100**	**#28**	**160**

Note that wire size is larger than required. Wire diameters are fixed so it is best to go over the minimum required. Should be no problem with this core size.

Given a core stack height of 1.50 inches and a tongue width of 1.50 inches, the core cross section is then 2.25 square inches. Using (16-1) the number of Primary turns should be 252 turns. Output voltages will reflect on the turns ratios of secondaries relative to the primary turns at the nominal AC power line input of 115 VAC RMS. First, though, is a check of power absorbed in the primary winding wire resistance at full secondary current load along with primary voltage drop.

From (16-1) the number of turns on the primary winding is 252 for 115 VAC at 60 Hz. From (16-5) the wire resistance would be 1.53 Ohms. At maximum secondary current load the voltage drop from 0.55 A RMS is 0.84 VAC RMS and wire resistance loss would dissipate about 0.46 W.

Secondary winding voltages under light load current would be equal to the turns ratios of secondary windings to the 252 turn primary winding. Each secondary has its own wire resistance and all of that information can be tabulated as follows:

Secondary	**VAC**	**Turns Ratio**	**Turns**	**AWG**	**Ω/1000**	**Winding Resistance**
1	**130**	**1.130:1**	**285**	**#32**	**164.1**	**27.99**
2	**20**	**3.750:1**	**44**	**#20**	**10.15**	**0.268**
3 & 5	**16**	**7.188:1**	**35**	**#28**	**64.90**	**1.359**
4	**7**	**16.43:1**	**16**	**#24**	**25.67**	**0.246**
Primary	**115**	**-**	**252**	**#24**	**25.67**	**4.181**

See Table 16-1 for Ohms per 1000-feet for the different wire gauges, formula (16-5) for wire resistance. An approximate dimension of D in (16-5) is about 0.188 inches out from the core.

Secondary full current load voltage drop from wire resistance can be tabulated as:

Secondary	**Load current**	**Winding Resistance**	**RMS Voltage Drop**
1	**60 mA**	**27.99**	**1.68**
2	**1.5 A**	**0.268**	**0.401**
3 & 5	**0.2 A**	**1.359**	**0.272**
4	**0.6 A**	**0.246**	**0.148**

At full secondary loading the primary winding resistance voltage drop can subtract from the primary applied AC voltage. In line with the *Turns Radio* tabulation, it will reduce each secondary AC voltage by that winding's own winding resistance voltage drop.[14] The final output voltages at -10% AC mains voltage input should still be high enough to fit within the *headroom* of all series voltage regulators.

[14] This is a gross approximation, not borne out by theory.

Some Tips on Bobbin Construction

Any thin, ***rigid*** material that is both non-magnetic and non-conductive will do for the bobbin. Epoxy-fiberglass substrate material for PCBs works out well for this as long as it does not have any foil on it.[15] As a jig for cementing the edges with more epoxy, the core laminations could be stacked as in assembly and then wound with white ***plumbers Teflon tape.*** That tape is very thin and will not adhere to any epoxy.

Rigidity is necessary for end pieces. Windings will tend to push sideways during winding pressure. The center piece gets pressure too, but a square piece of wood of the winder's center can keep the wire from distorting the bobbin center piece.

Epoxy-fiberglass material can be scored, then snapped apart for long, same-width strops, then cut to final length with a *Dremel* ***cut-off wheel.***[16] That cut-off wheel is also good for cutting out the rectangular hole in bobbin end-pieces.

Bobbin Winding Stacking Determination

Individual windings can be totalized first to test for fitting into the Window, approximately 2.06 inches long by 0.656 inches high, accounting for the bobbin dimensions using 1/32 inch thickness epoxy-fiberglass stock. Those will be as follows:

Secondary	Turns	Wire AWG	TPI	Wire Dia.	Layers	Practical Layers	Total	
1	285	32	70	0.008	1.976	2	0.017	
2	44	20	24	0.033	0.890	1	0.034	
3	35	28	51	0.014	0.333	1	0.015	①
4	16	24	35	0.021	0.222	1	0.022	
5	35	28	51	0.014	0.333	1	0.015	①
Primary (115 VAC)	252	24	35	0.021	3.495	4	0.100	
						Total	0.203	

① Secondary 3 & 5 both wound on same layer.

There will be 0.485 inches for inter-winding insulation and layer insulation, sufficient room in this large core size. An outer insulation material (for esthetic purposes) can be included. Note that some layer thicknesses of wire have an additional buffer for wire diameter tolerances and, perhaps, not-quite-tight winding by beginning hobbyists.

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[15] Foil on one side only could be used for a secondary having only a few turns on it. A problem having to use it as the first winding next to the core tongue and then doing good soldering to traces that are slightly skewed at an angle on the PCB.

[16] A Dremel rotary hand tool is probably the easiest to get at do-it-yourself stores and chains in the USA. There are several different brands on the market. Various tool bitts are available. A cut-off wheel is like a miniature grinding wheel and originally intended to cut screws or bar stock of small sizes.

Winding and Inter-Layer Insulation

Going by a rule-of-thumb of 0.020 inches thickness of kraft paper for 80 VAC peak-to-peak insulation, the individual windings' total build-up can be calculated approximately.

Secondary #1 (130 VAC), winding start is directly on the epoxy-fiberglass center piece, needing no extra kraft insulation. It is two wire layers with 0.050 inches kraft paper between, then 0.100 inches of kraft paper over that. Total insulation is 0.150 inches.

Secondary #2 (20 VAC) is one layer with 0.050 inch kraft paper on top of it. Potential between #2 and #3/#5 is relatively low. Total insulation there is 0.050 inches..

Secondaries #3 and #5 can be together on one layer, separated by at least 0.250 inches end to end between them, again with 0.050 inches of kraft paper over them. Insulation is 0.050 inches.

Secondary #4 (7 VAC) is one layer with 0.100 inches kraft paper over it. Primary winding will be on top of this winding's insulation and is at 115 VAC potential, therefore thicker. Total insulation for #4 is 0.100 inches.

Total of all windings and insulations is 0.203 + 0.350Combined secondary winding/insulation thicknesses will be 0.438 inches.

The single primary winding will require a minimum of 3 1/2 layers so 4 total layers will be expected. Since successive layers have about 1/4 of the 115 VAC input, inter-winding insulation need only be about 30 VAC with 0.020 inches kraft paper between them. Top-most insulation layer should be 0.100 inches kraft for safety from component conductors external to transformer. Total insulation for primary is 0.160 inches.

Total insulation thickness and all windings is 0.410 + 0.203 = 0.613 inches. Allowing for 1/16 inch thickness of bobbin center piece, there will be a 0.074 inch gap at the outside of the core window.

To a transformer manufacturer the insulation will probably be excessive. This is not a design for manufacturing, just a one-of-a-kind hobby project. Manufacturers have the capability to use ***odd*** AWG wire sizes and have their own techniques to make things slightly smaller. As a hobby project, this will take a fair amount of time to complete and lots of different *fudge factors* have been added on dimensions to make certain of safety.[17]

Split-Voltage Primary for Either 115 or 230 VAC

[17] A ***fudge factor*** is colloquial for values or dimensions that are larger (or perhaps smaller) than required to allow for non-optimum construction conditions.

This has become popular in this new millennium on finished goods, primarily to sell in countries with different AC mains voltages. The method is quite simple but requires a very accurate count of primary winding turns. All that is required are ***two identical primary windings*** configured as shown in Figure 16-3.

For the parallel connection (lower AC voltage) it is absolutely essential that the ***polarity dots*** marking the correct start or end of each winding is observed. If one winding's polarity is reversed, the two windings become essentially a short circuit. It is also essential that the ***same number of turns*** exist on each primary winding for a parallel connection.

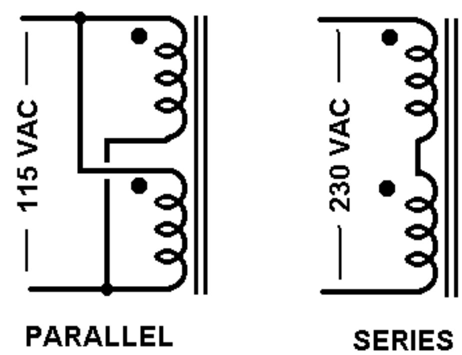

Figure 16-3 Connections for dual primary voltage input using two primary windings. Dots mark polarity of AC phase of windings.

With series connection, wrong polarity will result in nearly zero AC volts on any secondary. Phase of each seriesed primary winding will buck the other and the result is not catastrophic, just a high-impedance connection to AC mains input.

Winding a double primary is best done with ***two*** wire sources, playing out both of them onto a bobbin on a winder that has a turns counter of some sort (even if that is only a wood stick). Wire can be tensioned by hand and a rag (to avoid scratching enamel insulation) onto two separated bobbin areas. One must allow some distance between the two windings to allow for the higher voltages between the two, say approximately 0.25 inches to allow for a 230 VAC input and 650 Volts peak-to-peak possible potential between winding ends.

Shorted-Turn Effect

Any conductor, either a winding or a mechanical support, that is within the magnetic field of the core that has a shorted turn is the same as having a low-voltage secondary with a zero-Ohms load. The resulting current could be tens or hundreds of Amperes and would likely destroy the winding from excessive heating.[18]

A common hobbyist mistake is mounting toroidal power transformers with a large bolt and oversize metal washer through the center hole. ***Do not have any conductor contact the mounting bolt on the outside of the toroid to the mounting surface.*** Such a condition would be the equivalent of a one-turn winding terminated in a short circuit.

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[18] The bigger the transformer, the bigger Wattage it can handle. AC power source impedance is very low so there is little effect on lowering the AC input voltage.

Appendix 16-1

Transformer Paper Products

Transformer components have always been special-order and in bulk. Paper tapes of the widths useful in transformers are not stocked at distributors. There are too many width differences and thicknesses to make such stocking profitable. Bulk supplies in small sizes can be obtained from paper products distributors, photographic material suppliers, office supply chains. None of them are cheap in small quantities but neither does one have to buy a minimum-order of a one metric tonne roll of specialty paper.

Glassine - Thinnest paper. Is ***super-calendared*** so that appearance varies from transparent to translucent, seen in photo envelopes (as protectors) and as envelope address windows. Sometimes it is confused with cellophane (a polymer) which is definitely transparent. Calendering refers to paper run through very smooth rollers while still moist to set the finish of paper. Super-calendaring means running through rollers many times. Glassine is pure cellulose with fibers aligned and flat. It is generally water-proof and has been used in food product wrapping. Thicknesses vary from 0.0007 inch to 0.005 inch (140 to 700 VAC peak-to-peak), Glassine is not very porous but allows varnishes to stick to its surface.

Kraft - Cheapest, most widely-used. Name comes from the production process of removing lignin from cellulose fibers of trees. Light-brown to medium-brown color, has been used as carrying bags at food supermarkets and anywhere a very economical bag is needed. Basic kraft paper is used in corrugated cardboard boxes. Thicknesses range from 0.002 to 0.016 inch although production standards allow greatest tolerance range. Withstanding voltage is 80 VAC peak-to-peak for 20 mils to 450 VAC peak-to-peak for 160 mils. Kraft stock will absorb liquids somewhat (as in gummed tape for old package mailing).

Bond - Not normally used inside commercial transformers. Easily obtainable as common 8 1/2 x 11 inch letter stock, sold by ***pound weight*** as stationery. ***20 pound bond***, the most common as printer or copy paper is about 0.004 inch thickness and withstands approximately 140 Volts peak-to-peak.

Varnish Paper - Almost extinct in the new millennium but mentioned in old transformer texts. It is cured, varnished paper or cloth for higher voltage insulation from 0.002 to 0.010 inches thick.

Fish Paper - An old semi-colloquial term for translucent glassine mentioned in old transformer texts, also almost extinct by about 1980s.

Measuring Characteristics of Unknown Insulation

Thickness may be measured by taking 10 to 30 sacrificial pieces, stacking them, measuring the total thickness with a resolution of ± 0.002 inch. Dividing by the number of pieces yields the thickness to fair accuracy.

Withstanding voltage can be done on several pieces individually. A quick set-up uses an old discarded ***plate supply*** transformer, a microammeter, and an insulated clamp with conductive contacts on its jaws. A series resistor with a value to drop the total transformer voltage at maximum current rating of the microammeter will limit the amount of power applied at breakdown.

A *Variac* or *Powerstat*[19] variable auto-transformer can be used to adjust the discarded plate supply primary voltage. Diode, capacitor and bleeder resistor form the normal rectifier assembly. Bleeder is to discharge the capacitor when turned off.

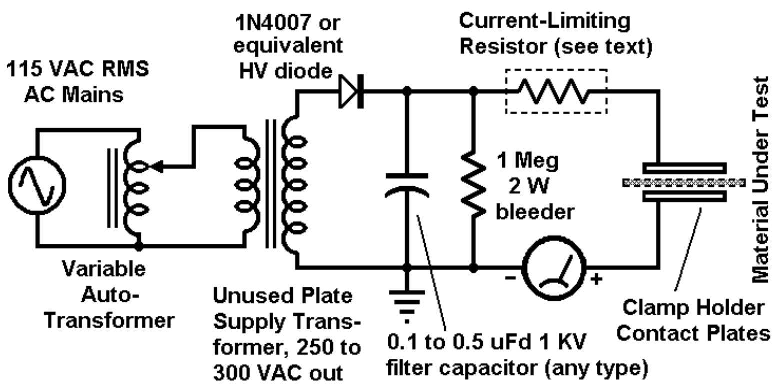

Figure 16-4 Insulation test schematic.

Insulating film sample and meter are not conducting at low voltages. When the insulating film breaks down under voltage, the microammeter conducts to show that has happened. Current-limiting resistor (enclosed in dash-line box) has a value to limit meter current to its maximum. Maximum diode cathode voltage will the 1.414 times the rated secondary voltage. For a 250 VAC rating, peak diode output voltage will be 358 V; for 300 VAC secondary diode peak output will be 423 V. Meter can be anything, including a digital VOM or a d'Arsonval meter.

To operate the test rig, set the variable autotransformer to zero output and slowly increase the primary voltage. As breakdown occurs, the meter current suddenly jumps to full scale. At that occurrence the variable autotransformer dial reading can be read and converted to the insulation breakdown voltage.[20] It is preferred that the rectifier circuit capacitor be a lower value for some peak ripple on it to *hurry-up the rise of each peak waveform of the ripple.*

Material holder should have flat surfaces for contact with material under test. Holders should not have any projections that can pierce material and give erroneously-low readings. Both resistors call out higher Wattage that needed. That is for voltage breakdown of resistors, typically rated 500 V for 1 W, 800 V for 2 W resistors.

Current-limiting resistor values should be 820 KOhms for 1.0 mA meters or 4.3 Meg for 200 μA with 300 V secondaries; 560 KOhms for 1.0 mA meters or 2.7 Meg for 200 μA meters with 250 V secondaries.

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[19] *Variac* is a trade mark of General Radio Company, *Powerstat* that of Superior Electric Company. Both are toroidal form inductors with output connection to a rotating contact along a conductive flatted edge on the rear of the toroidal wiring.

[20] Presumption is that the test rig has been precalibrated for output voltage versus the autotransformer dial. It the breakdown is sustained, then the voltage across the sample will be near zero.

Chapter 17

Wide-Band Transformers, BALUNs, RF Couplers

Wideband Transformers refer to operation over an octave or more in frequency. In the vacuum tube era it was common to use audio-range iron-core transformers for stage coupling. At RF in the solid-state era it has been common to use transformers for coupling impedance levels; those can cover a decade of frequency at HF. Such Z-coupling has been done by ***Baluns,*** a general term from ***Bal***anced to ***Un***balanced). Variations by the term ***hybrids*** allow sampling with isolation between input and output, including the ability to tell power-flow direction.

General

Transformers have the ability to ***transform*** AC voltages and currents at their outputs with a relatively minimum of power loss from input to output. They are all ***passive devices*** and therefore work in either direction insofar as AC or RF energy is concerned. All that was explained in the previous chapter apply to such wide-banded transformers. Energy coupling is essentially magnetic. Wideband frequency operation requires different kinds of ***core*** material plus some differences in winding connections as will be shown later.

In here there are two words occurring frequently: ***balanced*** and ***unbalanced.*** Balanced circuits (sometimes called ***symmetric***) are two-wire with both wires ***above a common ground. Unbalanced circuit*** (sometimes called ***asymmetric)*** have the common ground as one side of their energy-transfer circuit.

Auto-Transformer, the *Missing Link*

This type of transformer has only one winding and output voltage is obtained by a sliding contact on exposed wires of the single winding. It is commonly used to ***adjust*** AC line power voltage to the input of a power supply. Two common trade names are ***Variac*** (General Radio Company) and ***Powerstat*** (Superior Electric Company). Their simple schematics are shown in Figure 17-1. Output voltages are proportional to the sliding contact positions on the exposed winding wires. These are generally made on iron lamination toroidal forms.

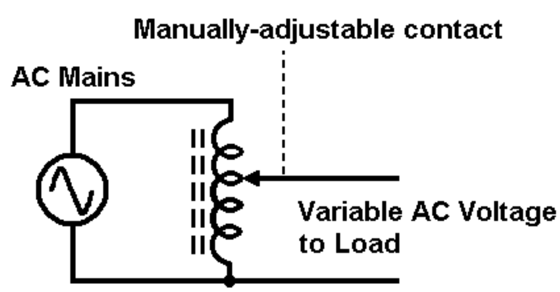

Figure 17-1 Variable auto-transformer to set AC Mains voltage to a supply.

An auto-transformer works by the single winding creating the magnetic field. The tap approximates a secondary winding. While not intended as an impedance adjustment device, the toroidal form allows a simple rotary manual adjustment of the tap position to achieve a desired AC output voltage.

Audio Frequency Range Transformers

These were widely used as *interstage transformers* in the vacuum tube era of about pre-1970. They can create voltage gain by stepping-up to a higher impedance on the secondary side (via having more turns on the secondary relative to primary winding).[1] They could also effectively provide a DC block between the high voltages of the plate driving stage and the grid connections of the secondary, essentially at zero volts relative to ground. More important was their ability to provide a *push-pull* grid input as well as voltage gain by doubling the number of secondary turns and adding a center-tap to ground as in Figure 17-2. Those were eventually displaced by cheaper vacuum tube *phase-splitter* circuits that could drive push-pull inputs with a single triode.

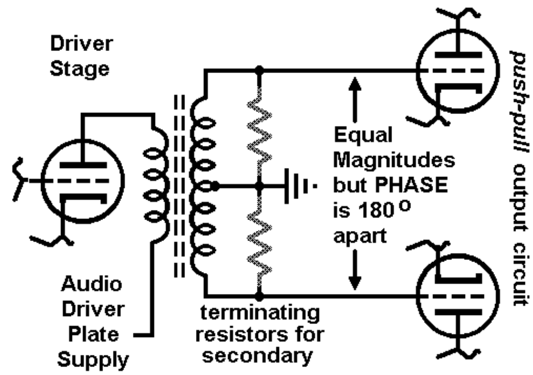

Figure 17-2 Old-style audio coupling, single-ended to push-pull output.

Final output transformers were kept in new designs for vacuum tube output amplifiers for a much longer time. Tubes needed a relatively high plate load impedance and speakers were almost all in the 4 to 16 Ohm impedance region. Impedance-matching by transformer was still a cheaper way to design audio power amplifiers until semiconductor circuits were developed for low impedance loads. Such semiconductor power stages could become *more high-fidelity* than even the most expensive quality transformers offered.

Microphone and other very-low-level inputs can still use low-power transformers into single-ended audio amplifier stages. Such transformers with balanced-input primaries and unbalanced outputs can effectively conquer ***common-mode*** noise pickup. ***Common-mode*** coupling is where noise and other interference are coupled to an input via differential coupling on balanced inputs. A balanced input primary plus effective shielding around the transformer greatly reduce such common-mode pickup. But, by about 1980 the use of op-amp circuitry could equal the common-mode rejection of small transformers and have greater bandwidth for better sound fidelity.

Eventually the lessening demand for audio transformers made manufacturers either drop those product lines or raise their prices. There are many texts available on how to design high-fidelity audio transformers but those are stymied by lack of good core material. Core material was always sold in bulk to manufacturers by specialty core material makers. Trying to make one's own few transformers using iron alloy laminations is possible but one has to have a large storage area to keep about a half ton of unused core material.

Modulation transformers were used when AM did its rather brute-force method of AM by

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[1] Vacuum tubes always in a negative voltage (relative to cathode) condition behave essentially as very high resistances (high MegOhm region), therefore allowing the output stage to have ***extra voltage gain*** through the turns ratio of primary to secondary windings.

literally swinging RF power amplifier plate voltages at the modulation rate. Power demand for the audio capable of handling that was about half the power of the RF output. A 100 W RF output AM transmitter needed a minimum of 50 W of audio to do amplitude modulation. Not efficient.

How Transformers Can Block Out some Distortion

Audio power output transformers couple *push-pull* voltages out to (generally) single-ended outputs (speakers). They can be said to be Balanced-to-Unbalanced impedance matching devices. Power output circuits (specifically vacuum tubes) are not perfect ***differential output*** voltages and currents can be quite good. A transformer's magnetic field is induced from AC currents in a center-tapped, balanced winding.

At low-level microphone inputs, the most noise-free input would be the differential input of the microphone output, both leads being balanced above ground. Any noise pickup from outside sources could couple the **same-polarity noise** to ***both low-level inputs equally.*** In early days a transformer was good to, in effect, cancel out most equal-phase and equal-magnitude noise pickup, that later being dubbed ***common-mode pickup*** when semiconductor op-amps became popular. ***Common-mode rejection*** specifications are common on operational-amplifier IC datasheets.

Hybrid Transformers in Telephony

Telephone subscriber sets existed well before the semiconductor era.[2] To create more comfort to subscribers, the relatively loud sound of a subscriber's microphone, relative to the other subscriber's voice, was reduced by use of a ***hybrid transformer*** capable of separating incoming and outgoing signals. Note that telephones were always ***full-duplex***, meaning one could simultaneously talk and listen at the same time.

When vacuum tubes became practical, ***hybrid transformers*** were developed to allow these new amplification devices to boost the signal of telephone circuits attenuated by miles of copper wire. Again, these *improved hybrid transformers* could ***separate signal paths.*** Without that signal path separation, the amplifiers would feed back on themselves and go into oscillation. *Long-distance* telephoning over hundreds of miles was possible using repeater amplifiers connected to *hybrid transformers.*[3]

The basic *Wheatstone Bridge* is explained more in detail in Chapter 37 on Metrology. Here that it operates by ***comparison*** of bridge arm impedances such that, at ***balance,*** any signal across points A and B will **not** appear at points C and D. Any signal into points C and D will not appear at points A and B. This is more apparent if all bridge arm impedances are the same.

The *Intermediate* form of the bridge is shown in the middle of Figure 17-3, this time with bridge arms Z_2 and Z_4 replaced by a center-tapped transformer primary. If arm Z_3 is an impedance exactly equal to the impedance of a telephone line and that telephone line is connected ***in place*** of of Z_1, then any signal coming into the intermediate circuit will appear at points A and B. Those two

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[2] They existed before the first vacuum tube was invented in 1906.

[3] The word ***hybrid*** is a general-purpose descriptor. The author has not found any etymological basis for its use with transformers. Nonetheless, mentioning the word *hybrid* in many an electronics lab is sufficient to mean just the transformer circuit application described here.

points would be the place to connect an earphone/receiver device. If a signal is introduced across points C and D, then it will ***not*** appear at points A and B because of balanced impedances and the equal turns of the center-tapped primary. The C-D signal would appear across each impedance at half its original value if all windings on the transformer had the same number of turns. That would be for the microphone/transmitter device.

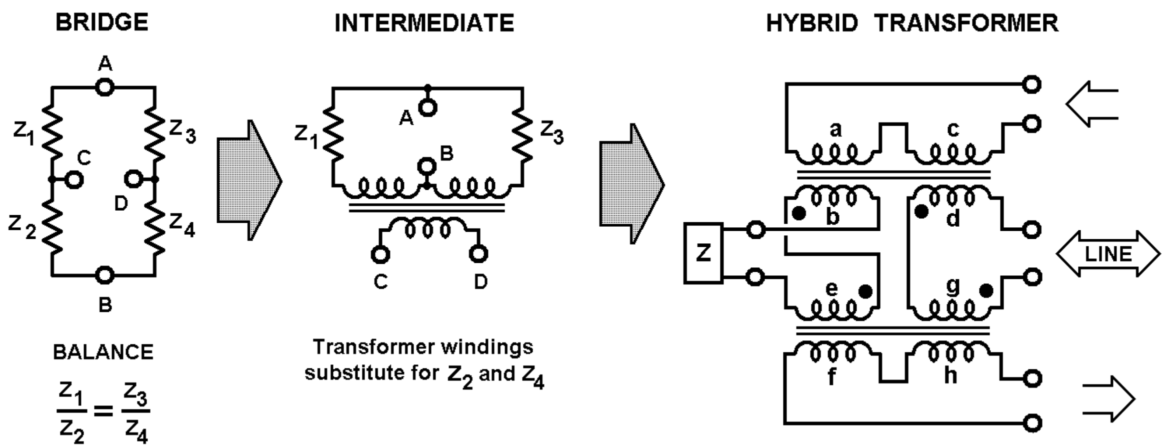

Figure 17-3 Evolution from the basic *Wheatstone Bridge* (at left) to final telephone hybrid transformer (at right). Telephone lines are always balanced above ground. Dots in right-hand hybrid transformer are *phase-polarity markers* to indicate the voltage phase as a result of the magnetic field through the transformer winding. Unshaded arrows indicate direction of signal propagation. Block marked *Z* is a passive balance impedance the same as the *Line* impedance.

Evolution of the Hybrid Transformer

While the intermediate form of hybrid in the middle will work successfully on the bench, it has some shortcomings for application on a long telephone wire circuit. Impedance of the long line varies slightly and there is a need to allow for that. Phase and magnitude of the transformer response will vary over a particular passband and two of them for a telephone repeater might oscillate. For 4-circuit frequency-multiplexed simple ***carrier*** equipment on long lines, the frequency response of the entire long-lines circuit, including hybrids and repeater amplifiers, must be at least 12 KHz.

Final telephone-application is the one to the right in Figure 17-3. All transformer windings are the same to fit the North American 600 Ohm characteristic impedance of long lines. Design of the transformers used in the hybrid can be optimized for the desired frequency range, there is opportunity to connect DC power for repeater amplifiers (with blocking capacitors), and the manufacturing process can make highly-identical transformers in large lots.

The secret to the Hybrid Transformer at the right of Figure 17-3 is the cross-connection of the lift-middle windings ***b*** to ***e.***. Any signal ***going into*** the hybrid transformer will be opposed in phase at windings ***f*** and ***h.*** That signal will not come out the bottom connection but does come out the middle (***Line***). In the same way, any signal going into the ***Line*** (middle) connection will come out the bottom connection. ***Directionality*** of signal propagation for full-duplex[4] operation has been assured in this hybrid transformer.

Of What Uses Are *Hybrids* Other Than in Telephony?

In microwave frequency work, a form of a hybrid is called a ***directional coupler.*** Single output port directional couplers can take a sample of the ***incident*** power (straight-through), usually 20 db less than the incident power to avoid taking too much from the load. That sample is from ***one direction of propagation.*** This can be enormously useful in testing a microwave system.

Microwave directional couplers are available as coaxial or waveguide structures, single or double output sample ports, at least an octave in frequency range. A few single directional couplers are available in decade operating bandwidths. Their ***directivity***, that is the difference between power sample leakage from the wrong direction and the power sample from the desired direction is at least 20 db down. While directional couplers are governed by physical laws considerably more complicated than hybrid transformers, they effect the same thing: Directionality of signal propagation.

A much simpler, but also very narrowband hybrid transformer arrangement made solely by coaxial cable in the 1940s was the so-called ***rat-race hybrid.*** It has been used mostly in textbooks rather than in real radio life and consisted of a wavelength-and-a-half of coaxial cable in circumference, one half the circle made up of a 3/4-wavelength line. The other half of the circle had three sections of coax each 1/4-wavelength long. Over a very narrow bandwidth full-duplex operation was possible at limited transmit power.

The ***Magic-T*** was a form of hybrid transformer made out of waveguide, good from about 8 GHz and higher. It could operate almost at octave bandwidths. Many older textbooks cover those.

Perhaps the most-used hybrid transformer-like device is the HF power output monitor and SWR indicator sold to radio amateurs. This is basically an RF voltage sampler and an RF current sampler with their outputs arranged to indicate forward power (output going to the antenna) and reverse power reflected back at the transmitter, usually displayed as SWR or Standing Wave Ratio.[5]

Very Wideband Transformers

By far the most numerous examples of these are the tens of millions of 75 Ohm coaxial cable to 300 Ohm twin-lead adapters shipped free with most TV receivers made from the 1970s into the new millennium. Their internal structure varied widely but externally they were simple and straight-forward, simple 1:4 impedance changers for the two most-popular signal input cable types used

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[4] ***Duplex*** and ***simplex*** are ancient hold-overs from the half-century before the first radio was demonstrated. ***Simplex*** refers to a circuit that is always one-way. As an example, the circuit from a ceiling lamp to its control switch is ***simplex.*** A radio circuit is mostly ***half-duplex*** between two stations in that one station transmits while the other listens, then the other station transmits while the first station listens. If the two stations are far enough apart in frequency they may go ***full duplex,*** each station listening to the other stations transmit frequency and vice-versa. Telephone circuits are always ***full-duplex.***

[5] The electronics industry uses the term VSWR for Voltage Standing Wave Ratio, a more accurate term since the RF voltage is almost always sensed on a line, not the “wave.”

worldwide.

Their useable bandwidths varied from 52 to about 240 MHz on up to 50 to 500 MHz, depending on the market area. North America and the USA in particular had the widest frequency range for TV, including closed-circuit cable distribution. Since these were no-extra-cost accessories, performance might have suffered some from manufacturers' budget constraints. They were too numerous over a four-decade time span to categorize. They seemed to be generally designed with a ferrite core and a few turns of wire through a small *binocular core.*[6]

A more technical source of information on wideband transformer design and fabrication is from a series of Application Notes from Motorola, authored by Helge Granberg.[7] These were essentially sales pitches for Motorola RF power semiconductors useable over all HF bands and on up to the middle of VHF. Some were quite wideband, covering a decade of frequency span without tuning and effecting a moderate impedance matching (roughly 10:1) to a 50 Ohm coaxial line.

From about 1980 onwards, HF transceivers with transmitters running about 100 W PEP maximum had final amplifiers that did not require tuning by users over a decade of frequency. They did this with wideband transformer structures similar to the Motorola-Granberg Application Note descriptions.

Getting Wide-Band Characteristics

For iron-lamination cored transformers it is basically a choice of iron ***alloy*** plus the construction and possible use of ***gaps*** between edges of laminations. Next, it is the minimization of inter-winding and intra-winding capacitance. It is possible to achieve broadband operation up to 25 to 30 KHz maximum frequency as witness the *high-fidelity* music system electronics of the 1950s and 1960s. The exact techniques seem to be buried in corporate-confidential data since it is surmised that much of it was achieved through trial and error.[8]

Ferrite and powdered-iron core transformers have perhaps the widest bandwidth and can carry KiloWatts of RF power. Ferrite material is an iron-alloy powder that has been sintered (heated to near melting point) and pressed together. Powdered-iron-alloy cores are molded with a plastic binder, not fabricated with much heat. Resonated powdered-iron core inductors are hard to beat for narrowband responses at high impedances due to high Q. Both can be respectable wideband transformers in toroidal or binocular core form. For either core material the higher the μ or *permeability,* the wider the bandwidth. This can be important for transformers used in *switching regulators*. Those require bandwidths into the low MHz region.

Bandpass filters can be an attractive alternative to moderate bandwidth transformers and also provide selectivity. They can be designed with unequal terminating impedances, thus providing an approximate impedance transformation.

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[6] Rounded edge ferrite core with two lengthwise holes for wires, vaguely resembling a miniature binocular, hence the name.

[7] Communications Concepts (www.communications-concepts.com) has several of Granberg-authored AppNotes available on their website as well as offering kits of essential parts for most of those. Application Notes and Engineering Bulletins numbers are: AN-758, AN-762, AN-779, AR305, AR313, AR347, EB-104, EB-27A, AB633 (Motorola document designations).

[8] Personal conversation with those involved in local transformer industries.

BALUNS

A Warning in General

The word ***balun*** is a contraction of ***BALanced to UNbalanced*** and describes a ***passive*** device intended to connected a balanced-above-ground differential pair of wires to an unbalanced wire such as a coaxial cable...or a balanced pair to another balanced pair having a different characteristic impedance. Baluns use ferrite or powdered-iron cores, core shapes as toroids or binocular shapes to contain their RF field. Core-set windings are connected to achieve rather specific input/output impedance ratios..

In amateur radio there has sprung up a new set of words to describe their use, such as ***Voltage Baluns*** or ***Current Baluns,*** both terms being technically incorrect. Transformed RF energy always involves both voltage and current, regardless of the resulting impedance characteristics.[9] Remember that a balun is a ***passive device*** like any transformer and couples in both directions. What some call an *input*, others call an *output,* yet both would be correct.

A very general definition of Voltage versus Current Balun is that a Voltage Balun at the balanced port has equal voltage magnitude but opposing phase; Current Balun has the current at the balanced port with equal current magnitude but opposing phase.[10]

One category of Baluns has no transformer connection, just a length of coaxial cable wound around a former or strung with large ferrite beads to break up any RF currents on the outer conductor of the coaxial cable. It has been called a ***choke*** balun and is generally categorized as a current balun. RF energy between inner and outer conductors of is unaffected provided the cable is not physically deformed to change the dimensional ratios of those interior conductors. It is not strictly a Balun, just an inductance of the outer conductor whose inductive reactance inhibits RF current flow in just the outer conductor.

Lengths of transmission line, principally coaxial cable, have been used to create the equivalent of balun action over very narrow percentage bandwidths. In general, balun designs are very broadband, ranging from an octave to a decade-and-a-half in commercial products.

Ferrite and iron-powder cores increase the magnetic field coupling and make for smaller physical size. Those can be wound with coaxial cable or with bifilar, even trifilar or quadrifilar wires. A bifilar wire pair is itself a transmission line, generally of low characteristic impedance. The difference with coaxial cable is that **both** wires couple with the core.

The above resulted in a new designation: ***Transmission-Line Transformers*** or ***TLTs*** to encompass everything, be it baluns or transformers handling RF energy. To increase the inordinate number of names of these devices, an ***Un-Un*** represents a transformation device to change

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[9] The author is not happy with such ignorant disregard for technical facts and is appalled at similar descriptors of antennas being *"electric"* or "*magnetic,"* the latter term incorrectly used with loops or other antennas whose dimensions are less than a half-wavelength. Unfortunately, several words have come into use and are now common; one such is ***hybrid*** in this chapter. Originally an adjective or descriptor for a noun or phrase (such as a *hybrid gas-electric auto),* it has been changed into the noun itself.

[10] From ***Radio Works*** website at Portsmouth, Virginia, www.radioworks.com. The differentiation in baluns between ***Voltage*** and ***Current*** types occurred in 1984 to 1986 although both types had been around for many years prior.

Unbalanced lines of one impedance to another Unbalanced line to a different impedance.[11]

Baluns Used Outside of Strict Radio Applications

Wired Local Area Networks or **LANs** have adopted balanced pairs in cable to carry very wideband digital signals. ***Category 5*** or, familiarly, ***Cat5*** lines have a characteristic impedance of 100 Ohms balanced. As of 2010 more and more peripheral adapters have Baluns to adapt Cat5 balanced pairs to 50 Ohm coaxial cable inputs found on some LAN peripherals or personal computers..

Very small Baluns are on the market to interface low-level audio devices to various audio equipment, everything from microphones to other audio devices.

Origin of Baluns

The word ***balun*** is just a contraction-name for a special-application transformer.[12] The first variation appeared with ***link coupling*** where a narrowband resonant L-C circuit was coupled to a low impedance via a short-turn linking coil. That was widely used in radio receivers and transmitters prior to WWII. With the advent of improved iron-powder and ferrite cores more abundant after WWII, the simple Balun transformer become more useful with a bandwidth of a decade or more in frequency.

As with all transformers that are tightly coupled with a high-μ core, ***source and load end impedances reflect to their opposite ends modified only by their designed impedance ratios.***..plus whatever losses and reactances exist within the Balun itself.[13]

The ***Current Balun*** or ***Guanella*** Balun was the first to appear in literature in 1944 and, apparently, the last to appear as a product 40 years later.[14] The second method or ***Marchand Balun*** also appeared in 1944 but its size relegated it to UHF or higher frequencies.[15] The last method, but

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[11] All the naming and re-naming seems endemic to amateur radio activity. This may be due to baluns being experimented with ***outside*** of their amateur radios and therefore easier to handle. Baluns are simple devices in terms of hardware therefore a greater number of amateurs can pretend to understand them.

[12] In amateur radio literature the subject of ***baluns*** have taken on an almost mythical status as if in a cult of devotees.

[13] Well-made Baluns don't add much reactance nor have much insertion loss. However, whatever is connected to each end will definitely exist at the other end, transformed only by the impedance ratio. The author has been astounded to learn that many radio amateurs (who should know better) think that inserting a Balun will somehow "cure" all mismatches, no other matching required.

[14] **"***New Method of Impedance Matching in Radio-Frequency Circuits,"* by G. Guanella, Brown Boveri Review, September 1944, pages 327-329. This short paper did not receive much electronics industry attention at the time and that may account for the long time before acceptance.

[15] *"Transmission-Line Conversion Transformers,"* by N. Marchand, ELECTRONICS magazine, Volume 17, December 1944, pages 142 to 145. This was displaced by methods of laying out whole microwave circuits on ***stripline*** by 1980, quickly followed by methods of synthesis and analysis of such things via computer, all in one package.

probably the most produced was the ***Voltage Balun*** or ***Ruthroff Balun,*** appearing in 1959.[16] What is interesting is that various RF-range transformers were made and used in the electronics industry before 1959 but were neither named-for nor identified with Ruthroff, much less as Baluns. Such were simply used as such, usually labeled simply as *transformers* because of the familiarity with transformers and link-coupling from resonant circuits that had been used three decades prior.

Basic Hobbyist Baluns

Figure 17-4 shows the schematic symbols and pictorials of a toroidal core Balun wound with bifilar wire. Depiction with bifilar wire is for visual clarity. Individual wires can be used instead of bifilar. ***Close coupling*** is stressed here and ferrite or iron-powder toroidal cores are an excellent way to achieve this close coupling. This helps extend the useful bandwidth.

Both types are shown with 1:1 winding turns ratio. That yields no impedance change but does allow one side to be unbalanced while the other is balanced. Note one thing in Figure 17-4: ***It is the connections to transmission lines which make them different.***

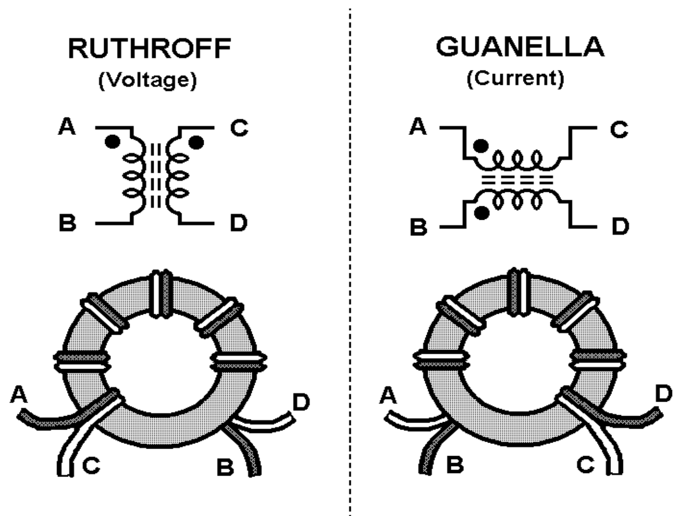

Figure 17-4 Two principal types of Baluns as wound on toroidal cores with bifilar wire. Dots on schematic symbols are RF phase equivalents.

Changing Impedance Ratios

Close-coupled transformers can change input/output impedance terminations through the ***square*** of their turns ratios. This works well in toroids, such as in balun construction. If the output winding has twice the number of windings as the other, provided both have the same number of turns, then the output terminating impedance is ***4*** times the smaller. If that one winding is three times times greater, then its terminating impedance is ***9*** times larger. Reverse the connections and the input termination is 4 times and 9 times smaller in impedance. See Figure 17-5 for a 1:2 voltage step-up balun with a symmetric 1:4 impedance change.

With only three windings, the Figure 17-5 pictorial is easy to get with a toroidal core. The same is true for a 1:9 impedance change using four windings, although it is more difficult to get the side with three windings center-tapped.

Figure 17-5 is good for a receiver antenna input or for a transmitter final into an antenna. For the receiver the secondary winding will be differential and good for a push-pull input. For a

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[16] *"Some Broadband Transformers,"* by C. L. Ruthroff, Proceedings of the IRE, Volume 47, August 1959, pages 1337 to 1342. Ruthroff had articles in ELECTRONICS magazine following the 1958 date with variations of bifilar twisted-wire windings with responses over frequency. The author does not have cites for those articles in the former McGraw-Hill biweekly magazine..

transmitter the windings are reversed and the differential input is to the transmitter collectors (usually the lower impedance).

Any combination of windings can be wound for whatever impedance ratios are desired. One word of caution: Baluns are not a sure cure for everything and a balun construction must be tested over a wide frequency range first to prove it out with low-reactance resistive loads.

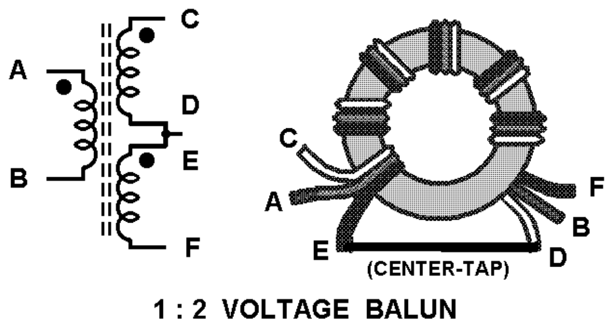

Figure 17-5 A 1:2 Voltage-step-up BALUN shown in schematic and pictorial form. This has a 1:4 impedance change with center-tap.

A Typical Simple Balun

Figure 17-6 has a Guanella 1:1 impedance ratio balun commonly used in HF amateur radio connected as ***asymmetric*** relative to ground. This is good for feeding a dipole antenna with coaxial cable while the output goes directly to the two dipole elements, ungrounded. The non-symmetric balun is wound with as many turns of wire as possible, consistent with carrying the expected RF current.

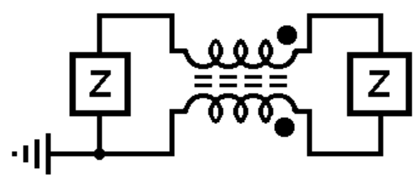

Figure 17-6 A simple current balun for a dipole antenna.

Impedance-Changing Baluns

Figure 17-7 has a 1:4 (or 4:1) impedance ratio, shown in both the symmetric form (both sides off ground) and asymmetric form (both sides grounded). Again, use as much wire as the core will hold consistent with expected maximum RF current. Both core sets have the same number of turns.

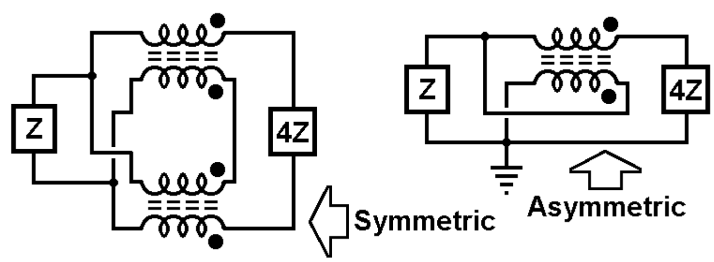

Figure 17-7 Two versions of a 1:4 Z-ratio balun.

Note the ***Z*** terms. As an example, a 1:4 balun can transform a 50 Ohm line (such as a coaxial cable) to 200 Ohms impedance.[17] With the impedance change, the RF voltage at the 200 Ohm side is twice that of the 50 Ohm side while the RF current is half that of the 50 Ohm side. In a lossless balun, the power level on each side remains the same.

Figure 17-8 has two versions for a 9:1

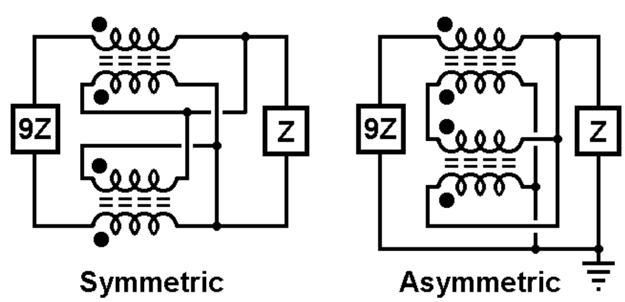

Figure 17-8 Two versions of a 9:1 Z-match Balun arrangement.

[17] In old broadcast TV, the common impedance was 75 Ohms. Four times that is 300 Ohms, a good value for the (cheap) ***twin-lead*** balanced line between external antenna and residence TV location.

impedance ratio balun. Major change between the two is the common ground on the right side. Again, this has two sets of core windings each.

Figure 17-9 is a redrawn version from Michael G. Ellis from his Internet *Tutorials* collection. It begins with a pair of Guanella windings for a 1:9 impedance ratio. By reconnecting the lower-impedance port, the ratio changes to 1:(9/4) Z. If Z were 50 Ohms at the output, then the input would be 22.2 Ohms. Conversely, if the lower impedance input were 50 Ohms, then the output would be 112.5 Ohms.

If a 1:9 impedance ratio were retained, then an input of 50 Ohms would be changed to 450 Ohms. Likewise, if the input was 75 Ohms, then the output would be 625 Ohms.

Taking them in reverse, a 50 Ohm output could come from a 5.56 Ohm impedance input with a 1:9 ratio. If a 1:4 balun were used, an output of 50 Ohms would match a 12.5 Ohm input. These are both mentioned since several semiconductor HF power amplifiers have low impedances.

There are several ways to connect core-sets but all involve multiples or fractions using integers. For low-power work, small binocular cores can be used with isolated primary and secondary windings.

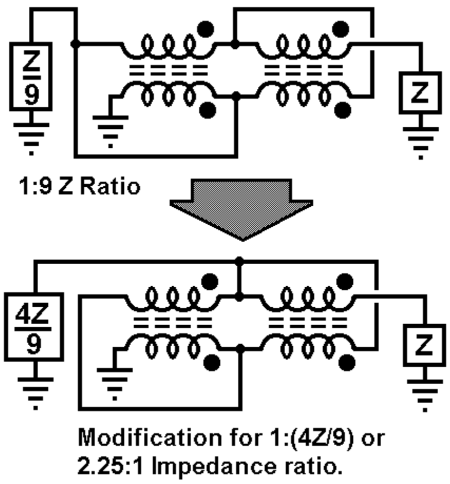

Figure 17-9 A variation from a standard 1:9 ratio at the top and bottom having a ratio of 1:2.25 in impedance.

Balun Construction

General

Due to the great variation in ***core material***, much of the design work has to be done with the aid of complex impedance measurement tools. What is required is a relative permeability or ***μ*** that is reasonably flat over the passband of interest.

Lacking specific core material data is no drawback. Voltages and currents can be figured out by pencil-and-paper. The little ***dot*** in these figures stands for voltage polarity. Current is the reverse polarity of the voltage dot. In essence, current that is magnetically coupled is simply shifted in phase by 180°.[18] For outputs relative to inputs follow Kirchoff's Laws to see the relative impedance, knowing that transformed voltages and currents each change by the square-root of the impedance change.

The most convenient component is a toroidal core of powdered-iron or ferrite material. The next type of core is the ***binocular.*** A *binocular* core can be built of several toroidal cores, using two rows of toroids, each row side by side with the other.[19] The important thing there is to keep the RF

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[18] That follows basic ***magnetic coupling*** rules.

[19] Often used in higher-powered transmitter wideband matching. See the Motorola-Granberg application notes for some convenient structure details.

magnetic fields confined to the toroidal structure. That will increase their wideband response.

Frequency compensation can be done by appropriate permeability of the core at the lower frequency end. Higher frequency end is influenced by the wire stacking. Addition of small capacitors can usually help the higher frequency end response. Figure 17-10 is an example of a medium-power HF semiconductor output stage matching to a 50 Ohm load. Frequency compensation capacitors C1 and C2 are shown in grey.

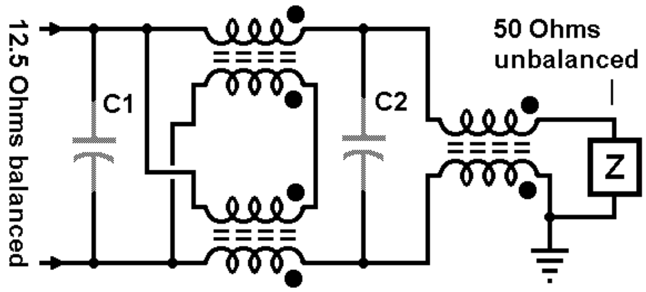

Figure 17-10 12.5 Ohm balanced to 50 Ohm unbalanced Balun with frequency compensation capacitors C1 and C2.

While this was using a particular core and wire size, it was intended to match an HF medium-power amplifier output to a 50 Ohm load. From NXP application note ECO7213, C1 = 270 pFd and C2 = 22 pFd. The resulting VSWR was less than 1.5:1 from 1.6 to 28 MHz. Without the capacitors the VSWR was slightly higher than 3:1.[20] For other core material and frequencies, the circuit can be tack-soldered together on the bench with small variable capacitors substituting for fixed values. Those can be measured and fixed capacitors substituted.

The Guanella Balun added allows the input (at left) to be balanced above ground while the final load (at right) is unbalanced, as for a coaxial cable.

Binocular Core Construction

Figure 17-11 shows a section-through of a binocular core with an approximate 1:9 Z-ratio. The core is depicted cut *lengthwise* to show the pattern of wires. The core itself is colored dark grey with its holes as medium grey.

With a sample long binocular core, the author was able to make a Balun with 1.75 turns primary and 8.75 turns secondary with the secondary center-tapped. That yielded a 50 Ohm input to an approximate 1000 Ohm output into a balanced band-pass filter for 4 to 18 MHz. Response was within ±2 db over that range.

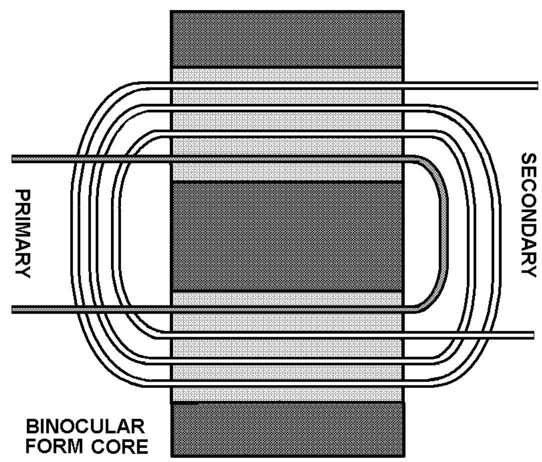

Figure 17-11 Winding a binocular core.

It should be noted that an *odd* number of turns allows a center-tap of the secondary winding to the left while an *even* number of turns allows the tap to be to the left. Depending on the core material, this one acted more like a 2:9 turns ratio or 1:20 Z-ratio. No data was available on the core material and no further investigation was done on that.

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[20] From NXP (old Philips) Application Notes ECO6907 and ECO7213, originally released in 1969 and authored by A. H. Hilbers of (then) Philips, they were re-released in March 1998 and this author obtained them in 2012 from the NXP website. It is a straight-forward explanation of Baluns and Figures 17-6 through 17-8 are redrawn from that. The English translation gets a bit wordy in places but that can be eventually understood.

Number of Turns of Wire

Wire size in AWG is chosen for the ***absolute worst-case current*** carrying capacity. It seems to work out best if a pair of wires is first twisted for ***bifilar*** lay-in.[21] That can be done with a non-electric hand drill, approximating total length plus about 20% (for length decreasing from winding the bifilar). On a toroidal or binocular core a substitute is to wind ***two*** wires together.

For those of more-theoretical bent, inductance can be measured at lower frequencies, final measurement near the approximate middle of expected passband in frequency. To find an equivalent coupled-inductor T-structure, see the ending of Chapter 13 on measurements on a coupled inductor with the opposite winding open and shorted.

Wire insulation of enamel, as with magnet wire, is good for low-power receiving. For higher power levels, the expected ***worst-case peak-to-peak voltage*** in the application must be used. For outdoor mounting, such high-power uses can use *Teflon* insultation. It should be noted that voltage gradients distribute fairly evenly across a winding so a turn-to-adjacent-turn voltage is about equal to the total peak-to-peak voltage divided by the number of turns of a winding.

For outdoor use with antennas, ***never*** expect Baluns to withstand a direct lightning strike. The voltage and current will far-outdo the worst-case voltage and current peaks. It is helpful to include a higher value resistor of roughly 100 times or more than the highest RF impedance to bleed-off natural electrostatic charges to a good ground. That is at DC. It applies to normally-balanced antennas such as horizontal dipoles but need not be done for vertical monopole antennas that are already at a DC ground potential.

Balun Measurements

This requires loads which are nearly perfect resistors and a calibrated signal generator, primarily for RF power. Older carbon composition resistors of the 1 Watt category seem to have the least internal parallel capacity if precise RF load devices cannot be used. The author has found that 51 Ohm carbon composition resistors to have about 0.8 to 1.5 pFd of internal parallel capacitance, good to VHF. A 1.5 pFd capacitance has a reactance of 106 KOhms at 100 MHz.

RF power levels are usually the bane of home workshop measurement. Refresh your thinking by perusal of the two Metrology Chapters 36 and 37 in regards to RF power levels. An excellent RF power indicator, one of a few based on ***successive-detection logarithmic detectors***, aided by a microcontroller, can be found on the Internet. Calibrated against a good source of power, they can be rather accurate. If the RF generator is reasonably stable, its RF power can be checked directly, then the Balun output can be checked with determination of Balun performance based on that.

Couplers of Various Kinds

General

Couplers is a general word for a number of different types which can split power flow and

[21] Three wires twisted is called ***trifilar*** and four wires twisted is called ***quadrifilar.***

provide isolation between output ports. These are most useful in microwave work and most operate over an octave of bandwidth. Isolation between outputs is called ***directivity*** and its value is given as the power flow (in db) of the normal output port ***plus*** the coupled output. The various types of common couplers are shown in Figures 17-12 for a directional coupler, 17-13 for a Wilkinson divider, and 17-14 for a 4-port hybrid coupler.

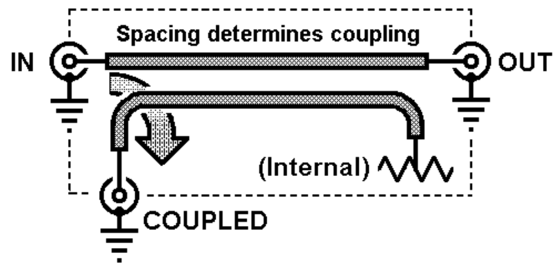

Figure 17-12 Single coupled-output directional coupler.

The directional coupler has its coupled output interacting with the main In-Out line over its quarter-wavelength coupled line, center-to-center spacing determining coupling. Single couplers have an internal termination for the coupled line. Dual couplers with 4 ports are available.

Most nearly made in an interim coaxial-microstrip structure, these are usually just machined parts and quite sturdy. A typical coupler is 20 db, losing only 0.05 db in the main line, with 40 db minimum directivity over an octave of frequency.

The Wilkinson Divider splits an input into two isolated ports using two lower-impedance, quarter-wavelength lines. As-is, the simple Wilkinson form is relatively narrowband and more suitable for near-fixed-frequency radios. It requires a non-reactive resistor whose value is equal to twice the In and Out RF impedances. With usual 50 Ohm structures that would make it 100 Ohms.

A Wilkinson is bilateral and can be used to sum two inputs, the reverse of the labels of Figure 17-13. As such it is more often built into microstrip or stripline assemblies.

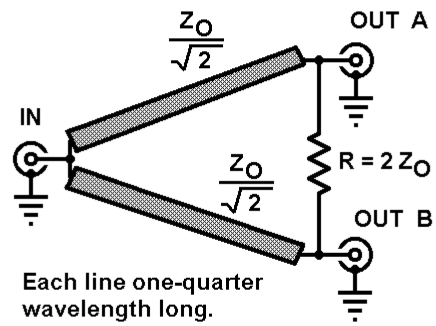

Figure 17-13 Wilkinson Divider.

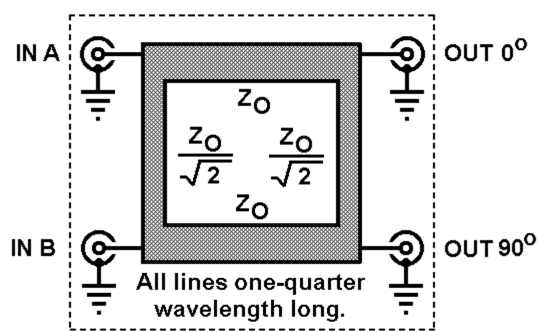

Figure 17-14 A 4-port coupler, sometimes called a *4-port Hybrid Coupler.*

The 4-port coupler of Figure 17-14 is also relatively narrowband. It has both Inputs and Outputs isolated from each other. It is made in microstrip and stripline for most applications although a few varieties have used lumped-constant L-C parts in place of the line paths at lower frequencies (under 300 MHz).

It is also bilateral in power flow and typically used in ***tee*** arrangements in higher-power VHF-UHF power amplifiers, both at inputs and outputs of modular, identical power amplifier structures.

The directional coupler of Figure 17-12 has been modified to work over a decade of frequency. There have been many papers presented on the Wilkinson and 4-port varieties to extend their frequency span plus allow for more (and some unequal) power-flow types. The three types of couplers require more precision in manufacture than is available to the average hobbyist. They are included for identification since they were available from the 1950s and still used in many microwave labs.

What has been left out are ***Isolators,*** permanent-magnet biased ***one-way power flow*** devices first available in the early 1960s. These are generally 3-port devices with octave bandwidths and, because of internal ferrite material, used in microwave work.

The Bruene Coupler for VSWR Indication

The ***Bruene Coupler*** was invented by Warren Bruene at Collins Radio in the late 1950s.[22] It remains (with slight modifications) as the detector in nearly ***every*** HF automatic antenna tuner today. It is expanded in explanation because it can be built by the average radio hobbyist, does not need precision structures, yet has about a decade of frequency response. See Figure 17-15.

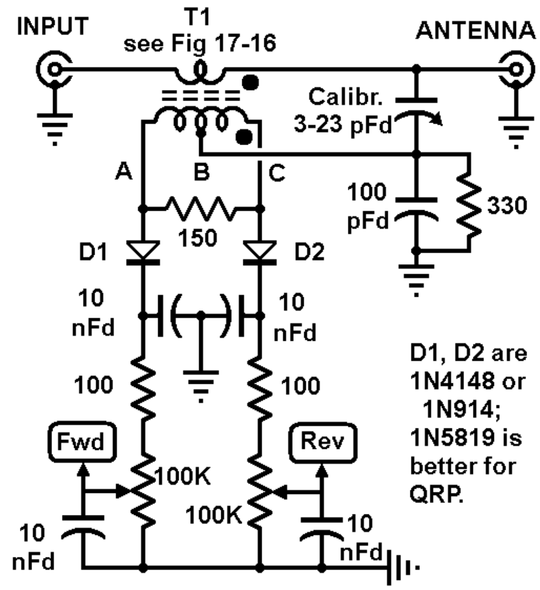

Figure 17-15 A Variation of the Bruene circuit shown in QST, January 1996.

This circuit, from the January 1996 edition of QST magazine, authored by Dwayne L. Kincaid, develops a DC voltage for the ***Fwd*** and ***Rev*** outputs, representing the Forward to and Reflected power from an the Antenna. It is the sensor of an automatic antenna tuner. In that tuner a microprocessor, using A-to-D inputs, uses the detected Reflected power, relative to Forward power to switch in an inductance (in binary steps) and capacitance (in binary steps) until there is ***less*** detected Reflected power versus Forward power.

It works from about 3 MHz to 30 MHz at an input power level of 2 W to 100 W. Detected power maximum is at about 4.5 VDC at 100 Watts.

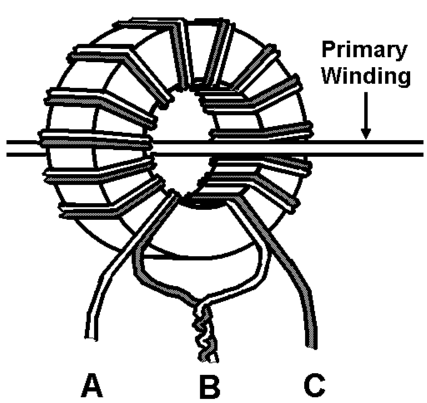

Figure 17-16 Pictorial to show T1 with an excellent center-tap on Secondary.

T1 of Figure 17-15 requires a good center-tap to take the sum of the RF line voltage and add to the detector inputs. Figure 17-16 shows this, relative to the A, B, C points on Figure 17-15. The original used an FT-37-42 toroid core which has only a 3/8 inch outside diameter. It might be easier to use an FT-50-42 toroid core, slightly larger, using only 13 ½ turns of bifilar wire. Secondary inductance will be about the same.

The Primary winding, a direct line from Transmitter to Antenna, goes through the toroid form center hole, thus making any toroid winding voltage proportional to ***RF current***. For high-power versions it is about a quarter-turn, going through the

[22] Warren Bruene is co-author of the ***Collins SSB Book,*** formally titled ***Single Sideband Principles and Circuits,*** McGraw-Hill 1964 and worked at Collins Radio for over four decades. His RF Bridge was described in the April 1959 issue of QST under the title ***An Inside Picture of Directional Wattmeters*** and later analyzed by Gary Bold of New Zealand in a 2009 paper entitled ***Bruene Coupler and Transmission Lines***, available on his website.

hole in the toroid. In this lower-power version it is approximately a half-turn of #18 AWG. Figure 17-16 shows the secondary winding with 13 ½ turns for an FT-50-42. Inductance for 13 ½ turns will be the same as 14 turns for an FT-37-42 core. With a light twisting os fhe secondary winding wires, this is a good way to get a center-tap on the secondary.

Mismatch losses and VSWR are relative things in this application. Calibration, as such, is fairly easy with a standard 100 Watt RF output transmitter. Referring to Figure 17-15 and using a dummy load in place of the Antenna, calibration can be done as follows:

1. Initial settings:
    A. Set both 100K potentiometers to center position, transmitter to about 10 Watts.
    B. Adjust the Trimmer capacitor marked ***Calibr*** for minimum voltage at ***REV.***
2. Set transmitter to 100 Watts and adjust the ***FWD*** 100K potentiometer for about 4.5 VDC at the ***FWD*** test-point.
3. Final setting.
    A. Reverse transmitter connection to Antenna and dummy load to Transmitter port.
    B. With 100 Watts at the transmitter, adjust the ***REV*** 100K potentiometer to 4.5 VDC.

If 100 Watts cannot be used, set the transmitter to 10 Watts, using about 1.5 VDC at the ***FWD*** and ***REV*** test-points. For amateur radio use, a low-power setting is preferred for antenna tuning so as to avoid unnecessary RF radiation. A Schottky diode, 1N5819, may be substituted for 1N4148s for lower-power QRP operation. With less forward voltage drop, a Schottky provides more output voltage at lower powers. Exact VSWR is not an issue here, only that the ***relative*** reverse power flow is at a minimum compared to forward power flow.

Almost any microcontroller with a built-in ***comparator*** circuit can be used for the solid-state relay driver to switch-in and switch-out series fixed inductors and parallel fixed capacitors. In the Kincaid ATU there are 8 inductors in a binary progression from 80 nHy to 10 µHy; there are 8 capacitors from 5 pFd to 640 pFd. The relay-switched arrangement is shown in Figure 17-17.

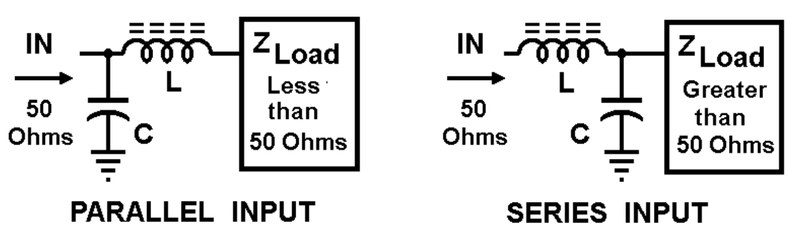

Figure 17-17 The two 2-component arrangements of relay-switched L and C per Chapter 8. At rest, Inductors are each on normally-closed relay contacts; capacitors are on normally-open relay contacts, closing to ground when energized..

A single Form-C contact can select between Parallel input and Series input. Relay contact sets are such that non-energized, the ATU has minimal components between Input and Load.

Programming is such that any ***REV*** input voltage higher than ***FWD*** input voltage will select both L and C relays. Program flow requires that intermediate-tuning steps must dwell on a relay combination position for at least 50 mSec to allow for relay contact bounce to finish. Automatic tuning can begin by selecting either Parallel or Series Input with L and C in roughly mid-position. From there it is a matter of selecting C for a minimum ***REV*** input DC voltage. If that will not reach a minimum, then L is switched and C switching repeated. A probable cycle time is about a minute, roughly the time of manual adjustment. Adding a rough frequency-measurement capability allows some pre-selection of settings. It can be expected that an average antenna will remain roughly stable so that a tuning cycle can be shortened to a few seconds.

Monimatch and *Transmatch*

Both are *invented* words. ***Transmatch*** is a manual L-C antenna tuner while ***monimatch*** is the Forward and Reverse power flow detector-indicator.[23] The *monimatch* is basically a dual directional coupler based on the illustration of Figure 17-12. It is seen in typical form in Figure 17-18. Being a physically-shortened directional coupler, It is somewhat frequency sensitive, thus requires a calibration chart for HF on up to about 60 MHz.

The main line, in grey, has an impedance of nearly 50 Ohms. Coupled lines, typically spaced from main line for -30 db coupling, may be slightly higher. Terminations of each coupled line can act as DC returns for a diode detector. This can be a stripline structure or part of a small box (for shielding).

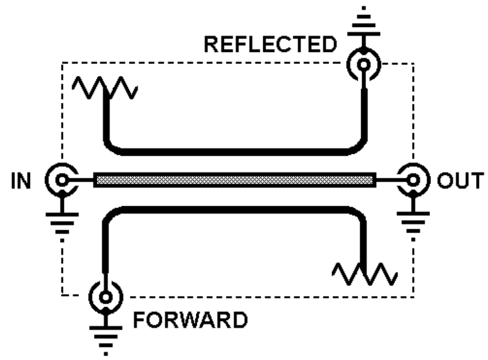

Figure 17-18 A dual directional coupler sometimes called a *Monimatch.*

A Tandem Match Forward-Reflected Coupler

This was used by Larry Phipps in a product on his amateur radio website called the ***LP-100*** Wattmeter.[24] This forms something akin to a hybrid directional coupler to the Forward and Reflected outputs. This is a separate assembly. The Wattmeter contains two Analog Devices AD8307 log detectors, each producing a DC voltage of the RF logarithm in power. That DC is processed by almost any microcontroller to whatever display is desired.

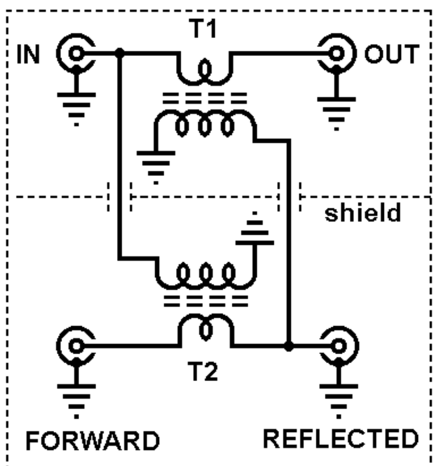

Figure 17-19 Coupler from LP-100 Wattmeter, taken from Figures 1 and 10 of the QEX article.

The advantage to such processing is that a built-in correction can be used, correction stored on internal ROM. Microcontrollers can add a frequency meter (reading off the Forward port) to handle ROM-stored settings for an antenna coupler. That can switch-in various L-C settings

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[23] Both invented names date back to the late 1950s in amateur radio literature.

[24] QEX magazine, January-February 2006 and advertised (as of 2013) at www.telepostinc.com in ready-built form.

automatically to enable very quick HF band changes.

Experimental Work

All wideband transformer and coupler devices require a fair amount of calibrated RF sources and response read-outs. The amount of literature available on them all is rather astonishing. To attempt anything ***new*** in wideband RF work requires considerable time and effort. Also, one has to know the basic theory behind each development in order to understand how each device is supposed to operate. The average hobbyist probably has to take an article's write-ups as-is in order to use any one of them.

Some Important References on Automatic Antenna Tuning Units

[49] ***An Inside Picture of Directional Wattmeters*** by Warren Bruene, QST April 1959.

[50] ***Low-Cost RF Wattmeter*** by A. C. Prescott and W. C. Louden, General Electric Ham News, May-June 1961, Volume 16, Number 3.

[51] ***An Automatic Antenna Tuner, the AT-11*** by Dwayne L. Kincaid, QST January 1996.

[52] ***The LP-100 Wattmeter*** by Larry Phipps, QEX January-February 2006. Also on Phipps website at www.telepostinc.com as the LP-100A and with improvements.

Note: All are considered informative and well-written, worthy of study and for building duplicates.

Chapter 18

Transmission Lines

Transmission lines for RF are broadband lines of controlled fields that transport signals from one point to another. They are either unbalanced with a ground surrounding an inner conductor as in coaxial cable, or they are balanced above ground with or without a ground shield. Included here is some discussion of Smith Charts and Line Measurement methods with a Noise Bridge.

What is *Characteristic Impedance*?

Characteristic impedance is the complex quantity of resistance and reactance of a line**.** At low frequencies there is only a resistive part. That resistive part is the descriptor of a line. If terminated in a resistive load, an infinite-length line would still appear to be resistive. It has been described as being equivalent to an infinite series of L-C sections of a lowpass filter. In one way it is since there is always an increasing loss of power, loss increasing with frequency. The infinite series of lowpass elements is the ***distributed-constant*** description. Most of this Chapter talks about the ***lumped-constant*** descriptor, that of a transmission line at one frequency.

The physical quality for *impedance* is, for an unbalanced line, the ratio of the inner conductor outer diameter to the outer conductor's inner diameter. This is the basic formula:

$$Z_O = \frac{138}{\sqrt{\varepsilon_R}} \; \mathrm{Log}_2\left(\frac{D}{d}\right) \qquad \text{Where:} \qquad (18-1)$$

ε_R = dielectric constant (air = 1.0)
D = Inner diameter of outside conductor
d = Outer diameter of inner conductor

An approximation for *balanced lines is:*

$$Z_o = \frac{120}{\sqrt{\varepsilon_R}} \; \mathrm{Log}_2\left(\frac{b}{r}\right) \qquad \text{Where:} \qquad (18-2)$$

b = center - center distance between wires
r = wire radius

Note the similarity to unbalanced lines. Term ε_R is the same in both equations. Note also that the natural logarithm to base-2 is for ***ratios*** and thus the terms within parentheses can be anything as long as they are both the same value of dimension.

Transmission line *loss* is given as ***decibels of loss per hundred feet at specific frequencies.*** That is found in all coaxial cable specifications. For any length of line, determine the length based on 100 feet and multiply the db by that value. If the loss per hundred feet is 3 db and the length is

400 feet, then the total loss is 12 db.

How the *Standard(s)* Got Standardized

This was done beginning in the late 1920s and on throughout World War II. The best characteristic impedance turns out to be between 30 and 80 Ohms. For general use, 77 Ohms had the least error for lower power. For higher power the characteristic impedance is close to 30 Ohms for least losses. The ***50 Ohm*** lines associated with modern test equipment is a *compromise,* one that became a *standard.* By the time of World War II the USA military set up their ***RG-*** numbering system for cables which survives today.[1]

Prior to this time, the only real standard was ***600 Ohms*** impedance courtesy of the rapid expansion of wired telephony. The wider spacing of 600 Ohm lines suited the utility pole stringing for long-distance lines. That 600 Ohm impedance was also used in high-power Lines to HF transmitters to reduce RF current flow. The new technology of adding sound to silent films boosted the use of 600 Ohms for balanced lines in audio processing equipment and in broadcasting.

With the emergence of television and relatively-wide video-frequency bandwidths, ***75 Ohm*** standards were born. There is no defining moment nor movement towards 75 Ohm impedances, but for lower RF frequencies, 50 and 75 Ohm characteristic-impedance lines are essentially interchangeable for short lengths. What was needed for unbalanced lines was ***connectors.***

Connectors

The first specification for an unbalanced line connector was the ***PAL*** or ***TV Aerial*** type designed in 1922 by Belling and Lee in the UK. While in-use in Europe into the post-WarII period, it had a useful upper frequency of about 100 MHz.

The first US specification for relatively wideband RF service was the ***UHF Series***, which exist today mostly with amateur radio equipment and known by the old military designators PL-259 and SO-239. It must be remembered that, around 1940, any radio frequency higher than 30 MHz was considered ***ultra-high***. Originally intended for radar set IF interconnections, the UHF series is good to about 300 MHz. They are essentially dimensioned for 50 Ohms and will take RG-8 coaxial cable directly; depending on design, they can take higher impedance RG-9. Design patents have all run out. With some forcing, a banana plug will fit into a female UHF connector center pin.

The type ***N*** connector was the first commercial-military design of the post-WWII period. Designed by Paul Neill, it had a frequency response initially to 1 GHz but expanded to about 12 GHz in subsequent decades. It was designed for 50 Ohms impedance and could handle 500 Watts of RF power. Has screw-threaded locking of its outer sleeve.

The ***BNC*** and ***TNC*** types were designed by Paul Neill and Carl Councelman whose surnames are in the designator behind letters for ***Bayonet*** and ***Threaded.*** The *BNC* was first in 1951 and the *TNC* followed soon after. The quarter-turn-to-lock bayonet feature is excellent for test equipment. BNC upper frequency is about 3 GHz. The TNC is good to about 11 GHz. Mating dimensions are about the same as type N.

Type ***C*** is a physically-larger BNC type with bayonet locking. Good for approximately twice the maximum power of type ***N, BNC, TNC,*** it can go to 4 GHz. Seldom used now. Councelman

[1] This required some deep thinking to set up. There were few test equipments that were accurate enough to measure 50 Ohms at wideband RF at the time. Since then the basic formulas have worked accurately.

is written as being the designer.

APC-7 is a precision connector found on some instruments, an expensive connector but one which can mate often with predictable good results. Originally developed by H-P, it was improved by Amphenol and the name stands for Amphenol Precision Connector, the -7 standing for 7 mm. Not much used now except in network analyzers. Also known as ***GPC, GPC-7.***

SMA and ***SMB*** are described as *Sub-Miniature, type A or B,* are smaller than BNC and TNC and were first made during the mid-1960s. *SMA* is screw-thread locking while the *SMB* is push-on, push-off. Upper frequency about 18 GHz for the SMA. The SMA has a 5/16-inch across the flats locking and many an RF worker carried their own *SMA wrench* in their possession.

Type ***F*** got its name rather long after first appearance as a consumer TV antenna and set-top box RF connector. Millions have been made of this inexpensive-to-make connector and, from use, their upper frequency is about 1 GHz. It is roughly 75 Ohms impedance.[2]

RCA connectors were never designed as transmission line connectors, dating back to days of World War II for use with audio frequencies, generally in consumer electronics. Nonetheless, more modern versions are used as wideband video connectors in consumer market TV sets today. The name was popularized some time after first being used as shielded phonograph inputs by the RCA Corporation.

There are many other types of connectors, including a couple which can mate shielded balanced pairs, maintaining their pairing. This listing is for the most-used types.

Open-Wire Balanced Lines

These were used back when HF was standard for international radio communication. Some of their use is relegated to nostalgic mythos. Generally 600 Ohms impedance, they were installed by hand with spreader insulator bars every few yards to maintain their spacing. With such wide spacing they can handle high RF voltages without arcing, usually to 100 KW. Loss per hundred feet is quite low but upper frequency is limited to low-VHF.

Perhaps the most common type is 300 Ohm characteristic-impedance ***twin-lead*** (common name), easy to make with two wires separated from one another by polyethylene plastic insulation. TV *twin-lead* has been in use for over half a century and supported a small off-shoot industry of specialty items for insulators and mountings.

An offshoot of TV lead-in balanced pairs, ***ladder line*** is TV line with (usually) rectangles of the polyethylene insulation punched out. This changes the impedance to approximately 450 Ohms but limits the upper frequency again to low-VHF due to punch-out sizes. The name comes from the appearance of being a *ladder* with remaining plastic holding dimensions of the wires seemingly like the steps of that ladder. Also low loss as with open-wire lines but generally limited to HF and lower.

Coaxial Cable Construction

RG-8 (half-inch outside diameter cable, 50 Ohm) generally has a woven braid of solderable copper for the outer conductor, a solid plastic dielectric, and (for 50 Ohm cables) a center conductor of a few strands of solderable copper. Good-quality variants use a PTFE (PolyTetraFluoroEthylene

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[2] Very approximate. The author has never tested one but has cable TV in his residence that can reach 850 MHz without much problem.

or *Teflon)* dielectric. RG-9 is a companion type for 75 Ohm characteristic impedance.

Smaller RG-58 (quarter-inch outside diameter, 50 Ohm) has much the same construction and RG-59 (75 Ohm) has a solid-wire center conductor that becomes the center pin of type ***F*** plugs. For cable TV interior-exterior lines the outer conductor may be one or two wraps of foil as an economy factor.

Even smaller RG-174 (50 Ohm) cable, roughly an eighth-inch outside diameter, is made much like RG-58 with a woven copper braid outer conductor, plastic dielectric, and a few strands of center conductor. Such small coaxial cable has been used in *baluns* made with large toroidal core powdered-iron formers.

There are a few types of what appears to be coaxial cable that is really *balanced* and has an outer conductor, largely for protection and noise reduction. This type is more expensive and requires a special connector.

Any cable catalog will show many varieties of coaxial cable available and the specifications are good to use as a general guide. For most applications, flexibility is desired so manufactured coaxial cable is much cheaper to buy. It is easier still if the distributor sells cables with connectors mounted on each end.[3]

Rigid and Semi-Rigid Coaxial Cable

This is used for VHF-and-up on the interior of equipment. Outer conductor is a rigid tube of (usually) copper, PTFE or other plastic dielectric, and a solid-wire center conductor. Dimensions are generally governed by the coaxial connectors' drawings. Home maintenance stores carry smaller sizes of copper tubing suitable for use in building one's own semi-rigid, short coaxial lines.

The distinction between *rigid* and *semi-rigid* is vague. Generally, rigid coaxial lines can't be bent easily by hand while small copper tubing can be bent. Rigid coaxial lines are generally 1 5/8 inch diameter pressurized lines with dielectric generally air. Interior supports are usually plastic spacers about every half-foot or so. Rigid coaxial lines are used in microwave towers as feedlines. Semi-rigid lines have some limited flexibility and are used between assemblies of other electronics; to a certain degree they can be deformed slightly for mating their connectors.

More Specific Impedance Information

General

A ***matched*** transmission line appears to a source as a single resistor over a wide band of frequencies. The resistance value is that of its ***characteristic impedance.*** This allows maximum power transfer of a signal. When there is a ***reactance*** associated with a ***load***, either in parallel or in series, the maximum power transfer is ***reduced.*** Accordingly, if the source impedance is ***not*** used with a matching characteristic impedance, a ***mismatch*** occurs. That results in an interesting

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[3] Among hobbyists, being able to mount and solder PL-259 male UHF series connectors is some kind of male-to-male false distinction. In general, buying coaxial cable assemblies with attached connectors assures a good connector mounting. It is difficult and time-consuming to mount connectors of any kind and assure a good solder joint without melting the non-PTFE dielectric for most hobbyists. There should be no shame associated with buying an assembly with attached connectors.

phenomenon called ***standing waves.***

Imagine a wavefront traveling down a transmission line and meeting a *mismatch* at the load end. Part of the forward-going wavefront is ***reflected back*** to the source end. Measuring the RF voltage at any point in the line will show the total of ***incident*** (or forward-going) RF voltage and current with the ***reflected*** RF wavefront voltage and current. The end result is a number of peaks and valleys along the line representing the in-phase and out-phase RF voltage along with magnitude. With the frequency relatively stable, this has the appearance of ***standing waves.***

With a perfectly-matched source and load, there are ***no*** *standing waves.* RF voltage probed along the line will show a uniform RF voltage. *Standing waves* only exist with a *mismatch.* The greater the mismatch, the greater the peaks and valleys of RF voltage along the line. The ratio of maximum to minimum (the peaks and valleys) RF voltage is termed ***SWR*** or Standing Wave Ratio.

In the RF industry the usual term is ***VSWR*** for Voltage Standing Wave Ratio, a hardly-intrusive voltage probe into a transmission line is less likely to upset line conditions or bother a load impedance. *VSWR* is pronounced as *Viz-war* and is common to talking about the subject. Radio amateurs seem to favor *SWR* and pronounce it by the letters of the acronym or simply pronounce all the words.[4]

Discontinuity

Any physical change of conductor dimensions, even if the characteristic impedance is the same with either set of dimensions, will introduce a reflection. Such reflections can be seen using a ***TDR*** or Time Domain Reflectometer.[5] For HF and low-VHF such discontinuities are too small to be a problem unless there is an abrupt change in characteristic impedance. In the following the term *discontinuity* is used for any physical change.

VSWR Measurement

While it is possible to measure RF power to include phase, the technique is complicated. A voltage measurement turns out to be the easiest and least-intrusive, the most broad-band. That leads to using the ***magnitude*** of the complex quantity defining the forward, or ***incident*** power relative to the reflected power. Readers are cautioned that ***complex number*** quantities are explained in Chapter 2 along with how to manipulate them. For those who don't jump back, ***magnitude*** is:

$$\text{Magnitude} = \sqrt{(\text{Real Part})^2 + (\text{Imaginary Part})^2}$$

Note that, because both Real and Imaginary parts are squared, Magnitude is always positive. This is important since the ***detector*** may be anything, voltage or current or power, all with no regard to output polarity. Since most such *detectors* are broadband, they can be calibrated at lower, more convenient frequencies. Using *magnitude* is also good for calibration since a resistance value can

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4 The author isn't sure why the difference exists. VSWR was apparently first, but a few radio amateurs were rather clannish and wanted their own terms.

5 A ***TDR*** generates a very short pulse as well as generating a time-base waveform for an oscilloscope to show the characteristic impedance vertically with distance to a discontinuity horizontally. Quick and efficient, it is generally too expensive for hobby workshops.

be used as a reference with its reactance tuned out to zero.

Value of Mismatch

Using RF voltage ***in the direction along a line*** with *incident* power that which goes from source to load and *reflected* power that which comes back towards the source, it can be said:

$$\text{VSWR} = \frac{E_{INC} + E_{REF}}{E_{INC} - E_{REF}} = \frac{1 + \sqrt{(P_{REF})/(P_{INC})}}{1 - \sqrt{(P_{REF})/(P_{INC})}} \qquad \text{Where:} \qquad (18-3)$$

E_{INC} = Forward - direction RF voltage
E_{REF} = Reflected - direction RF voltage
P_{INC} = Forward - direction RF power
P_{REF} = Reflected - direction RF power

By using an identity of: $\rho = \sqrt{\frac{P_{INC}}{P_{REF}}}$ we can also $\text{VSWR} = \frac{1 + \rho}{1 - \rho}$

From these formulas, it is important to keep in mind that VSWR is ***always a number between 1.0 and much higher.*** Even at this much later time in the technology some insist that their equipment or measurements are so good that their VSWRs are *less than one.* Such less-than-unity claims are ***false.*** A VSWR = 1.0 is perfect, anything higher is progressively imperfect.

Measurement Methods

Probably the most common method is to use two power meters using a *hybrid transformer* or a ***directional coupler*** to separate the two power quantities, at least at HF and low-VHF. With a calibrated ***RF power ratio*** (as from the *rho* of the preceding formula), it is a relatively easy task to indicate the VSWR, usually by a meter on the front panel.

Hybrid transformers are covered in Chapter 17. A ***directional coupler*** has a lower end at about the 2m band. Such couplers are at least a quarter-wavelength long if made from a coaxial structure but have a ***directivity*** (difference of forward versus reflected power) of at least 20 db.[6] *Directional couplers* can be made in *lumped constant* style but have limited bandwidth.

Calibration of power-indicating VSWRs can be done with resistors measured at DC, usually with 1/4 or 1/8 Watt resistors, mounted in connectors with shortest leads possible. Since the formulas involve *magnitude* of power ratios, a reactance-less calibration standard will work fine. This leads into ***normalization*** of impedance.

Normalizing Impedances

[6] Most directional couplers cover at least an octave of frequency. They can be made if one is a machinist with machine-shop equipment in the workshop. Calibration can be lengthy although done with relatively simple test equipment.

If you are working with a ***50 Ohm system***, that is all components having a 50 Ohm characteristic impedance, then the magnitudes can all be ***normalized*** by dividing all magnitudes by 50. *Normalizing* impedance magnitudes doesn't change the polar form of phase angle and power ratios will always be the same. This eases manual measurement recording and presents a more condensed form of information...provided all viewers are aware of *normalization* being done.

Note: If using a polar form of a complex impedance, then normalization need only work with the magnitude. For a rectangular form of complex impedance, ***both*** real and imaginary parts must be divided by the normalizing value.

SMITH CHARTS

General

Smith Charts were first done by Phillip H. Smith beginning in the 1930s. By the end of World War II they had evolved into a two-dimensional form of representing a complex quantity of R + jX (or G + jB) that should normally take three dimensions. A *Smith Chart* has become so standard in the RF industry that some test equipment displays are done in the way of a *Smith Chart.*[7]

Interestingly enough, Smith's invention began while he was working on measurement of long-distance telephone lines operating at frequencies audible to humans.[8] A few evolutions later, they became the popular form used today at RF. The essential form has remained relatively unchanged since the 1939 version originally published in McGraw-Hill's *Electronics* biweekly.

Construction of the Chart

To those encountering a Smith Chart for the first time, it seems a confusing and (somewhat) contradictory *graph.* It is basically a set of ***circles*** of varying radii. An impedance plotted on a Smith Chart will have coordinates describing the rectangular form of complex impedance. This is found by the intersection of a resistance radius and a reactance radius.[9]

The basic Smith Chart has radii of ***normalized*** values, that is, the two impedance parts are divided by the ***system impedance.*** By standard use, the *system impedance* is usually 50 Ohms. The resistance and reactance radii are both valued from 0 to infinity. Each measured impedance of a line and/or load occur at a single frequency. Several frequency impedance measurements at different frequencies may be connected to form a curve of the *trend* of impedance change.

To best illustrate a basic Smith Chart, examine Figure 18-1. It shows the two sets of circles' radii each representing the separate parts of a complex-form impedance. The first is a series of ***constant-resistance*** circles with their center on a horizontal line through the middle, origin from the

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[7] For its uniqueness and importance at RF, a ***Smith Chart*** will have both the name and word capitalized.

[8] From ***Electronic Applications of the Smith Chart*** *in Waveguide, Circuit, and Component Analysis,* by Phillip H. Smith, Member of the Technical Staff, Bell Telephone Laboratories, Inc., published by McGraw-Hill Book Company 1969, Library of Congress Catalog Card Number 69-12411. This large-format hardbound book of 222 pages serves as an excellent source of information about transmission lines, waveguide, and applications of many things that can be measured in complex notation.

[9] This also applies to ***admittance*** and that will be covered a bit later in this Chapter.

extreme right to the middle. The second set is ***constant-reactance*** radii with their center on a vertical line along the right side with origin sliding vertically on the right side, inductive reactance on the top half with capacitive reactance on the lower half. Reactance radii are mirrored top and bottom with the only difference being the ***sign*** of reactance: by convention positive sign on the top, negative sign on the bottom.. End result of this type of plotting is a coordinate of two items of information: Resistance and Reactance, both related to the ***curving coordinates***. There is no straight-line Cartesian coordinate system of x-horizontal and y-vertical typical of other electronics. Once able to deal with curving coordinates, much of the confusion about a Smith Chart will disappear.

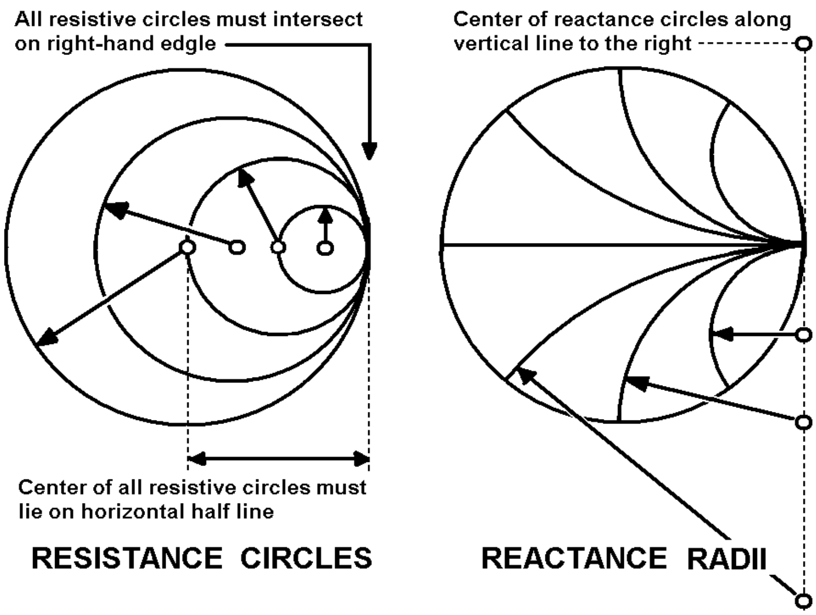

Figure 18-1 Construction of the basic Smith Chart

Built-In VSWR and Some Other Things

A simplified Smith Chart is shown in Figure 18-2. This is how it will appear on copies and overlays. Figure 18-3 is the halftone Smith Chart with a perfect impedance plotted, the point at the Chart center. That plot will have a normalized impedance of 1.0 + j 0 Ohms. Of course, that simple an impedance doesn't need a chart but it does orient the viewer to the beginnings of Smith Chart markings.

Figure 18-3 (less the center plot point) shows the coordinate curves of a simplified Smith Chart for the various examples in this Chapter. Resistance radii are at 0.25, 1.0, and 4.0 while reactance radii are at 0.5, 1.0 and 2.0, all left to right.

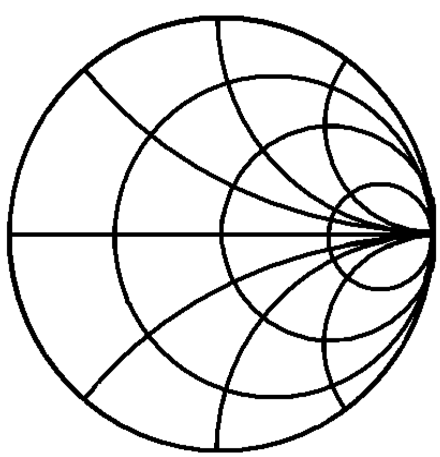

Figure 18-2 A simplified Smith Chart to be used in here.

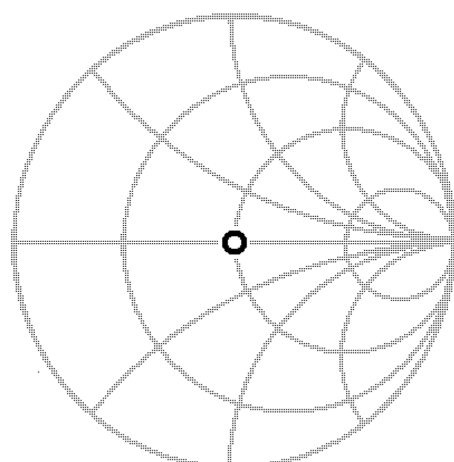

Figure 18-3 A perfect 1.0 + j 0 impedance plotted on a Smith Chart at its center.

Figure 18-4 is a bit different. There we have four normalized impedances of 0.25+j0, 0.46-j0.88, 4.0+j0, and 0.46+j0.88 identified by alphabet characters A through D. If these are ***normalized to 50 Ohms,*** they become (approximately) 12.5+j0, 23.0-j44, 200+j0, and 23.0+j44 Ohms. All four points will indicate a VSWR of 4.0 relative to 50+j0 Ohms. That includes any that fall on the centered dark circle.

Rotation is important. Those indicate rotation of impedance plot points along the transmission line. It is ***clockwise*** rotation to go ***towards*** the generator/source, ***counter-clockwise*** to go ***towards the load.*** What is most interesting here is that only a specific length of transmission line can help to match the impedance to be resistive, with no reactive component at a single frequency!

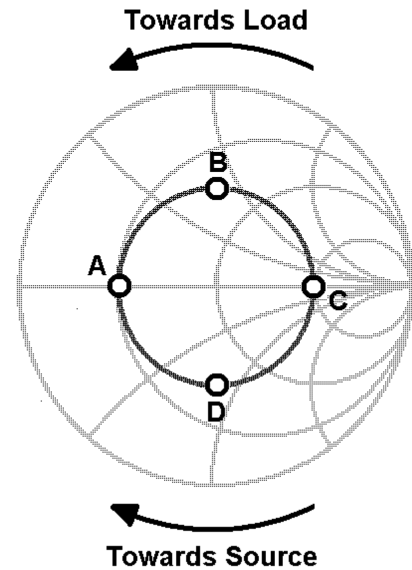

Figure 18-4 Four impedance values separated by 1/8 wave-length sections of line.

Rotation can also be an advantage in a physical structure. As an example, take a line that is exactly 0.25 wavelengths long and short the open end. Connect the other end to the line. Does it affect the main transmission line at that frequency? ***No.*** At that frequency it will appear as a very high resistance to RF, absorbing nearly nothing in power from the source. This is a good way to use a mechanically-shorted line as an ***insulator...***but only for this narrow frequency band.[10] At other frequencies it will appear as a finite complex impedance.

Another important thing about this *rotation* is that a shorted or open transmission line at higher frequencies, from high-UHF to microwaves, is that these lines can act very similar to ***lumped-constant L-C tuned circuits.*** Large filters can be made this way, all types.[11]

Familiarity and Strangeness

Some manufacturer's datasheets show Smith Chart plots for devices. While those are strange to those not familiar with Mr. Smith's remarkable diagram, they are simply rotational-scale plots of conventional, separate complex-quantity impedance plots. What is nice for manufacturers is that a Smith Chart can replace two separate plots of R and X components of a complex quantity, saving the manufacturer from supplying too much paper or diagrams. Once the reader has encountered the Smith Chart and practiced using it, its usefulness will be demonstrated.

A Case History of a Quick Match

Some time ago, a newly-moved radio amateur wanted to get on the amateur 20 meter band in a hurry. The mover had temporarily mislaid some of the belongings and the antenna tuner was

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[10] This has been used in several narrowband radio installations from low-HF on up to UHF.

[11] Scope of this subject is itself very large and not a part of this book. It can be found in any Internet Search, usually enmeshed with rather arcane formulas centered on laws of ***Field and Wave Theory.*** Just the same, it has been used in consumer electronics up to the end of this new millennium such as in (old-style) UHF converters having PCB-structure bandpass or lowpass filters. Where specifications are not strict this is a very economical way to do RF filtering.

not available.[12] The transmitter did not seem to load up as well with the owner's vertical so enter the author with his trusty *RX Noise Bridge.*[13] This author also had some sections of RG-58 (with connectors) and some spare HV, high-current capacitors. Some quick measurements at 100 KHz increments resulted initially as follows:

Frequency MHz	Beginning Impdedance	Calculated VSWR	Impedance with 12' added coax
14.0	39.3 - j50.8	3.04	25.7 + j34.2
14.1	27.8 - j41.3	3.27	31.7 + j46.6
14.2	23.3 - j31.6	3.15	45.2 + j57.4
14.3	24.5 - j17.5	2.36	80.7 + j47.0
14.4	27.7 - j 2.8	1.81	90.5 - j 2.7

The *beginning impedance* was resistive with a series capacitance. Adding almost a quarter wavelength of coaxial line (12 feet or about 0.239 wavelength in RG-58) changed that to resistive with a series ***inductance.*** Junk box capacitors totaling 200 pFd would have a reactance of -56.04 Ohms at 14.2 MHz. Putting that capacitance in series with the added line resulted in:

MHz	Impedance at added coax	Reactance of 200 pFd	New Z at Source	New VSWR
14.0	25.7 + j34.2	-j56.8	25.7 - j22.6	2.45
14.1	31.7 + j46.6	-j56.4	31.7 - j 9.8	1.67
14.2	45.2 + j57.4	-j56.0	45.2 + j 1.4	1.11
14.3	80.7 + j47.0	-j55.6	80.7 - j 8.6	1.64
14.4	90.5 - j 2.7	-j55.3	90.5 - j58.0	2.74

The end result was a rather low VSWR around 14.2 MHz and the transmitter loaded up nicely without any trip-off due to high VSWR. Note: After all this was done, a finaly measurement resulted in slightly-less VSWR which could have been several things.

Graphical Determination of Line Length

In the preceding example, there was a guess at the line length based on previous work with transmission lines. Such a length can be determined more exactly by simple work with a Smith Chart and an ordinary school compass. See Figure 18-5 for a simple example.

For an example, assume a frequency of 3.9 MHz and a measured impedance equal to 22 - j17 Ohms. Plot that point on the Smith Chart. Using the compass, draw a circle until it intersects the 50 Ohm resistive circle on the Chart. Note the clockwise rotation in fractions of a wavelength from the original plot point. The impedance at this second point is 50 + j84.5 Ohms. Putting a series capacitance of -j84.5 Ohms at that second point will result in a total of 50 + j0 Ohms. The problem now is getting the length of line needed to reach that second plot point. Given an approximate formula:

[12] Lifted from the author's article in ***Ham Radio*** magazine, January 1981, entitled ***A Quick and Simple Antenna Match.*** The mover eventually got all things to the owner. Number of frequencies reduced from 50 KHz for easier reading here.

[13] ***Improvements to the RX Noise Bridge*** by Robert A. Hubbs and A. Frank Doting, ***Ham Radio,*** February 1977. Author's Bridge based largely on that.

$$Length(feet) = K \cdot \lambda \cdot vp \, / \, F(MHz)$$

Where: (18-4)

K= 984 for Length in feet or 300 for Length in meters
λ = Rotation in wavelengths obtained from Smith Chart
vp = Velocity of Propagation of line (0.659 for PVC-dielectric RG-58A coax)

Given a rotation of 0.236 (for λ) and using RG-58A, the Length in feet comes out to:

$$Length = \frac{984 \cdot 0.236 \cdot 0.659}{3.9} = \frac{153.04}{3.9} = 39.24\, feet$$

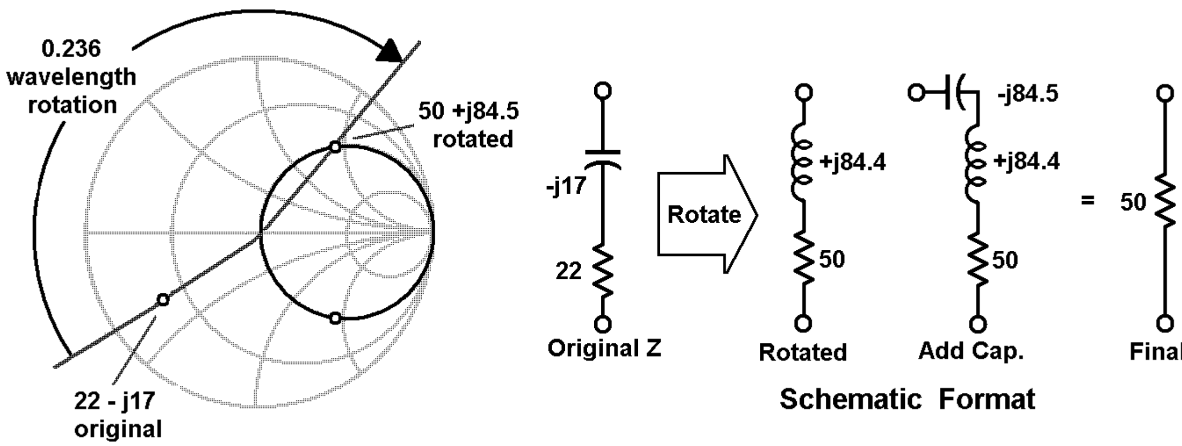

Figure 18-5 The 3.9 MHz example pictured in Smith Chart plot and in schematic form.

Series reactance of -j84.5 Ohms at 3.9 MHz is satisfied with about 483 pFd. A match to a ***resistive load*** can be done with not quite 40 feet of RG-58A and a single capacitor. For a limited monoband operation this is both economical and simple.

A Warning

While this simple match can get to 50 Ohms resistive ***does not mean*** it will be so over the entire transmission line. VSWR is still there, like it or not, and ***peak*** RF voltages and currents ***still exist*** in the line as follows:

$$E_{PEAK} = E_{NOMINAL} \cdot \sqrt{2 \cdot VSWR}$$
$$I_{PEAK} = I_{NOMINAL} \cdot \sqrt{2 \cdot VSWR}$$

Where: Nominal subscripts are RMS values at perfect match.

Allowing for *fudge factors,* multiply each *PEAK* value by two.

Using a Simple Antenna Bridge for Complex Quantities

The RX Noise Bridge

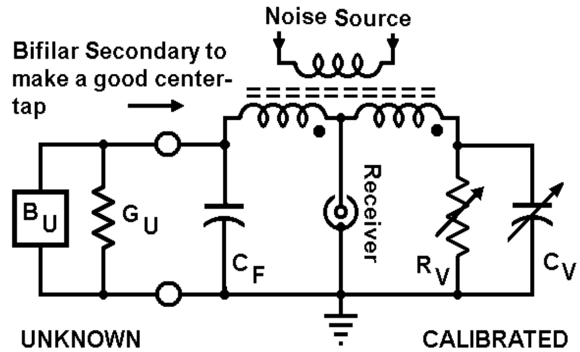

Figure 18-6 A basic Noise Bridge for measuring complex admittance. Rv is a 250 Ohm potentiometer, Cv can be anything from 200 pFd to 500 pFd. Cf is half the value of Cv.

The basic ***Noise Bridge*** is often dismissed by many, most of whom do not have any hand-held scientific calculator. That simple *Noise Bridge* is illustrated in Figure 18-6. Basically, this bridge measures ***unknown admittance.*** Since Z = (1/Y), the scientific calculator can easily do the inverse calculation to present the unknown parallel admittance in more-familiar series-form impedances.

It requires a good center-tapped secondary to make even voltages on the *calibrated* side and *unknown* side. The secondary windings should be done similar to Figure 17-16 in the previous chapter. The detector is any receiver tuning to the frequency being checked. The bridge is balanced when R_V and C_V are set for a ***null*** in the receiver.[14] The signal source is a wideband noise generator of low power. This is convenient for measuring antennas since the RF radiation is very low. It is unlikely to re-radiate and upset any on-going radio conversation.

Modification for Accuracy

C_F is approximately half the full-capacitance range of C_V. It is physically locked-down but measured. The C_V dial is calibrated positive through its entire range. The accompanying program has the term (C_F - C_V) so that polarity is correct for impedance. Making both from the same physical variable keeps a balance for the Bridge.

R_V can be a dual 1000 Ohm potentiometer, wired in parallel. It ***must*** be a continuous carbon type, preferably linear in taper. It has a calibration dial done directly by measurement of the dual potentiometer. This is good for up to the top of HF in frequency and a little beyond.

Calculation of null settings turns the parallel R-X of the Bridge into the answer of a series form of R-X of impedance. Bridge parallel-form allows the best symmetry to limit stray inductance and all have the same ground point. Wiring should be as short as possible and symmetrical about the transformer for best accuracy up to 30 MHz.

Choice of a Calculation Device

Calculation programs were originally published for an HP-25 and a TI-58 handheld calculator in ***Ham Radio*** magazine.[15] In the 35 years between publication (1978) and time of this writing (2013) there was an explosion of calculator models and features. Rather than restrict

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[14] The noise source can be modulated on-off at up to 1 KHz for easier identification.

[15] May 1978 issue of ***Ham Radio*** magazine, pp 34-42. See footnote 6 in Chapter 37.

equations to a specific family of calculators, it was decided to present only the equations in a generic BASIC listing. This avoids some obsolescence. Given small *lap-top* computers (not available in 1978) allows portability. Value scaling is done with all frequencies in MHz, Ohms, and pFd.

The Program in Generic Basic

Start Up

This assumes processor initialization is done, usually specific to a particular BASIC dialect. There are no Input-Output statements since those may vary with BASIC dialects. All output is by numbers. Programmers can code in whatever they want for Input-Output format. It is assumed that all variables in subroutines are ***global***; that is, they are unchanged wherever they are in a total program. In the program listing, anything to the right of a *semicolon* is a ***Remark.*** Remarks do not execute; they are there to help explain the program to a user. There are three parts to this program:

1. ***Setup***, primarily for part 3, sets internal constants for up to 50 MHz.
2. ***Direct*** input of admittance from the Bridge; program handles polarity signs.
3. ***Indirect*** input where a known transmission line is between Bridge and Load; program handles polarity signs.

Polarity of reactance is signed for ***impedance*** results**,** that is, a positive reactance is inductive and a negative reactance is capacitive. Outputs are in series-form impedance. Variable names try to stay with the original (calculator) nomenclature given in the 1978 *Ham Radio* publication.

Stopping a program, entry of Bridge data, output of impedance depends on the dialect of BASIC used. Six-digit-accuracy floating-point decimals are assumed.

Program

```
          TWOPI = 6.283185
;     Enter a steering variable for Feet or Meters for line length.
          FLAG = 0         ; <- Assumes a Line is used
;     Enter the fixed capacitor (variable = CF) in pFd.
          CF = ???         ; depends on value used
;              FOLLOWING INPUTS ASSUME VARIATIONS
;     Characteristic impedance of Line, variable name RO
          RO = 50          ; <- Assumes 50 Ohm Line, can change
;     Velocity of propagation, variable name = VP
          VP = 0.6594      ; Solid polyethylene, can change.
;     Enter the physical length of line in Feet or Meters, variable
;     name = LEN
;          For length in Feet:
                KAY2 = 0.73204 * LEN / VP
;          For length in Meters:
;               KAY2 = 2.4017 * LEN / VP
;     Enter Attenuation of Line whose length was just calculated,
;     variable name = ATT
          ALPHA = ATT / 4.343
;
;                          MEASUREMENT
;
;     Enter the frequency of measurement in MHz, variable name = FREQ.
```

```
;          Note that this could be part of an array, but that is not
;          shown here. This generic program assumes one reading equals
;          just one frequency.
MEASURE    FREQ = ???            ; <- Always in MHz
;
;     Enter CV and RP from Bridge readings when receiver is nulled.
;
           OMEGA = TWOPI * FREQ * (10^(-6))
;                            Above assumes FREQ is in MHz
;
           XP = OMEGA * (CF - CV)
;                Above assumes CV is calibrated in pFd and CF is very
;                much locked down and included as a constant in program.
;                That results as the proper polarity sign for XP in
;                resulting impedance calculation.
           T1 = RV * RV
           T2 = XP * XP
           T3 = T1 + T2          ; <-  (Common Denominator)
           RS = RV * T2 / T3     ; <- Real part of Generator-end ZG
           XS = XP * T1 / T3     ; <- Imaginary part, Generator-end ZG
;
;     There is just one specific integer flag variable for steering,
;     variable name = FLAG. It will bypass the following two equation
;     sets if NOT equal to 0.
;
      IF (FLAG.NE.0) GOTO SKIP
;
;     CALCULATION OF TRANSMISSION LINE FROM GENERATOR TO FINAL LOAD
;          [Adapted from Program 3 in Ham Radio May 1978 article]
;
           EPS = (2.719282)^ALPHA     ; <- Raise e to the power ALPHA
           T4 = EPS * EPS
           BETA = KAY2 * FREQ         ; <- FREQ in MHz
           T3 = T4 + 1 + (2*EPS*COS(BETA))
           XA = ((EPS+EPS) * SIN(BETA)) / T3
           RA = (T4 - 1) / T3
;
;     ROTATION OF TRANSMISSION LINE FROM GENERATOR TO FINAL LOAD
;          [Adapted from Program 4 in Ham Radio May 1978 article]
;
;          RO = Real part of ZO, characteristic impedance of Line;
;               Imaginary part is zero.
;          RG = Real part of direct Bridge reading.
;          XG = Imaginary part of direct Bridge reading
;          RA = Real part of Line between Bridge and Load
;          XA = Imaginary part of Line between Bridge and Load
;
;               (RG - RA*RO) + j (XG - RO*XA)
;     ZL' = ----------------------------------------     ZL = ZL'*RO
;          (RO - RG*RA + XG*XA) - j (RA*XG + RG*XA)
;
           T1 = RG - (RA*RO)
           T2 = XG - (RO*XA)
```

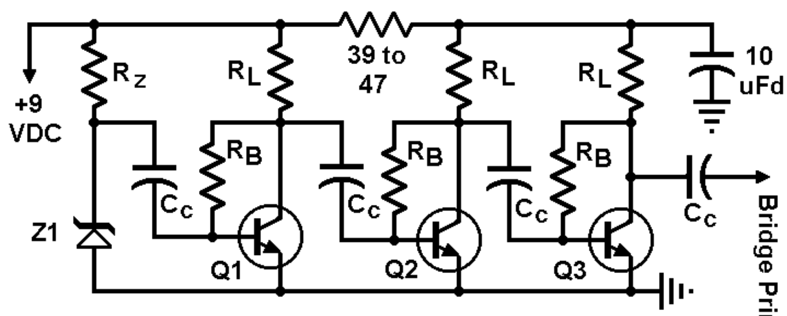
```
            T3 = RO - (RG*RA) + (XG*XA)
            T4 = (RA*XG) + (RG*XA)
            TD = (T3*T3) + (T4*T4)
            TR = RO*(((T1*T3)+(T2*TR))/TD)   ; Real part of solution
            TM = RO*(((T2*T3)-(T4*T1))/TD)   ; Imaginary part, solution
            GOTO VIZWAR
;
;     Direct Load impedance with NO Line between Bridge and Load.
;     Temporaries TR and TM used to make it common for OUTPUT routine.
;
SKIP        TR = RG
            TM = XG    ;=> Flow to Output of Impedance at Frequency
;
VIZWAR      T1 = TR - RO
            T2 = TR + RO
            T3 = TM * TM
            RHO = SQRT (((T1*T1)+T3)/((T2*T2)+T3))
            VSWR = (RHO+1)/(RHO-1)
;
OUTPUT      ???              ; <- [whatever desired]
;     OUTPUT of Frequency, Impedance data here (Real and Imaginary) and
;     VSWR at a single frequency, then a loop back for a New Frequency.
            GOTO MEASURE
            END              ; End of program.
```

A Noise Source

A noise generator is shown in Figure 18-7. It takes the small amount of noise present in a zener diode and amplifies this for the Bridge.

This is not a critical circuit. Powered from a +9 VDC battery, the wideband amplifier must have a minimum of 60 db voltage gain at 30 MHz with a minimum of 70 db voltage gain at 1.6 MHz.

If desired, an astable multivibrator can *key* the Q1 stage at about a 400 Hz rate. That is for easier identification when trying to null the Bridge.



Transistors are 2N2222 or similar. Z1 is 5.4 to 6.3 V type with Rz selected for slightly less current. RL is 330 Ohms. RB is 100K. 9 V battery drain is 30 to 35 mA. Voltage gain of Q1 to Q3 is about 90 db at 1.6 MHz, down to 70 db at 30 MHz.

Figure 18-7 A noise source for the Noise Bridge. Cc is 4.7 to 22 nFd, disc type. Bridge Primary winding is grounded on one side. PNPs can be used if the battery supply polarity is reversed. Put a switch on the battery.

The amplifier of Figure 18-7 draws 28 mA, exclusive of the Zener. It is a modification of the Hubbs and Doting article illustration for broader bandwidth, hence the increased current. This is normally acceptable since most 9 V batteries can handle 45 mA for hours.

Other Types, Not Quite Transmission Lines

Waveguide

Waveguide belongs with a category of guided transmission, having a whole new set of characteristics and parameters governed by *Field and Wave Theory*. It is too complicated to be covered in here and various subjects on it belong in its own book. The most common type of waveguide exists as a 1x2 dimensioned cross-section rectangular tube with a particular Electric and Magnetic field arrangement. Next follows circular waveguide, itself different from rectangular.

G-Line

G-Line, the colloquial term, is a sort of in-between of coaxial transmission lines and waveguide. There is only one wire, covered with a thin coat of dielectric, and the fields and waves travel along the wire, the dielectric, and immediate space around it. Launching and capturing a wavefront uses a horn-like structure, usually fed by coaxial cable, the outer coax conductor connected to the horn. Coaxial cable center conductor goes to the G-Line wire.

References for Chapter 18

[53] Theory and Problems of TRANSMISSION LINES, a Schaum's Outline Series of McGraw-Hill Book Company, by Robert A. Chipman, PhD, University of Toledo, 1968, 236 pages, 8 ½ x 11 format with 165 solved problems in detail. The author considers this book as one of the ultimate texts on transmission lines over frequencies of audio through UHF. It was apparently out of print by 2013 but may be found at used-book stores. It can be math-heavy in places but tells almost everything in detail. Much of the formulas were taken from this reference. Highly recommended.

[54] Electronic Applications of the Smith Chart, by Phillip H. Smith, McGraw-Hill Book Company, 1969. 222 pp, 8 ½ x 11 format. The inventor of the Smith Chart explains it all and some more for wired and waveguide, including Negative Smith Charts, Rieke Diagrams, and Network matching solutions that can apply to Chapter 8 of this book. Hardcover version includes overlays of basic Smith Chart diagrams. Highly recommended.

[55] ***How to Use the Smith Chart,*** **Ham Radio** magazine, November 1970 and (reprint) March 1978, by James Fisk, W1DTY. A clear and no-nonsense detailed explanation of a Smith Chart principally for VHF and below frequency use. Recommended.

[56] ***Improvements to the RX Noise Bridge,*** **Ham Radio** magazine, February 1977, by Robert A. Hubbs and A. Frank Doting, beginning page 10. One of the first accurate Bridges utilizing measurement of the Load as an admittance for better accuracy.

[57] ***Antenna Bridge Calculations,*** **Ham Radio** magazine, May 1978 by the author (pp 34-44) for a Hewlett-Packard HP-25 handheld calculator; In the same issue, pp 45-47, is ***Noise Bridge Calculations*** by T. J. Anderson (no relation to the author) for the Texas Instruments TI-58 handheld calculator.

Chapter 19

Physical Design, Homebrewing

Schematics are nice but the parts have to be ***built.*** There's an infinity of choices, all depending on the skill of the *homebrew designer* and ability to work with tools. Presented here are some guidelines with only a ***few*** choices and styles. There are some recommendations on hand tools and a way to hand-craft etched circuits on PCB stock, plus some elementary guides of fasteners and general drill-hole and fastener dimensions..

General

Homebrewing has nothing to do with potable beverages; it means (in the colloquial sense) ***making things at home.***[1] To do that requires that parts ***fit together physically.*** You need dimensions of the parts along with ways to mount them. That takes ***planning ahead.*** That, in turn, takes drawing or sketches with legible information on how things fit together. While doing that there are thousands of choices of arranging and mounting, all with sturdiness of structure to fit a particular physical environment.

Breadboards[2] are fine for prototyping circuits for their *electronic function* but are usually overly large and cumbersome. Once a breadboard prototype has been proved, the planning can begin to design and make it into a useful electronic device. In the electronics industry the term ***pre-production prototype*** is often used. This is to make a trial for factory-assembly and test, all for the purpose of checking viability of test procedures and *reducing assembly costs.* Home designers do ***not have to copy commercial designs.*** They can make their own and eliminate this in-between step between laboratory and production. Neither does the home designer need to follow certain styles of fastening things which may be fine for a factory to cut production costs to a minimum.

Think of the entire design process as a sort of jig-saw puzzle. Usually, for a device that has manual controls, the arrangement of front-panel parts must fit a physical style familiar to the builder. If this is for mobile or hand-held use, it must be more robust physically than something that will remain in a residence. Add to that temperature extremes and variations in AC line power Voltage as those apply.

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[1] Alcoholic beverages such as mead, wine, beer were once ***brewed*** by individuals. As civilization progressed, groups formed to make the beverages in bulk and sell that. Those groups became *brewers* with self-imposed titles denoting their profession. Individuals could still make their own in small quantities and such were called ***home-brewers.*** *Homebrewing* as a term expanded to denote anything made by individuals in any activity.

[2] Once upon a recent time, every home kitchen had ***bread-boards.*** These were simple pieces of flat boards, usually mounted along with drawers in kitchen furniture, usually with the original purpose of *slicing bread,* stopping a knife travel without causing it to lose its edge. After the appearance of already-sliced bread, the *breadboard* lost its purpose. At the same time home electronics could be made on such now-useless-in-the-kitchen devices using wood screws to hold down parts. The term ***breadboard*** became synonymous with electronics prototypes and the name stuck as part of the jargon in allied industries.

Planning Ahead With Sketches and Drawings

An Example, a Digital Dial Retrofit

What was desired was an increase in the accuracy of an old HF double-conversion commercial receiver to replace a mechanical slide-rule style dial with a digital dial to show carrier frequencies to a resolution of ± 10 Hz. Done with discrete logic at some time in the past, it was very similar to that in Chapter 48 but with limited bands. The available space within the old receiver was at the top, to a limitation of about 1 ½ inches thick. Width was limited to about 7 inches. This new digital dial would take the place of an old conventional string-and-pulley type of slide-rule dial.

Physical support would be largely at the back end, screwed to the rear panel. The front end would go into the back of a special dial bezel molded out of plastic. That was done using commercial elastomeric RTV as the mold, based on a plaster form. The plastic was a transparent acrylic (with hardener) bought at a local *Michael's* arts and crafts store. The visible bezel portion would be sprayed with glossy black enamel when done so as to match knobs on the front panel. A slight to-and-from slippage was built-in to the bezel back. This provided greater ease in mounting within the receiver.

Somewhat-large LED 7-segment numeric indicators would be used with numeral heights of about 5/8 inches. Six would be required, only the lower five decimals actually counting. Nearly all digital circuitry was logic device to logic device, the planning could go directly to a double-sided commercial prototype PCB. This prototype PCB could hold up to 30 16-pin DIPs. By some auxiliary sketches, the two LO Preamps could be built with separate passive components and transistors, taking the space of slightly more than two DIPs, each. Using an old 100 KHz quartz crystal (tall can), that would take three DIP spaces, largely on a daughter board of phenolic-paper. The counter circuitry would require 19 DIPs.

Rummaging through the non-electronic junk turned up a dishpan-style sheet metal box with an opening on the largest side. Originally a shield cover, its interior size was 5.50 by 8.75 inches, 1.00 inches deep. It had two flanges pointing outward along the edges, each about 0.75 inches. Using some PCB stock that was 1/16 inch thick, the opening could be covered so as to keep any radiated EMI down from the rest of the receiver.

A number of 22/44 pin edge connectors would fit the Prototype PCB. There was also a small piece of 1/8" thick transparent green Lucite which would make a nice optical filter for the green glow of the 7-segment LED digits. There was also an assortment of metal spacers, both threaded and through-hole, which could be support items.

The final drawing is shown in Figure 39-1. This used a xerocopy of the commercial prototype PCB. It was deliberately turned *upside-down* to present a good appearance when mounted in the receiver. The box (aluminum) was nicely Iridited.[3]

Physical support begins at the rear. Threaded spacers hold the PCB edge connector firmly to the rear. Two small pieces of oak are cut and slotted, the slot holding the PCB intact with some epoxy to hold it fast. The epoxy can also enter the corner holes of the PCB to make sure of physical support. A small piece of PCB was made for the LED numeric assemblies. Their LED current-setting resistors (not shown) are soldered between the LED backs and the prototype PCB. The oak wood blocks can be drilled and tapped to hold the green optical filter and LED PCB to the blocks.

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[3] ***Iridite*** is a chemical treatment of aluminum, puts a light yellowish color to aluminum.

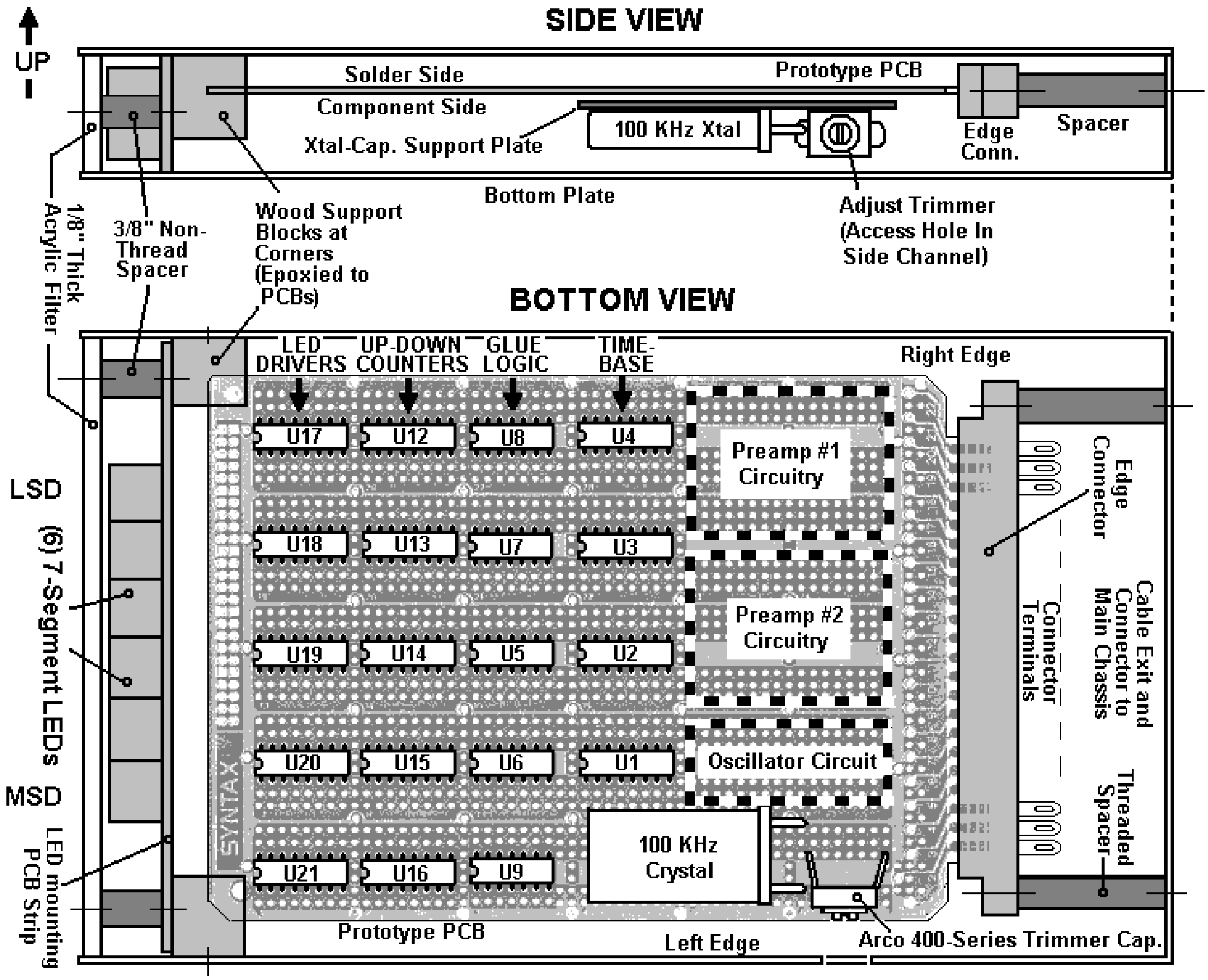

Figure 19-1 A finished (cleaned-up) sketch of the old *Digital Dial* used as an example.

Space at the back of the box has a grommet for wires to the main receiver. There was also space for a 2716 EPROM to hold counter preset information, addressing coming from main receiver bandswitch. At first thought, the EPROM could have been mounted on a small PCB daughterboard under the main logic DIPs. That obstructed many of the pins of the logic and was changed to the back end. There is enough room back there to hold a couple of 2716 EPROMs if needed.

There is a small access hole on the Left side to tweak the small Arco trimmer capacitor for time-base calibration after the digital dial retrofit receiver installation is complete.

There was some trouble of a slight shifting of intensity of the LEDs from various combinations of numerals. This was traced to the existing power supply in the commercial receiver. Using another power transformer winding, the LED and CD4511 latch-decoder-driver packages were powered with an additional rectifier and series-regulator (TO-3 diamond case), those components mounted on an aluminum plate that replaced the *bottom cover* made of PCB stock. Those components were mounted in the relatively-open space under the U5 to U21 location below the prototype PCB on Figure 39-1.

By measurement, there was room for a 3/4-inch protrusion of the TO-3 case of the series regulator with a plastic film and small aluminum protective cover (for the regulator IC case). The new aluminum plate bottom panel acted as a *heat sink* to draw heat away from the regulator case body. A full-wave diode rectifier and its input capacitor (made from smaller electrolytics in parallel) along with a couple of 1/4-Watt resistors allowed the circuit to be laid rather flat. Wiring was laid flat against the Right side in the spare space to the back and cable exit to the main chassis.

Current demand for the LEDs at +5 VDC was rather high, varying from 290 mA to 670 mA depending on te number of segments lit. By later estimate, each segment took about 18 mA when on.[4] This was a bit large but the optical green filter (also from the Junk Box) was a bit denser than desired. Note: Green LED segments took a bit more voltage than the Red variety and also had a slightly-less luminous intensity. An LCD would have been better but those were not as available at the time of design and PIC was just getting started on producing microcontrollers. The end result was a good fit into the commercial receiver.

Sketch Materials and PCB Etching

A combination of *quadrille paper* and *vellum* (from a tablet) were used with sharp pencils. There was no need of any PCB layout in this example. Figure 19-1 was considerably cleaned-up for this drawing. Such neatness is not necessary, but ***exact dimensions are necessary.*** The prototype board was already etched with triple through-hole lands to allow conventional Teflon-insulated wire interconnections. LED pins were wired directly to their current-setting series resistors so an LED support plate wasn't needed to be etched.

There are several nationally-advertised PCB prototype houses that will take on small quantity production. At least two have their own layout software compatible with PCs. There are several kits on the market for making one's own PCBs, including etching of copper foil and a few with PC-compatible foil layout or photo-exposure of resist, ready for etching.

Working With Small Projects, to Scale

This is possible, such as making drawings and sketches to a 1:2 ratio. The final finished work will ***not be as neat*** as such drawn information, depending on the fabrication tools at hand. One good thing for eyesight is a head-mounting optical stereo magnifier. The arts-and-crafts variety (such as from a local *Michael's* store) is as good as a fancier, more-expensive type found in electronics distributor catalogs.

Inches or Metric Measurements?

That depends on your own work experience or mental flexibility. You will probably need to stay within ***one*** of them. Legally, 1.000 inch equals 2.540 Centimeters. Exactly. By treaty. The inverse is 1.000 cm equals 0.394 inches (by calculator). Most IC datasheets now have physical sizes given in both measurements.

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[4] Supply for the LEDs was from the old receiver's tube rectifier replaced with (then) new silicon diodes. The 5 VAC power transformer winding had a 10 Watt rating and was slightly greater in voltage. At first a voltage doubler was thought necessary but a full-wave diode rectifier could provide just enough voltage to overcome the series-regulator *headroom* voltage. That is not always the case with old plate-filament power transformers but it worked in this example.

Materials for Fabrication

Do-It-Yourself chain hardware stores are the places to start. They have all kinds of fasteners, insulating materials, sheet glass and transparent plastics, various aluminum extrusions, plastics for sprinkler pipe installations, paint, some solvents, and all sorts of hardware that might apply. Most carry tools of many kinds plus drill bits, tool chests, container cabinets. Fancy tools aren't a necessity; one can get by with a 1/4-inch electric drill and a rotary tool such as made by Dremel (largely for cutting).

The ***Internet*** has to be consulted for sheet aluminum and PC Board stock. In the USA a lot of PCB chemicals have been banned from small mail-order sales. Living in an urban area will uncover several stores that sell some of the banned material over-the-counter.

Metal

Aluminum is always good. It is conductive, it is light-weight, and can be cut and formed with most hand tools. If you are starting with bare aluminum sheet stock, you need a way to cut it to size, to bend edges. For cutting a ***metal shear*** is preferred**.** Bending should require a ***metal brake*** for things like box edges; one can get by using clamped-on wood pieces..

There are still ready-made aluminum cabinets and some chassis. You need a listing of sizes for such things at hand. Remember the visualization problem, the how-to-build-it and how to access the interior of boxes.

Depending on the manufacturer, aluminum may be marked by a four-number code. A 2024 type aluminum is good for drilling. A 6061 type is fairly rigid, good for un-bent cabinet pieces. There are many kinds of aluminum alloys so it is good to have a reference from a local supplier handy before commencing with a project.

Wood Products

Lumber yards are the source for plywood. The ***finish grades*** will increase the price as will the number of plies. Quarter-inch plywood generally has only 3 plies, is about the thinnest for prolonged use. Next is 3/8-inch thickness, generally rare. Half-inch thickness is good but begins to be bulky. Half-inch ply generally has 5 plies. Three-quarter-inch thickness generally has 7 plies but begins to be quite bulky. A ***select finish*** on one side is necessary to get of knots in plies.

Masonite is becoming less available but a smooth finish on one side is good for things like covers that will become front panels. Masonite drills and files easily. Most raw stock is 1/4-inch thick. It is almost mandatory that any finishing be done with a ***primer*** paint ***first*** on Masonite.

Wood products don't have physical stability of metal unless it is at least a half-inch thick.

Plastics

Polycarbonate and acrylic plastic panels can be found in small sizes at do-it-yourself stores under trade names of ***Lexan*** and ***Lucite.*** Those usually have an adhesive cover paper to protect the product. ***Keep that paper on*** until finished. Such paper is good for marking, such as for drill holes.

Because of lower melting point of most plastics, drilling and machining should go at a slower speed than with wood. Wood products generally need a slower speed than metal machining. One good thing is that machining bit pick-up from plastics can be snapped off by hand.

Transparent plastic sheet can have marking decals placed on the inside (in reversed order) to use the sheet as a protections from fingernails, rings, etc., that would normally smudge such marks. Inkjet-printed legends can also be in color to add some zip to appearance. ***Translucent*** plastic sheet has to be treated like metal since it is only partially clear.

Acrylic plastics are affected by most paint solvents, such as acetone. They can be painted but it requires a ***dust coat*** first (very light coating), then a slightly-thicker coat after that. Usually an enamel has little effect on either acrylics or polycarbonates; it is better to try that out on a scrap piece first to make sure.

Printed Circuit Board Stock

This is usually 1/16-inch thick, ***foil one side*** or ***foil both sides.*** Go for fiberglass-epoxy substrate rather than a phenolic-paper substrate. Phenolic-paper substrates are good for bread-boards but some of the cheaper kinds can warp over time. Fiberglass-epoxy is generally colored a light green.

Copper foil on such ***FR4*** stock is made of electro-formed copper and bonded with a high-temperature adhesive. PCB stock is sturdy enough to make whole enclosure boxes, using a large soldering iron to join edges. Note: Such structures need a physical jig to hold the PCB pieces in place until soldering is done. Larger soldering iron heat may cause some PCB stock to move slightly during edge-soldering. Some simple carpentry tools can aid in making such jigs. The author uses a small triangle tool for that, PCB pieces secured to scrap wood sides using existing, planned drill holes. It should be noted that a fair amount of scrap plywood will accumulate in time to make such jigs.

Metal Angle Stock

Most chain hardware stores have special racks of (mostly) aluminum extrusions. ***L-channel*** is the most versatile. Most racks have 1/2-inch and 3/4-inch sizes in bulk lengths of 4, 6, and 8 feet. Such can be cut to size with a Dremel rotary tool having a ***cut-off wheel.*** Wheels don't last long but are fairly inexpensive purchased in bulk of 30.

L-angle stock usually has a 1/16-inch to 3/32-inch thick side. It may be used with a small ***bench vise*** for more-precise drilling. As edges of cabinets, such L-angle stock is quite robust. They can be secured to one another for more strength.

Plastic Boxes

There are quite a variety for sale, all sorts of dimensions. Most are prototypes for production work, thus the sizing. They have quite a variety of material. The most robust of the plastic types are ***ABS*** or acrylonitrile-butadiene-styrene. A reasonably high-temperature plastic, ABS can be machined at wood-machining speeds.

It should be noted that all plastic boxes are ***quite transparent to RF.*** This must be taken into account on the first paper lay-out. If necessary, use some heavy kitchen aluminum foil (available at food stores) cemented into boxes and properly grounded for RF shielding. For RF shielding, use PCB stock; see this Chapter a bit later for how to make those.

Paint

Hardware store paint is generally of the ***enamel*** kind and in spray (aerosol) cans. That is, it does not have a solvent to affect acrylic or polycarbonate plastics. Note: If questionable, try it on a test piece first. Paint will not affect metal or wood. Most metal surfaces need a ***primer*** base coat. Primers seldom affect the finish color; if they do a second paint coat generally covers that up.

Sanding sealer is good for wood surfaces. Caution as this has become expensive by this new millennium. Sealers may be lightly sanded after at least three days of drying. That smooths the surfaces. Most sanding sealers have an acetone base, can be thinned with about a quarter to one-third ***acetone*** for brushing. ***Acetone*** solvent is available in hardware stores, usually in pint cans.

Varnishes (also become rather expensive) are good for a variety of surfaces, including inductors. It is a good light coating for holding inductor windings in-place, a substitute for so-called ***Q-Dope*** (an acrylic in an acetone solvent). By experience, the author has found that *Q-Dope* tends to ***lift*** from non-porous surfaces. ***Q-Dope*** does not improve inductor Q values; it tends to degrade Q slightly, common to all coatings. The author prefers ***McCloskey Gym Seal*** *varnish* (also expensive) even though it takes a few days longer to adequately dry.

Adhesives

Masking tape has been regarded for years as a *third hand* in laboratory breadboards. It is cheap and some brands leave little residue. There are many types. ***Masking tape is not permanent.***

Two-Part Epoxy is a good all-around cement. There are many kinds on the market, slow to fast drying. The two parts in squeeze tubes are good for storage, can remain good for several years. Most kinds can be mixed on pieces of bond paper, the paper discarded when done. Caution: Epoxies will stick to just about anything, be careful on discards.

Model hobby cements used with die-cast plastic models are a good all-purpose cement. The older type using acetone based solvents generally affects many plastics at joint edges. Some plastics may be affected by modern nitrate-based model cements so questions need a test piece done to see the effect.

Fasteners

This means ***nuts and bolts.*** These are purchasable in hardware stores in bulk, usually by the box of 100 or more. The cheapest are made of brass, unplated, usually graded ***UNC*** (Unified National Coarse). Next are the plated variety. Try to avoid those since the plating may actually oxidize after a few years. The best is ***stainless steel,*** graded ***UNF*** (Unified National Fine), also the most expensive. Stainless variety will last for years, no plating to come off or oxidize to leave odd residue on fingers and tools.

In addition to threaded nuts and bolts, ***flat washers*** and ***lock washers*** are needed to keep a tension on fasteners and to avoid scarring flat metal surfaces. Again, the stainless steel variety is best. Generally, ***lock washers*** should have ***internal teeth*** for ease of handling.

Threads, Sizes, Drill Holes

Threads are given as a dash number following a size number. Threads are measured ***per inch.*** Size numbers are arbitrary, lower numbers the smallest. Table 19-1 gives the common sizes, clearance diameter and Tapping drills for 75% of thread.

Table 19-1 Common Screw Fastener Sizes, Inches
[Drill ID number given in square brackets]

Screw Size	Thread	Clearance	Tapping
2-56	0.086	0.096 [41]	0.070 [50]
4-40	0.111	0.128 [30]	0.089 [43]
6-32	0.137	0.149 [25]	0.107 [36]
8-32	0.163	0.177 [16]	0.136 [29]
10-32	0.189	0.201 [7]	0.159 [21]
10-24	0.189	0.201 [7]	0.150 [25]

Thead **column is maximum to screw threading.** ***Clearance*** **column is minimum for good fit.** ***Tapping*** **column is for ~75% threading.**

The table assumes a steady drilling platform with jig-fastened work. In case of using a hand drill, use the next-size larger clearance drill and make up the mounting difference dimensions with flat washers under the nuts and screw heads.

Screw Head Shapes

There are a great number of different screw heads available. Figure 19-2 shows the most used types.

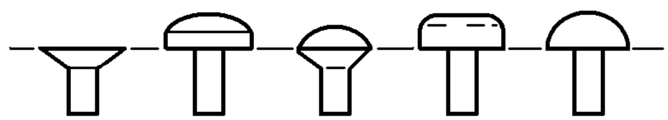

Figure 19-2 More-popular screw head types.

It should be noted that Flat-head types come in two head angles, 80° and 100°. It is difficult to tell the two apart by casual observation. One way is to have both counter-sinks in a scrap of metal and try their fit. Rather than use a counter-sinking tool cutter, a common oversize drill bit can create the 80° angle. Note that an *oval head* type requires a counter-sink similar to a *flat head* but also has a rounded top like the *round head.* Wood screws generally have *round heads.*

Driver Tool Sizes

Most popular heads come with *screwdriver slot* or *Phillips (cross-recess)* driver holes. Set-screws (headless) have either a *screwdriver slot* or an *Allen wrench* driving tool cutout. The Allen type is a 6-sided (hexagonal) shape and can be found in multiple-size tool wrench carriers.

For Phillips cross-recess cuts, a #1 Phillips driver is good for screw sizes 2 and 4. A #2 Phillips for screw sizes 6 and 8. Depending on the screw maker, either a #2 or #3 Phillips is for the screw size 10. A smaller-than-appearing Phillips driver works well, one of the features of the Phillips design.[5]

Nut Sizes

[5] The ***Phillips head*** is so named in honor of Henry F. Phillips (1890-1958) who purchased the rights from John P. Thompson about 1935. Phillips filed for a US Patent on it in the same year (2,046,837). First licensed to a Rhode Island company, Phillips cross-recessed screws were used on GM's 1937 Cadillacs.

The most common type of screw fastener receptacle is the hexagonal-outline nut. There are ***nut-drivers*** as hand tools, having hollow shafts to allow excess screw threads with (generally) standard-sized hexagonal receptacles which hold the nut for tightening. Common sizes are in Table 19-2 with dimensions shown being ***maximum expected.***

Table 19-2 Common Screw Fastener Nut Sizes
[Dimensions given are expected maximums in inches]

Screw Size	Across Flats	Across Points	Thickness
2	0.188	0.217	0.067
4	0.250	0.289	0.098
6	0.312	0.361	0.114
8	0.349	0.397	0.130
10	0.375	0.433	0.130

Maximum dimensions are given to allow sufficient space, such as on PCB structures.

Flat Washers

Flat washers are good for protecting a surface from abrasion by screw heads or nuts. Exact sizes will depend on the manufacturer.[6] Table 19-3 gives *maximum* outside diameter and *minimum* inside diameter. Thickness is approximate maximum. Flat washers are available in larger outside diameters than those shown here.

Table 19-3 Common Flat Washer Sizes and Holes, Inches
Dimensions are *maximum* for outside dia., *minimum* for inside dia.

Screw Size	Outside Diameter	Inside Diameter	Thickness
2	0.220	0.095	0.018
4	0.280	0.115	0.026
6	0.375	0.141	0.032
8	0.375	0.168	0.032
10	0.438	0.195	0.032

Lock Washers

Lock washers are intended to increase friction between threading and tapping, to hold parts together under higher shock and vibration. Only ***center teeth*** types are recommended. That cuts down on the amount of fastening hardware in the home shop. Table 19-4 gives *maximum* outside diameter and *minimum* inside diameter. Thickness is approximate maximum, given when internal teeth are fully closed.

[6] Fastener hardware manufacturing is highly automated since prior to WWII times. One operator can tend several dozen small machines endlessly cutting the parts. Machines are set to certain in-house standards which follow the ASME guidelines but there are slight differences in dimensioning between manufacturers.

Table 19-4 Common Lock Washer Sizes and Holes, Inches
Dimensions are *maximum* for outside dia., *minimum* for inside dia.

Screw Size	Outside Diameter	Inside Diameter	Thickness
2	0.175	0.095	0.015
4	0.270	0.115	0.019
6	0.295	0.141	0.021
8	0.349	0.168	0.023
10	0.381	0.195	0.026

Note that lock washers have slightly less outside diameters than flat washers. This allows all teeth to embrace the mounting edges for a good tension. Note also that this applies also to ***external teeth*** lock washers.

As an alternative to lock washers, a dab of lacquer can be applied to small fasteners. This was sometimes done in WWII military electronics manufacturing. It works but is a sort of *last resort.* That leaves a paint residue on surfaces, may even take off part of the surface paint on removal. It is not the best way to withstanding extremes of shock and vibration.

A Typical Joining

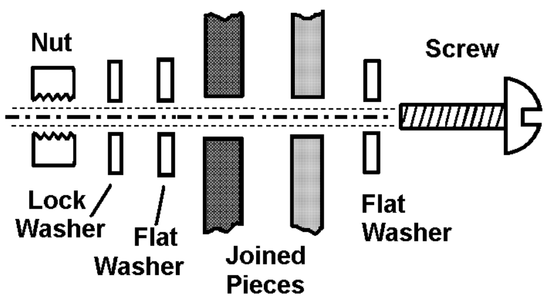

Figure 19-3 Exploded view of a typical joining of two pieces (shaded).

A typical joining is shown in the exploded view of Figure 19-3.[7] The flat washers both mask any off-marking holes and provide a better flat surface for screw heads and nuts, plus making the joined pieces stay in better alignment.

Some mechanical engineers insist on using two lock washers, but that seems a bit over-kill in technique. The main purpose of a lock washer is to provide a resistance to joining, center springs trying to push parts apart. Corners on the lock washer have been designed to both push outward and to dig into metal of nuts.

The dash-dot line marking denotes a ***centerline*** through which all parts align.

Rivets

These are in a category all their own. In general, a ***rivet*** is solely for making something. It must be ***drilled out*** to disassemble. It is not included in here.

Some Oddball Fasteners

Wing nuts have no flats, instead have two *ears* for ease of unscrewing them with fingers. They are sometimes called ***thumb nuts.***

Dzus fasteners are quarter-turn close and open fasteners, usually for often-removed covers

[7] ***Exploded views*** describe several parts which are normally together. It is as if they were all ***exploded***, then caught just after the explosion. The technique is over a half-century old but has lapsed after the drawing take-over by PCs in the mid-1980s.

on aircraft and other vehicle skins. Seldom used in electronics work but they may cover electronics access through to an interior vehicle body. It is pronounced ***Zoose*** and was born in the 1930s. These are made with all parts captive and take the same effort to disconnect as to connect.

Torx screws are similar to Allen heads except they use splines. Found mostly in some consumer electronics structures manufactured off-shore since about 1980. Most large screwdriver sets using interchangeable bits now include Torx heads.

Spline heads are similar to *Torx,* are found in older set screws. Rather than the six flats of an Allen, these have six splines.

Spring nuts are quite cheap, being made from a single piece of springy steel. They can take many kinds of threads or the self-tapping kind. Not for any kind of true precision construction.

Other Hardware

Spacers and Stand-Offs

Depending on the distributor, these are available in Hexagonal or Round exterior shapes, in length sizes to about 1 ½ inches maximum, through-hole or tapped for size 4 or 6 screws, usually made of aluminum or plated brass. Note that ***standoffs*** used to have a terminal on one end for component soldering; wording has slipped so that is also now applied to ***spaceers.*** These are good for stacking small PCB structures. Dimensions are generally on multiples of 0.250 inches in length. For dimensions one needs a listing from one to three vendors dealing with these. Only the very big distributors have large stocks.

Headers (of the Newer Kind)

These are in-line strips of contacts connected by a plastic dielectric.[8] They have mating contacts which are usually for direct installation on PCBs. Spacings tend to be 0.100 or 0.156 inch and some male strips are made to be snapped off to make smaller header strips. Mounting is generally by soldering at the pads of the PCB containing the header. A good friction lock but not good for sideways motion stability.

Solder Lugs

Good for aluminum chassis, but non-metal chassis limits them. Only a few kinds needed for latter construction. Old tube sockets sometimes have intrinsic solder lugs to a metal chassis.

Bar Stock

Square aluminum bar stock of 1/4-inch to 3/8-inch is handy for mounting things. These can be drilled and tapped (or just drilled for clearance holes) to support edge connectors or small PCB structures. Indeterminate supply sources, usually metal yards or surplus houses.

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[8] ***Header*** once meant a connection to a stand-alone device. Now it refers to a line of contacts in a straight line. They can be used to sandwich together PCB structures but they need a mechanical support as well.

Brackets (of Many Kinds)

Save all old brackets and small assemblies that have them. One never knows in advance when some old right-angle brackets can be used decades later.

Knobs and Panel Bearings

At one time a *standard* developed for front panel controls, namely that knob shafts were a quarter-inch in diameter. ***Panel bearings*** were sold to lead such shafts through brackets and front panels, generally through 3/8-inch mounting hole sizes. Such panel bearings were simple, no rollers, just a sleeve to hold long shafts in alignment. Knob prices have grown large so it is a good idea to save all set-screw-mounting knobs of favorite shapes.

Do-It-Yourself Printed Circuit Boards

Rather than explain fine details of *how to make a PCB looking very **commercial***, the following shows a quick way to make *one-of-a-kind PCBs* which are not fancy and limited to two-sided-foil PCBs. Remember that *electrons and circuits don't care about appearances.* Appearance of a finished product is solely for the maker.

Layout On Paper

This is where *quadrille paper* becomes versatile. The author would suggest a ***first*** layout in a twice-normal-size scale. Be *loose* in positioning. This will not be a photographic plan of any kind. Remember that you will ***hand-paint*** resist on the blank foil. Once the layout is reasonably satisfactory, re-layout the PCB in ***full scale.***

Cleaning Up OLD PCB Stock

This needs only a lemon (worst-case $1 each from the supermarket) and some salt. Any cheap salt. Cut the lemon in two. Lay the cut side on the foil and do a few swipes. Sprinkle on the salt. Re-rub ***vigorously*** with the lemon. After a few vigorous rubs, the surface oxidation will go away, leaving the foil bright and shiny. Wash the whole stock with soap and water, whether or not any cleaning was done. This works extremely well, believe it or not! Note: ***No sandpapering*** of the foil.

First Step, Drilling the Through-Holes

Lay the *full-scale* layout on the PCB stock, fasten with masking tape. Make sure to identify which side on the PCB stock if this is a two-sided-foil type. Using a small drill, drill through the layout paper at the through-holes, through the PCB stock. This aligns the back-side foil through holes with the front-side for two-sided-foil material.

A good general substrate is epoxy-glass with a light green color, generally called ***FR4.*** Being a fiberglass mat, it is tough on cutting tools and drill bits. Fiberglass tends to be tougher on drills than aluminum, so there must be an adequate supply of spares of often-used drill sizes.

Through clearance diameters should be 0.003 to 0.006 inches wider than measured component and wire diameters. Remove the full-scale layout paper but keep it.

Painting On Resist

A small brush is needed. The resist material should be diluted with solvent matching the resist paint. For a lacquer use acetone. For a nitrate-base paint, use what is appropriate. Model hobby stores have both although lacquers are rather scarce. Don't dilute too much. Dilute resist should be runny but some should still hang on the brush..

Copy the paper layout paths for conductors with the brush, using the dilute resist paint. This won't be perfect. Electrons don't care about *perfection.* It doesn't matter whether you start with the reverse side or the beginning side. Through-hole positioning is the ***same*** for each side.

A magnifier may be needed. A binocular magnifier is suggested, available at Arts and Crafts stores such as *Michael's.* Use plenty of light. Resist paint color should not clash with clean foil copper or the substrate. The author uses black model aircraft paint commonly called ***hot fuel proof lacquer...***although it is not strictly a lacquer but has a nitrate base.

Checking Over the Finished Resist

Carefully check the finished resist painting with the full-scale layout. If a couple traces got too close together, use a sharp knife blade along the conductor path to strip away traces which are too close. Do not press too much with the knife blade; you are not going to cut through the foil or the substrate. Note that this can be repeated as needed, even to the same location. Once the resist painting is considered correct, it is ready for etching.

Etching

Ferric chloride is a good etchant. It is available ready-to-go in various sizes up to a gallon or in dry form in a plastic bag. For the latter follow instructions to make it liquid.

The ready-for-etching PCB should have mounting holes at the corner. Those holes can take ***paint plugs*** (rubber, conical shape) to act as feet. The PCB should lay in the etching tray (any kind, plastic being best), PCB bottom surface elevated from tray bottom by the *paint plug* feet.

Time of etching will vary depending on the age of the etchant and any heat applied to the etching tray. For the first few PCB etchings it is best to watch and observe the progress. When all of the copper color has disappeared, the etching is complete. That is also a good reason for choosing a neutral black resist paint. You should see the substrate color on ***both sides*** with no copper foil color visible.

Leaving a PCB in etching too long will begin to *under-cut* the edges. Once the copper foil is gone, it is almost impossible to replace.

Heating the etching tray is a handy way to hurry up the etching process. The author uses a rather old photographic print dryer with some equally-old photographic trays for etching laid on top of it. This allows using old ferric chloride etchant in the half-gallon size for a few years.

Ferric chloride etchant is rather opaque for observation through it. There are other etchants, mostly in dry form which allows greater observation of work being etched.

Finishing Work

When etching is done, clean the PCB with soap and water. Thoroughly. Next process is to remove the resist paint. The first step can be lifting the resist lines with a sharp knife. After soaking in the etchant, most paint resists will thicken slightly, a result of absorbing water in the etchant. That also allows dried paint resist to *lift* off the un-etched copper foil traces. Not all paint resist lines will come off.

Final step is to wash the PCB in the correct solvent to dissolve the paint resist. Another tray can be used for that, no paint-plug feet needed. Another container is handy for used solvent. Given at least a 40:1 volume difference of solvent to resist paint, used solvent will not be too disturbed.

Once all that is done, there is bound to be some resist paint left *inside* through-holes. Most of that can be removed by hand, twirling a drill bit with the fingers while moving it in and out of holes. Any resist paint still remaining will char and move out during component soldering.

Storage

If the finished PCB will not be used for a few weeks, store it in a flat, ***closable*** plastic bag. Such bags are available in food stores in many different sizes. The point of this storage is to reduce the amount of foil surface oxidation as much as possible.

Commercial PC houses will add plating plus solder-mask covers, all of which represents a lot of extra work for a hobbyist. That doesn't need to be done. About the only thing to add is to run a fine bit of solder along traces. Just a slight amount. While foil adhesive has a high temperature, too much solder coating can cause dimensional warping of a PCB structure.

Absolute Final Foil Protection After Building

Once a PCB structure has had all its parts mounted and circuit checked out, there can be one added step to keep the foil traces alive and well. A light coat of varnish can be applied. Lacking that, a commercial solder-resist coating (from PC chemical suppliers) can be applied. The latter is expensive and usually doesn't help more than plain varnish coating. Remember, only a **light** coat of varnish. This may offer some protection against growth of ***tin whiskers*** over the years.

Alternate Resist Materials

Many nail polishes are lacquers. A sniff test can confirm that in a store. Brushes supplied with nail polishes are too coarse to use, too wide. A ***Sharpie*** pen (Sanford Company) found in many different stores can be used directly. The author has tried this but found the felt-tip pen point to be a bit coarse for normal work. One has to be careful with ***Sharpie*** resist applications. The amount of marking material applied is quite thin compared to actual paint resists.

Most ***sanding sealer*** paints are lacquers, can be removed with acetone. Their only problem is that they are clear or translucent after drying. It is hard to see a painted trace resist relative to an unpainted PCB. Some hardware stores carry ***dye*** (lacquer base) which can be mixed-in. Black dye is always good, contrasts well with any substrate or bright copper foil.

Warnings on Ventilation

All solvents are supposedly harmful to breathe. You have been warned. By experience, the author has found that normal residence airflow, while small, is usually good enough to work in, even bathrooms. The alternative is to wear masks or even face-plates, all of which distract from the work at hand.

Workshop Tools

The following are some suggestions for minimal (and optional) tools for use in the home shop. It is not imperative, just a handy list.

Minimal

Pliers, smallish, long-nose.
Diagonal wire cutters (sometimes called *dikes),* smallish for PCB work.
Wire strippers, the kind with half-circle guides for wire diameters.
Screwdrivers for slot and Phillips head screws.
Nut driver kits with interchangeable nut sizes.
Allen wrenches in kit form, either a collection or all on one holder.
Small-tip soldering iron; two for also handling ***ROHS*** lead-free solder.
Generous supply of copper wire wicking material with flux (*Solder-Wick).*
Spring-loaded, triggerable solder-sucking tool (*Solda-Pult).*
Small right-angle wrenches (sometimes called *Crescent* wrenches).
Metal-working files, smallish, for deburring and enlarging metal holes.
Electric hand drill, minimum chuck size 1/4 inch.
Ruler, minimum length 6 inches, steel type preferred.
Tool box to hold all tools.
Trays for PCB etching.
Solder, your choice for type, flux centers, at least one pound on roll.

Optional

Rotary multiple-use tool such as a *Dremel* set.[9]
Small drill-press holder for the *Dremel* to act as a PCB drill.
Cut-off wheels (for *Dremel)* to cut bar and right-angle stock, shafts.
Bench-mount drill press taking 3/8-inch bits
Bench vise used with small drill press.
Hacksaw with various blades for metal and plastic bar stock cutting.
Micrometer for measuring drill and wire diameter, precision parts.
Drill bit sets, one for small PCB holes, one for larger holes.
Tap and Die set for making special screws and threaded holes.
Larger diagonal wire-cutter for large wire cutting and metal snipping.
Long steel-tape measure in retractable holder.
T-handle Reamer for enlarging holes greater than a quarter-inch diameter.
At least 2 small C-clamps, capacity about 2 inches.
Container cabinets (with label space) for storing parts.
Hands-free head-worn magnifier for working with tiny parts.

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[9] Both ***Loews*** and ***Home Depot*** stock ***Dremel*** tools with many replacement bits and attachments.

Vendors

Besides the do-it-yourself stores, some arts and crafts stores carry off-shore made tools and aids for working, such as ***Michaels***. While these are low-cost, it should be noted that quality is not up to usual hand tool standard and material for pliers and cutters may be made of mild steel rather than stronger steel. This is especially pointed out for diagonal wire cutters.

Making Small Boxes From PCB Stock

Printed circuit board material is good for making small boxes, either single-sided or double-sided stock. The adhesive holding foil to the substrate can tolerate considerable heating so soldering provides a good bond plus it creates a continuous ground plane. Figure 19-4 shows the four steps. The first step is to make an ***Assembly Jig*** out of scrap plywood. Three-quarter inch thick ply is good for this. Plan the box to have at least two holes on each side. Those holes are for small wood screws to secure the box pieces to the ***Jig.***

The ***Assembly Jig*** **MUST** be made with three sides mutually perpendicular. If no square tool is available, use a piece of good bond paper glued onto its cardboard backing. Jig pieces should be machine-sawed to insure straight lines. Glue and screw the Jig pieces together. It is good for many small boxes.

One begins assembly with the (bottom) piece and two sides, as in Step B of Figure 19-4. The box pieces are secured to Jig sides and bottom using the holes just mentioned. Mechanical holding will keep the box pieces aligned through the soldering.

Using a large soldering iron, at least for the tip, run a bead of solder along each edge to be joined as in Step B. This does not have to be a large bead. The foil helps to conduct heat away somewhat. In the sketch, spacers are used at each corner. That is optional. Do a solder bead for the

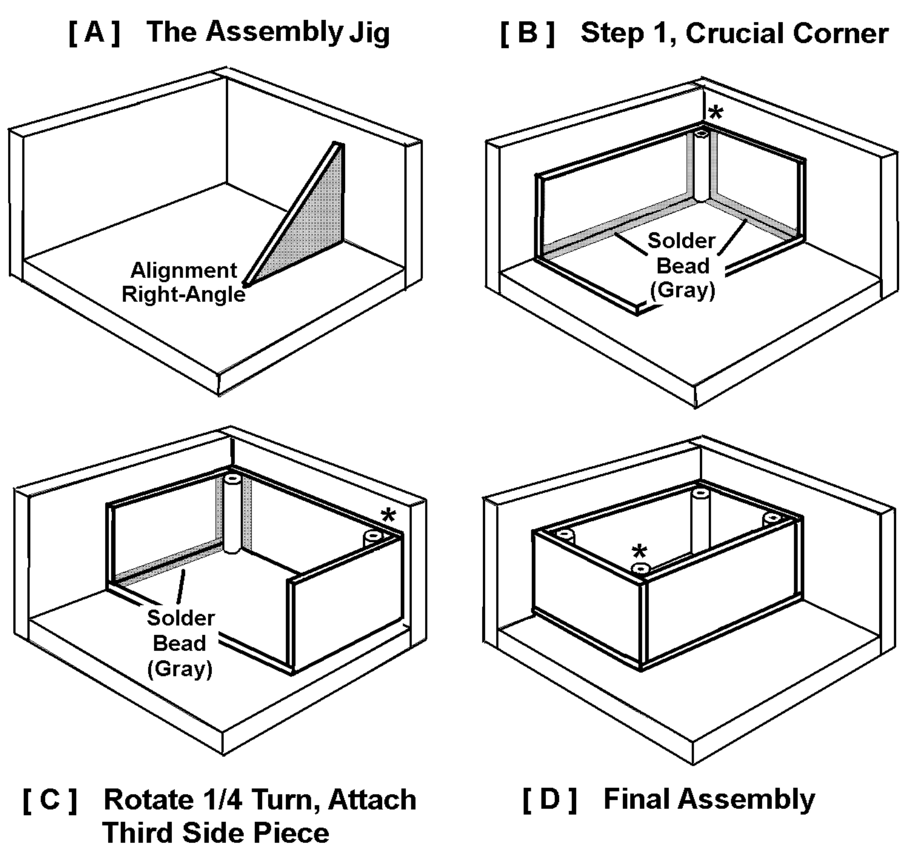

Figure 19-4 Steps in making a small enclosure from PCB material. The asterisk denotes the corner done in Step B. Solder bead is kept small.

edge normally covered by a spacer. Cheaper brass spacers can be used there. Hold spacers in place by a long screw and do a few solder beads of spacers to corners. Exact alignment isn't necessary here since holes for screws going into spacers can be obtained later.

Disconnect the PCB box pieces and rotate it a quarter turn. Secure the box pieces to the Jig again and include one more side piece. Run another solder bead along joining edges as in Step C. If spacers are used here, repeat their attachment for the second spacer.

Repeat the procedure for Step D and the final side piece of the PC Box. This will have the least opening for soldering iron insertion so plan that into construction. Make the final solder beads so that all sides and the (bottom) piece of the PC Box are fully joined.

Largest box size seen was 8 inches wide and deep, 2 inches high.

Using Double-Sided Foil PCB Stock

This works well with single-sided foil stock. For double-sided foil, there may be a lack of continuity for the second foil to the first foil. A way out is to use the mechanical-joining holes with a rivet or a short screw with a lock-washer on each end. A way out, less-satisfactory, is to drill small holes and connect the two foil sides with a short solid-wire piece, soldering each end to each foil.

Cutting the PCB Stock

A metal shear is the best. A very heavy paper cutter works next best. The point was to get all edges at exact right-angles. This allows all edges of the final assembly to have the smallest solder beads and line up looking like a box.

Lacking a shear or a heavy paper cutter, the procedure is to scribe marks on the foil, again with exact right-angles. Scribe another line about 1/16-inch away. Peel off the foil to reveal the PCB substrate. Cut into the substrate, preferably with a motorized jig saw. The cut would then be sanded with sanding paper over a large block. While that could be done directly on the total PCB stock, the thin foil tends to curl and/or not come off cleanly. It is better to cut PCB stock directly into the substrate and not touch the foil deliberately.

Wiring

The vast majority of solid-state circuitry is very low current, low-voltage in nature. The majority of hobbyist wiring can be done with #26 AWG solid wire with white ***Teflon insulation.*** That is also the most expensive wire, dropping slightly in cost by buying in 500-foot rolls. There is no point in using colored wire when an Ohmmeter is handy. Simple masking tape tags can be added to ends when the wiring work must be postponed to another day.

Teflon wire just doesn't melt to a soldering iron, not even a small soldering flame from a gas gun. At worst it chars slightly, something that can be chipped off with diagonal cutters. Cheap wire is almost always available in ***meltable*** insulation, colored or not. It just isn't worth it except for heavy current-carrying wires.

For flexible applications, go first for the larger number of strands per wire, then the wire size for current-carrying capacity. For highest-flexibility, use the *flat type strand* interwoven with fabric. The latter is also the most difficult to solder due to the fabric. A small gas gun can be of help there, able to burn away the fabric; use caution there to avoid damaging wire surface.

Table 19-5 Common Wire Characteristics
Dimensions are in inches.

Wire, AWG	Diameter	Ohms per 1000 Feet	Current, Amps
12	0.081	1.588	41
14	0.064	2.529	32
16	0.051	4.016	22
18	0.040	6.385	16
20	0.032	10.15	11
22	0.025	16.14	7
24	0.020	25.67	3.6
26	0.016	40.81	2.2
28	0.013	64.90	1.4
30	0.010	103.2	0.86

1. Interpolate for sizes in-between
2. Diameter is of wire only, excludes jacket.
3. Ohms per 1000 feet is at 25° C.
4. Current capacity is maximum for laying on a chassis; de-rate to half that for bundles up to 15 wires.

Making Your Own Resistors Under 10 Ohms

Small resistors less than 1.0 Ohm and to tolerances of (roughly) ± 5 % can be made by fine wire. Select a wire size and divide Ohms-Per-1000-Feet column of Table 19-5 by the resistance. Divide that by 1000 to get the length of wire. Maximum current capacity is about the same as the Table value. Wind wire on a MegOhm resistor body; parallel resistance will not be measurable

Appendix 19-1

Drill Sizes, Inches

Size	Diameter
80	0.014
79	0.015
78	0.016
77	0.018
76	0.020
75	0.021
74	0.023
73	0.024
72	0.025
71	0.026
70	0.028
69	0.029
68	0.031
	0.0313 = 1/32
67	0.032
66	0.033
65	0.035
64	0.036
63	0.037
62	0.038
61	0.039
60	0.040
59	0.041
58	0.042
57	0.043
56	0.046
55	0.052
54	0.055
53	0.059
	0.0625 = 1/16
52	0.065
51	0.067
50	0.070
49	0.073
48	0.076
47	0.078
46	0.081

Size	Diameter
45	0.082
44	0.086
43	0.089
42	0.093
	0.0935 = 3/32
40	0.096
39	0.099
38	0.102
37	0.104
36	0.106
35	0.110
34	0.111
33	0.113
32	0.116
31	0.120
	0.1250 = 1/8
30	0.129
29	0.136
28	0.141
27	0.144
26	0.147
25	0.150
24	0.152
23	0.154
22	0.157
21	0.159
20	0.161
19	0.168
18	0.169
17	0.173
16	0.177
15	0.180
14	0.182
13	0.185
	0.1875 = 3/16
12	0.189
11	0.191

Size	Diameter
10	0.194
9	0.196
8	0.199
7	0.201
6	0.204
5	0.206
4	0.209
3	0.213
	0.2188 = 7/32
2	0.221
1	0.223
A	0.235
B	0.238
C	0.242
D	0.248
E	0.250 = 1/4
F	0.257
G	0.261
H	0.266
I	0.272
J	0.277
K	0.281
	0.2813 = 9/32
L	0.290
M	0.295
N	0.302
O	0.316
P	0.323
Q	0.332
R	0.339
	0.3438 = 11/32
S	0.348
T	0.358
U	0.368
	0.3750 = 3/8
V	0.377
W	0.386

1. **Drill sizes rounded to three decimal fractions.**
2. **Not for precision manufacturing.**